From Independence of Clones to Composition Consistency: A Hierarchy of Barriers to Strategic Nomination

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Outline

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Overview

Similar alternatives (ideological, party affiliation) can hurt each others' chance of winning an election.

Charles Martin	Peter Zimmerman	Joe Dunne
116,677	95,519	86,923

Figure 1: 1934 Oregon governor election

Desirable axioms:

- Independence of Clones (IoC)
- Composition Consistency (CC) (stronger)

Clone sets

Definition 1 (Tideman 1987, §I; Laffond et al. 1996, Def. 4).
Given a preference profile $\boldsymbol{\sigma}$ over candidates A, a nonempty subset of candidates $K \subseteq A$ is a set of clones with respect to $\boldsymbol{\sigma}$ if for each $a, b \in K$ and each $c \in A \setminus K$, no voter ranks c between a and b.

Informally: clone sets are sets of alternatives that appear next to each other in all ballots in a profile.

IoC

We want to ensure that adding or removing candidates will not influence whether a candidate from inside or outside the clone set wins an election.

Definition 2 (Zavist and Tideman 1989). An SCF f is independent of clones (IoC) if for each profile $\boldsymbol{\sigma}$ over A and each non-trivial clone set $K \subset A$ with respect to $\boldsymbol{\sigma}$,

(1) for all $a \in K$,

$$K \cap f(\boldsymbol{\sigma}) = \emptyset \Leftrightarrow (K \setminus \{a\}) \cap f(\boldsymbol{\sigma} \setminus \{a\}) = \emptyset$$

(2) for all $a \in K$ and all $b \in A \setminus K$

$$b \in f(\boldsymbol{\sigma}) \Leftrightarrow b \in f(\boldsymbol{\sigma} \setminus \{a\})$$

IoC consequences

- Plurality and Ranked Pairs fail IoC.
    - **Ranked Pairs**: Given a profile $\boldsymbol{\sigma}$ over candidates $A = \{a_i\}_{i \in [m]}$, construct the majority matrix M, whose ij entry is the number of voters who rank a_i ahead of a_j minus those who rank a_j ahead of a_i. Construct a digraph over A by adding edges for each $M[ij] \geq 0$ in non-increasing order, skipping those that result in a cycle. The winner is the source node.
- STV satisfies IoC.

Example

3 voters	2 voters	4 voters	3 voters
a_1	a_2	b	c
a_2	a_1	c	a_2
b	b	a_2	a_1
c	c	a_1	b

$\Longrightarrow$ remove a_2

5 voters	4 voters	3 voters
a_1	b	c
b	c	a_1
c	a_1	b

Figure 2: (Left) Example profile $\boldsymbol{\sigma}$. (Right) $\boldsymbol{\sigma} \setminus \{a_2\}$.

Clone decomposition

Definition 3. Given a preference profile $\boldsymbol{\sigma}$ over candidates A, a set of sets $\mathcal{K} = \{K_1, K_2, ..., K_\ell\}$, where $K_i \subseteq A$ for all $i \in [\ell]$, is a (clone) decomposition with respect to $\boldsymbol{\sigma}$ if

1. $\mathcal{K}$ is a partition of A into pairwise disjoint subsets, and
2. Each K_i is a non-empty clone set with respect to $\boldsymbol{\sigma}$.

Composition product

Definition 4. The composition product function of an SCF f is a function Π_f that takes as input a profile $\boldsymbol{\sigma}$ and a clone decomposition $\mathcal{K}$ with respect to $\boldsymbol{\sigma}$ and outputs $\Pi_f(\boldsymbol{\sigma}, \mathcal{K}) \equiv \bigcup_{\mathcal{K} \in f(\boldsymbol{\sigma}^{\mathcal{K}})} f(\boldsymbol{\sigma}|_K)$

Informally: first run SCF on a clone decomposition for a profile, then on all winning clone sets separately; output union of all winners.

CC

We want our SCF to always select the best candidate from winning clone sets, which IoC does not guarantee.

Definition 5. (Laffond et al. 1996, Def. 11). A neutral SCF f is composition-consistent (CC) if for all preference profiles $\boldsymbol{\sigma}$ and all clone decompositions $\mathcal{K}$ with respect to $\boldsymbol{\sigma}$, we have $f(\boldsymbol{\sigma}) = \Pi_f(\boldsymbol{\sigma}, \mathcal{K})$.

Consequences of CC

- Plurality, STV, Ranked Pairs all fail CC.
- Ranked Pairs with tie-breaking in favor of some $i \in N$ satisfies CC.
  - Construct a tie-breaking order Σ_i over unordered pairs in A.

NOTE Not anonymous.

- CC implies IoC.

CC implies IoC

- Consider clone decompositions $\mathcal{K} = \{K\} \cup \{\{b\}\}_{b \in A \setminus K}$ for $\boldsymbol{\sigma}$ and $\mathcal{K}' = \{K \setminus \{a\}\} \cup \{\{b\}\}_{b \in A \setminus K}$ for $\boldsymbol{\sigma} \setminus \{a\}$.
- All winning clone sets still win under $f(\boldsymbol{\sigma}^{\mathcal{K}})$ after removing a.
- If K intersects with the winners of $\Pi_f(\boldsymbol{\sigma}, \mathcal{K})$, $K \setminus \{a\}$ still intersects $\Pi_f(\boldsymbol{\sigma} \setminus \{a\}, \mathcal{K}')$, by CC; otherwise, all winners outside the clone set remain winners.
- By CC, $\Pi_f(\boldsymbol{\sigma}, \mathcal{K}) = f(\boldsymbol{\sigma})$ and $\Pi_f(\boldsymbol{\sigma} \setminus \{a\}, \mathcal{K}') = f(\boldsymbol{\sigma} \setminus \{a\})$
- We can now derive the two conditions for IoC.

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Outline

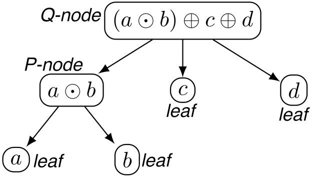
CC Transformation

- any SCF f can be transformed to *CC-transform* SCF f^{CC}
- f^{CC} satisfies CC, *and* satisfies all axioms* satisfied by f
- transformation takes polynomial-time
- **Input**: SCF f, preference profile $\boldsymbol{\sigma}$ over candidates A
- **Output**: winner candidates $W \subseteq A$, determined by f^{CC}

1. Construct a PQ-tree representation of $\boldsymbol{\sigma}$ that groups clones together as children of the same node
2. Recursively apply f to T, for each clone set based on the preference order among them

Construct PQ-tree

- elements of $A \Rightarrow$ leaves of T
- $\mathcal{C}(\boldsymbol{\sigma})$ has only trivial clones $\Rightarrow$ children of P-node $\odot$
- $\boldsymbol{\sigma}$ rankings in linear order or reversal $\Rightarrow$ children of Q-node $\oplus$
- e.g. for profile $\boldsymbol{\sigma}$ of $a \succ b \succ c \succ d$ and $d \succ c \succ a \succ b$
  - $\mathcal{C}(\boldsymbol{\sigma}) = \{\{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, A\}$
  - collapse $K_1 = \{a, b\}$ so $\mathcal{K} = \{\{K_1\}, \{c\}, \{d\}\}$
  - $\mathcal{C}(\boldsymbol{\sigma}|\{a, b\}) = \{\{a\}, \{b\}, \{a, b\}\} \Rightarrow P$-node
  - $\boldsymbol{\sigma}^{\mathcal{K}}$ is $K_1 \succ c \succ d$ and $d \succ c \succ K_1 \Rightarrow Q$-node
- if a clone structure isn't P or Q, it can be reduced to one by combining clones
- *Order of collapsing doesn't matter [Elkind et al., 2012]*



Recursively run f on the PQ-tree

- f^{CC} recursively runs f on PQ-tree T of $\boldsymbol{\sigma}$ starting at root
- at each P-node B, run f on children $\boldsymbol{\sigma}^{decomp(B,T)}$
  - apply f to winning children, continue
- at each Q-node B, run f on B's first two children $\boldsymbol{\sigma}^{decomp(B,T)}|\{B_1(B,T), B_2(B,T)\}$
  - if $B_1(B,T)$ wins, apply f to B's first child
  - if $B_2(B,T)$ wins, apply f to B's last child
  - if both win, apply f to all of B's children
- f^{CC} runs f on the summary of each clone set, top-down
- when we apply f to the members of the clone set, we still take into account information about the larger structure that the set is embedded in

Algorithm Results

For any neutral SCF f, f^{CC} satisfies

1. If $\boldsymbol{\sigma}$ has no non-trivial clone sets, $f^{CC}(\boldsymbol{\sigma}) = f(\boldsymbol{\sigma})$
2. f_{CC} is composition-consisted (satisfies CC)
3. If f itself is already composition consistent, then $f^{CC}(\boldsymbol{\sigma}) = f(\boldsymbol{\sigma})$ for any $\boldsymbol{\sigma}$
4. If f satisfies any of {anonymity, Condorcet consistency, Smith consistency, decisiveness (on all $\boldsymbol{\sigma}$), monotonicity^{ca}, independence of Smith-dominated alternatives^{ca}, participation^{ca}}, then f^{CC} satisfies it as well
5. Let $g(n, m)$ be an upper bound on the runtime of an algorithm that computes f, then $f^{CC}(\boldsymbol{\sigma})$ can be computed in $O(nm^3) + m \cdot g(n, \delta(PQ(\boldsymbol{\sigma}))$
6. If f is polytime-computable, then f^{CC} is as well

Relaxation of Axioms

- f^{CC} doesn't actually preserve monotonicity, ISDA, or participation
- Because adding/removing alternatives alters the clone structure of $\boldsymbol{\sigma}$, so its PQ-tree is different
- f^{CC} does preserve *clone-aware* relaxations of the axioms: robustness against changes respecting clone structures
- implicit assumption: clone structures are inherent (e.g. political affiliation), so changes to $\boldsymbol{\sigma}$ wouldn't affect $\mathcal{C}(\boldsymbol{\sigma})$
- e.g. an SCF f satisfies monotonicity^{ca} if $a \in f(\boldsymbol{\sigma}) \Rightarrow a \in f(\boldsymbol{\sigma}')$ whenever
  1. $\mathcal{C}(\boldsymbol{\sigma}) = \mathcal{C}(\boldsymbol{\sigma})$
  2. for all $i \in N$ and $b, c \in A \backslash \{a\}$, we have $a \succ_{\sigma_i} b \Rightarrow a \succ_{\sigma_i'} b$ and $b \succ_{\sigma_i} c \Rightarrow b \succ_{\sigma_i'} c$
- Similar for ISDA^{ca} and participation^{ca}

Outline

Game-theoretic modelling

- We can model elections as games where **candidates** play actions R(un) or D(rop out), with utilities over actions defined based on a distance metric $d_{\boldsymbol{\sigma}}(a, b) = |K| - 1$, where K is the smallest clone set containing a and b.
- The closer the winner is to a candidate in all ballots, the more utility they see in losing against them.
- $d_{\boldsymbol{\sigma}} = 0$ iff $a = b$ (all candidates like themselves the best)
- An SCF is **candidate-stable** if for all profiles, all players running is a Pure Nash Equilibrium.

Obvious dominance

If SCF is IoC, running is a dominant strategy for all candidates.

Still, candidates may drop out of the race due to not knowing whether an SCF is IoC or not in fear of hurting their clones.

Definition 29 (Li 2017, Informal). An action s is *obviously dominant* for player a if for any other action s', starting from the point in the game when a must take an action, best possible outcome from s' is no better than worst possible outcome from s.

Achieving strategyproofness using f^{CC}

- Ask each candidate individually if she intends to run, in order determined by PQ-tree traversal
  - if B is a P-node
    - ask candidate (leaf) children of B to pick between R or D
    - apply f to children other than those who chose D
    - if winner is a candidate (leaf), the game ends
  - if B is a Q-node
    - if $B_1(B, T)$ is a leaf, ask her to pick R or D
    - if she chooses R, game ends and she wins
    - if she chooses B, move on to $B_2(B, T)$... until P/Q node or winner is chosen
- For any $f^{CC}(\boldsymbol{\sigma})$, R is an obviously-dominant strategy for all candidates
- A candidate can decide to run or not after she learns whether her smallest clone set has won, not hurting the other candidates in the set. Best-case-D=worst-case-R=a_2 wins
- Nice result - we can get the same outcome of the election by replacing candidates on a ballot with party names, and only later holding internal primaries, irrespective of the voting rule.

Extensive Form Games

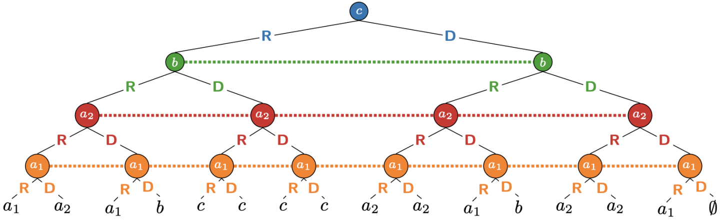


Figure 6: EFG representation of $\Gamma_{\boldsymbol{\sigma}}^{STV}$ for $\boldsymbol{\sigma}$ from Fig. 2. Terminals show the winner under that action profile. Information sets are joined by dotted lines. For a_1, the worst outcome of running is c winning, and the best outcome of dropping out is a_2 winning, so running is not an obviously dominant strategy for a_1.

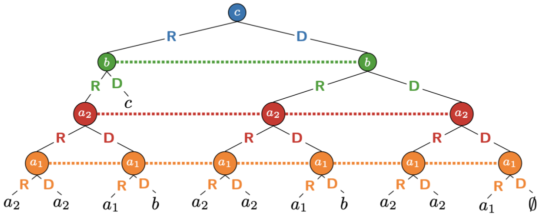


Figure 7: $\Lambda_{\boldsymbol{\sigma}}^{STV^{CC}}$, for $\boldsymbol{\sigma}$ from Fig. 2, the PQ-tree of which is in Fig. 5 (right). For a_1, best outcome of not running is a_2 winning, which is no better than the worst outcome of running, which is also a_2 winning. Therefore, running is an obviously dominant strategy for a_1. A similar analysis applies for all

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Bonus slide: Sausages

4.1 Background: Clone Structures and PQ-Trees

For a profile $\boldsymbol{\sigma}$, Elkind et al. [2012] define the *clone structure* $\mathcal{C}(\boldsymbol{\sigma}) \subseteq \mathcal{P}(A)$ as the family of *all* clone sets with respect to $\boldsymbol{\sigma}$. For example, for $\boldsymbol{\sigma}$ from Fig. 1, $\mathcal{C}(\boldsymbol{\sigma}) = \{\{a\}, \{b\}, \{c\}, \{d\}, \{b, c\}, \{a, b, c, d\}\}$. They identify two types of *irreducible* clone structures: a *maximal* clone structure (also called a *string of sausages*) and a *minimal* clone structure (also called a *fat sausage*). A string of sausages arises when each ranking in $\boldsymbol{\sigma}$ is either a fixed linear order (say, $\sigma_1 : a_1 \succ a_2 \succ \cdots \succ a_m$) or its reversal. In this case, $\mathcal{C}(\boldsymbol{\sigma}) = \{\{a_k\}_{i \leq k \leq j} : i \leq j\}$, *i.e.*, all intervals in σ_1. The *majority ranking* of the string of sausages is σ_1 or its reverse, depending on which one appears more frequently in $\boldsymbol{\sigma}$. A fat sausage occurs when $\mathcal{C}(\boldsymbol{\sigma}) = \{A\} \cup \{\{a_i\}\}_{i \in [m]}$, *i.e.*, the structure only has the trivial clone sets.

(a) Fat sausage (P-node)

(b) String of sausages (Q-node)