Expressive Ballots for Voting Systems and Political Analysis

Théo Delemazure

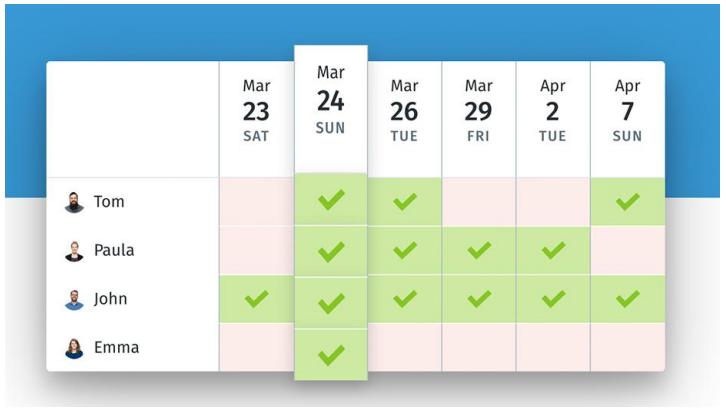
Supervised by *Jérôme Lang* and *Dominik Peters*

LAMSADE, Université Paris Dauphine - PSL

PhD Defense - June 11th, 2025

We vote all the time

Political election



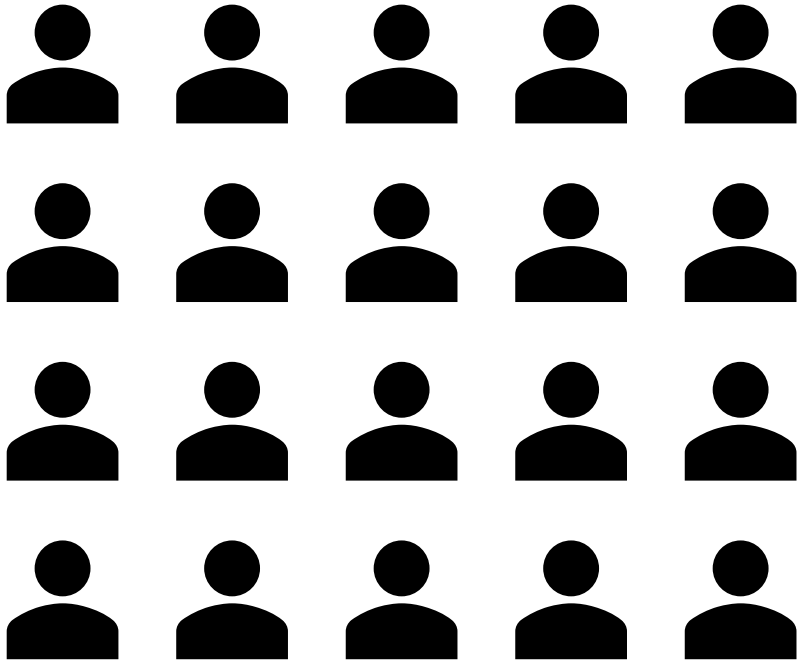
Decide on a date

Jury decision

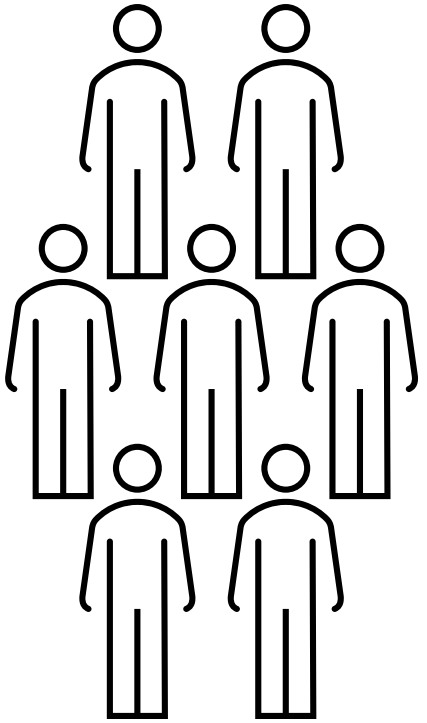
Choosing the pope

Voting systems

Voters

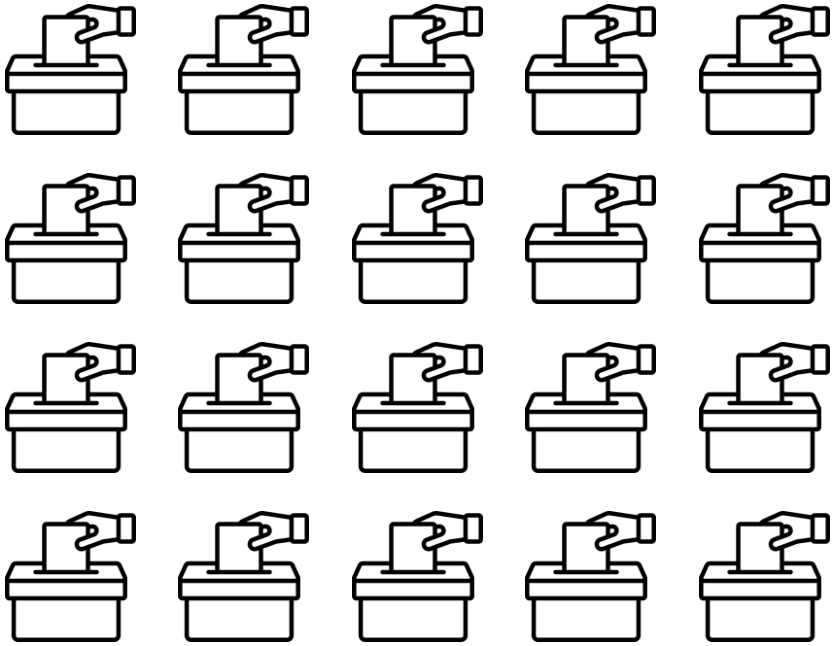


Candidates

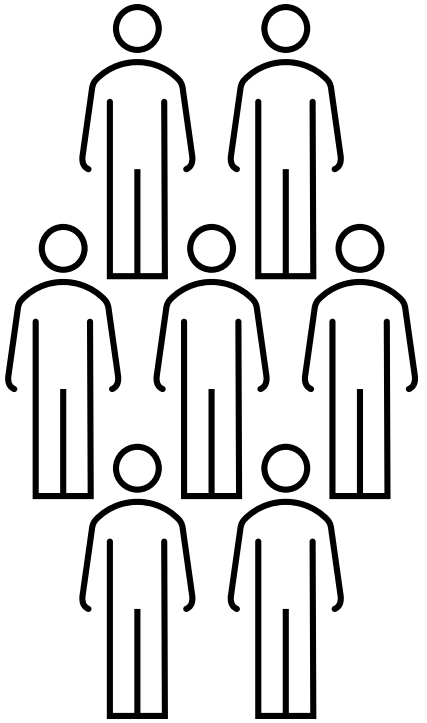


Voting systems

Voters give their preferences over candidates



Candidates



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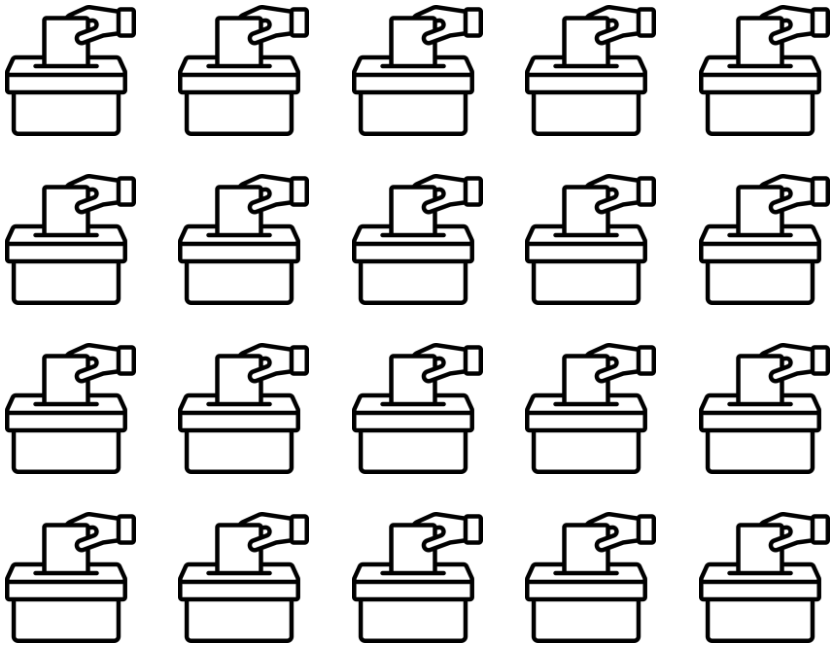
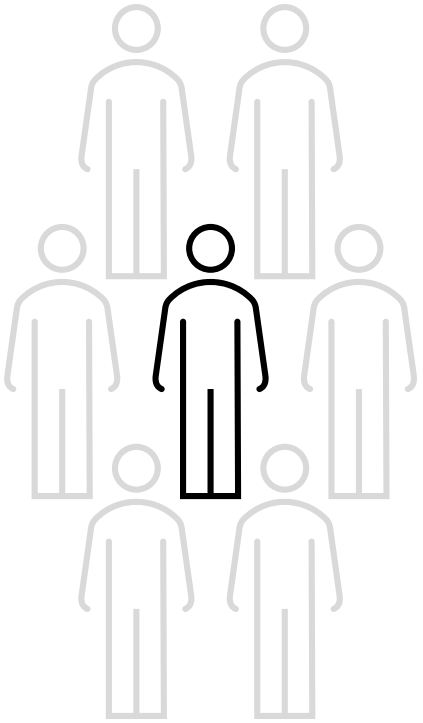
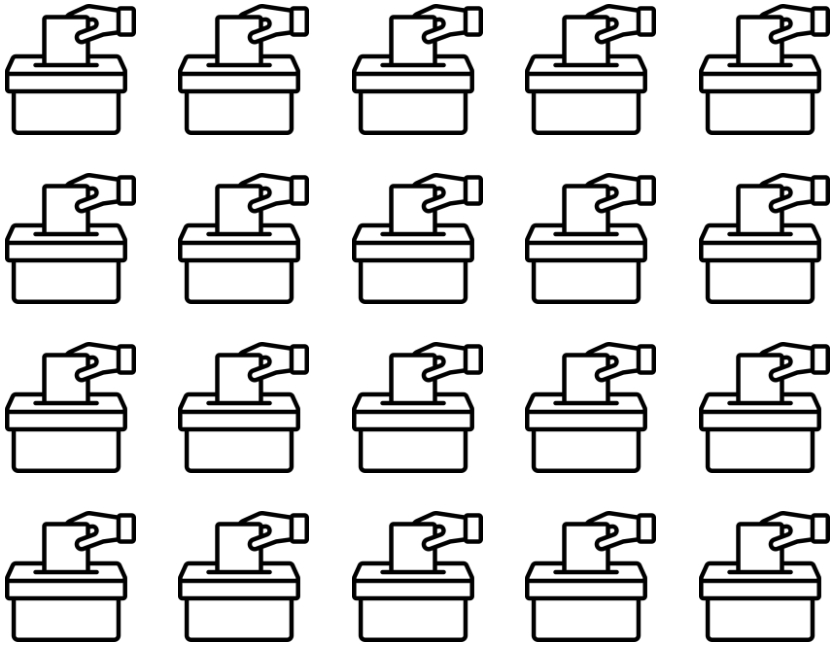


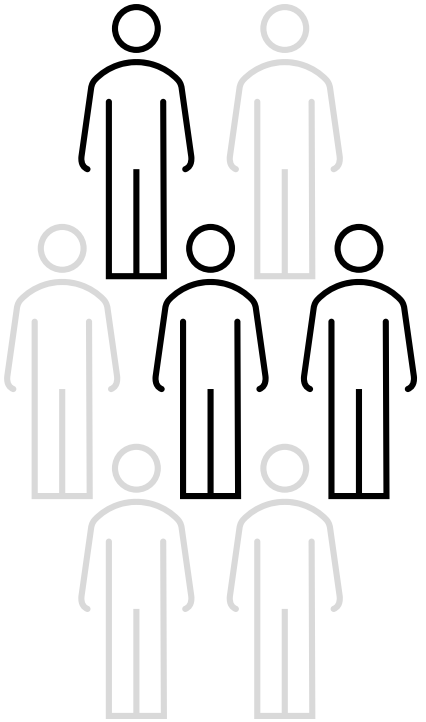
A **winner** is selected



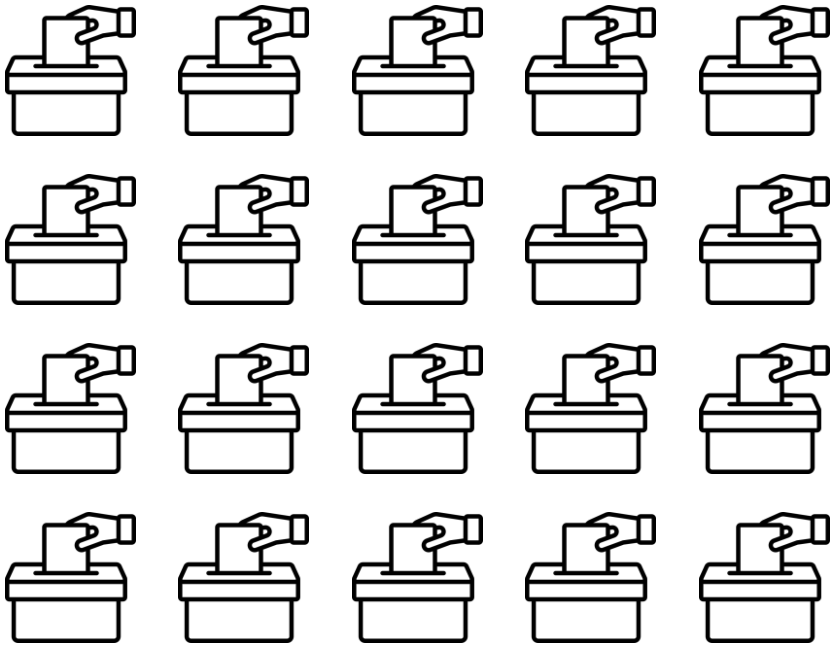
Voters give their preferences over candidates



A **committee** is selected

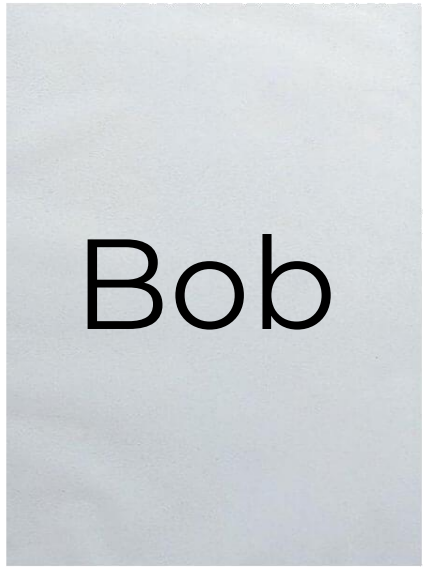


Voters give their preferences over candidates



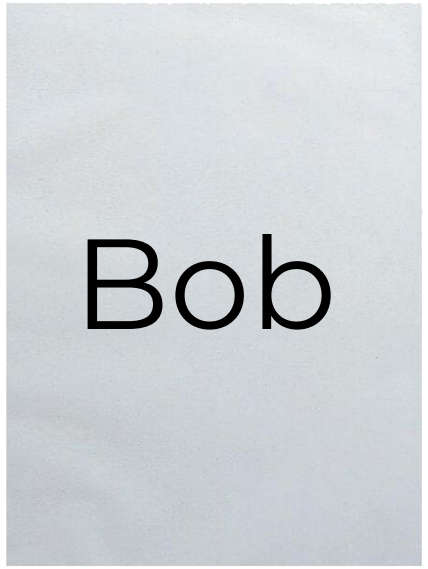
A **parliament** is selected

Voting ballots

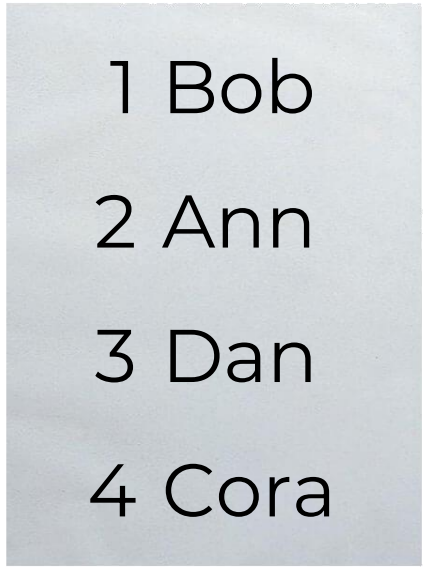


Uninominal Ballots

Voting ballots

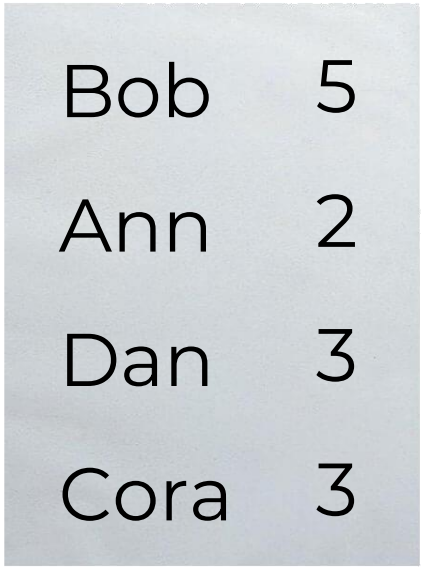


Uninominal Ballots



Rankings

Approval Ballots



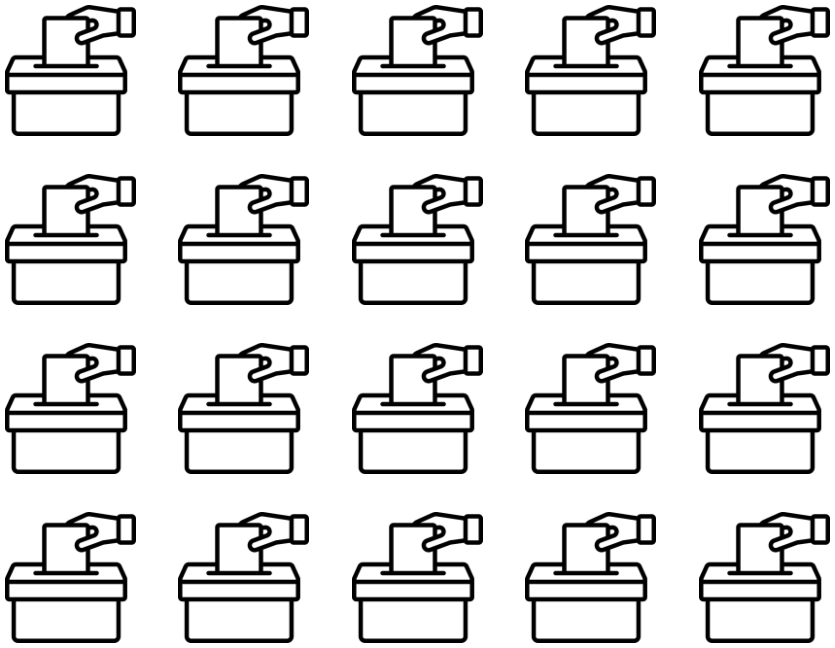
Scores

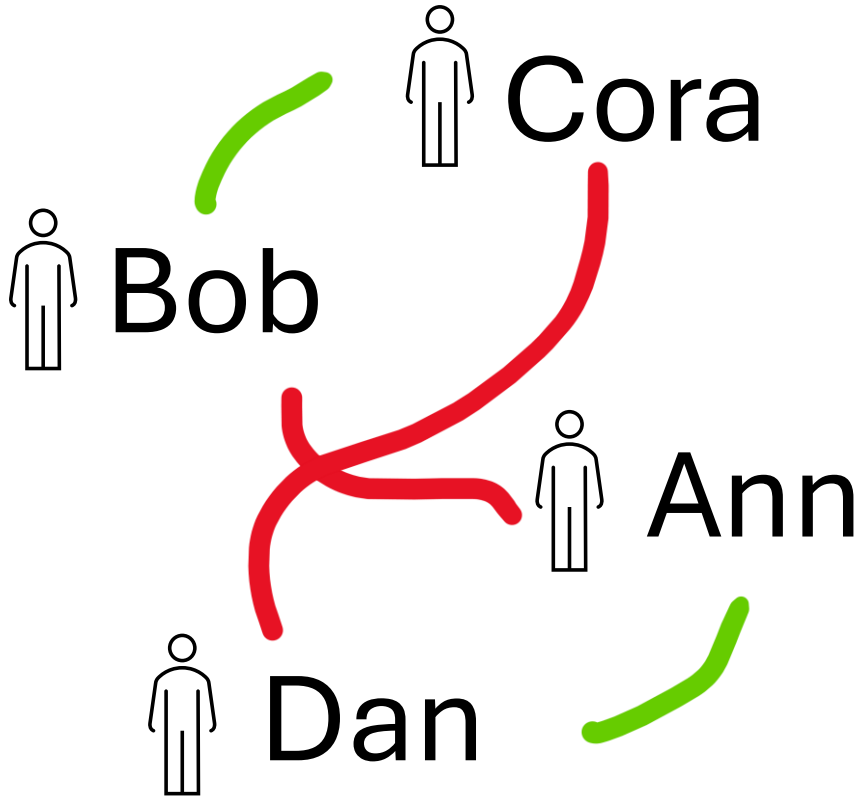
I. Expressive Ballots for Voting Systems

1. Approval with Runoff
2. Instant Runoff Voting with indifferences
3. Rankings in proportional election with thresholds

Deduce structure from preferences

Voters give their preferences over candidates





Outline of the presentation

I. Expressive ballots for voting systems

1. Approval with Runoff
2. Instant Runoff Voting with indifferences
3. Rankings in proportional elections with thresholds

II. Expressive ballots for political analysis

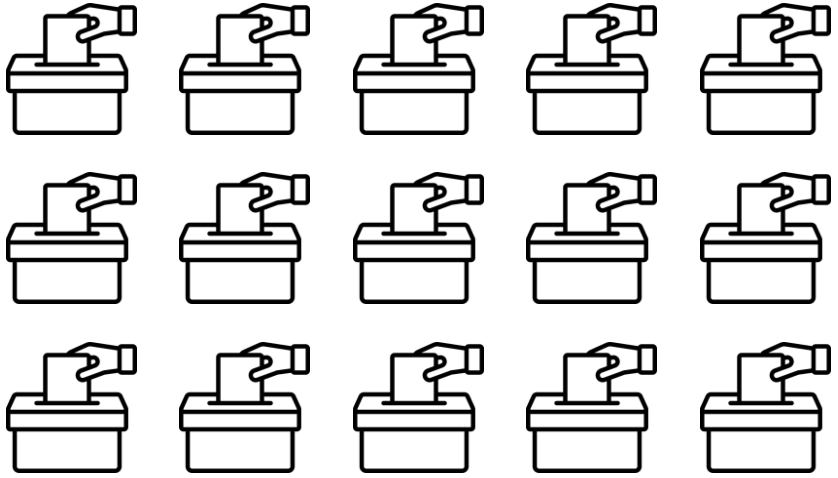
4. Learn candidate axes from approval data
5. Identify conflicting pairs of candidates via rankings

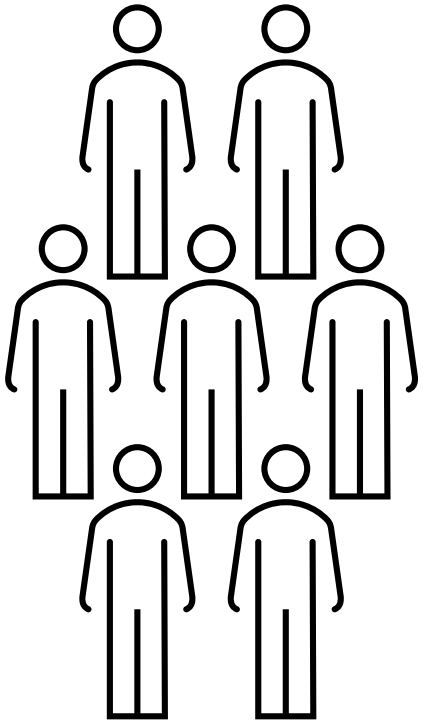
1. Approval with Runoff

Approval with Runoff
Théo Delemazure, Jérôme Lang, Jean-François Laslier, Remzi Sanver
IJCAI-2022

The *Plurality with Runoff* voting system

1st Round: Voters can vote for one candidate among the possible ones.

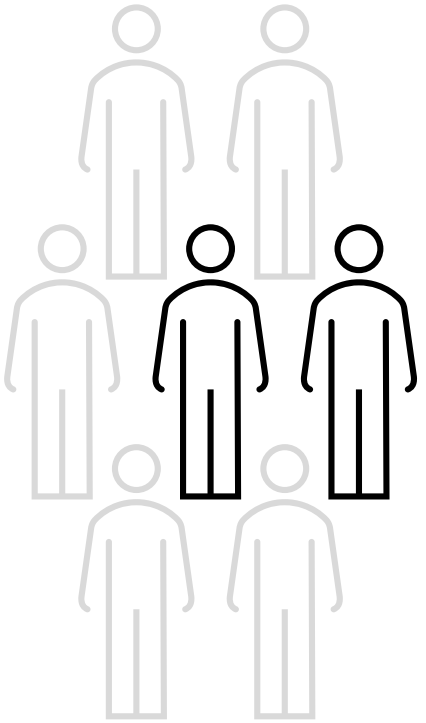




The *Plurality with Runoff* voting system

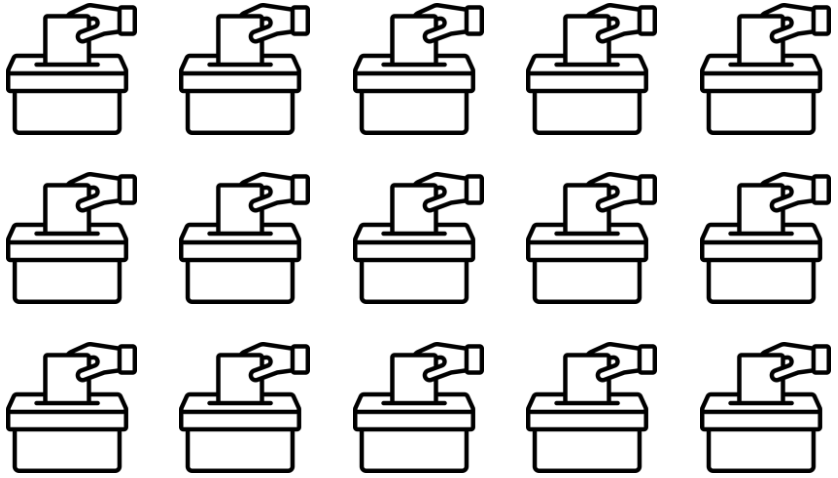
1st Round: Voters can vote for one candidate among the possible ones.

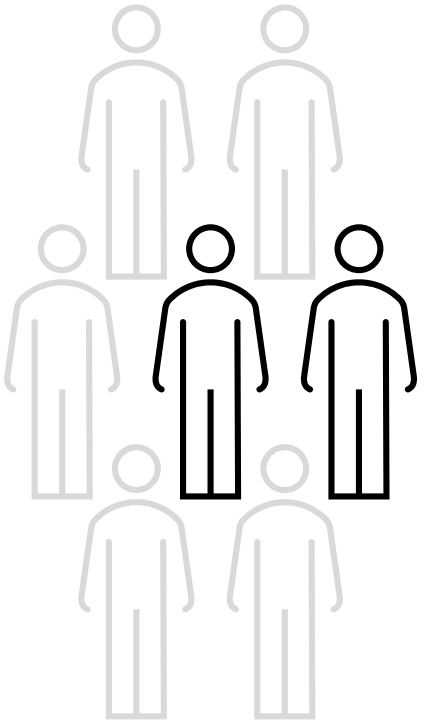
The **two candidates** with the most votes are selected.



The *Plurality with Runoff* voting system

2nd Round: Voters can now only vote for one of the finalists.





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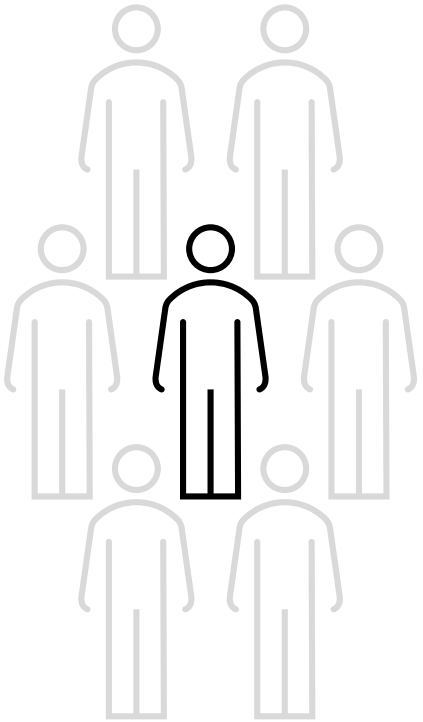
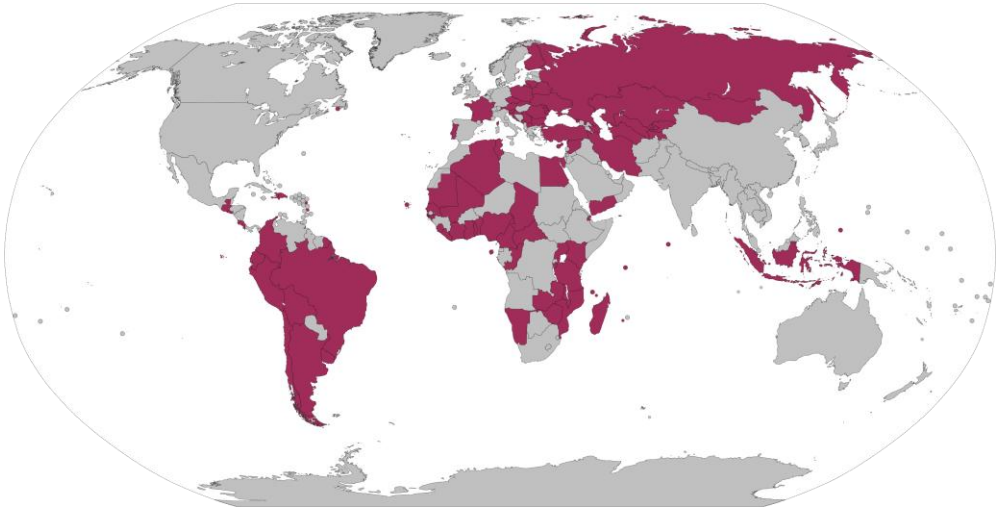
The candidate with the most votes is the **winner**.

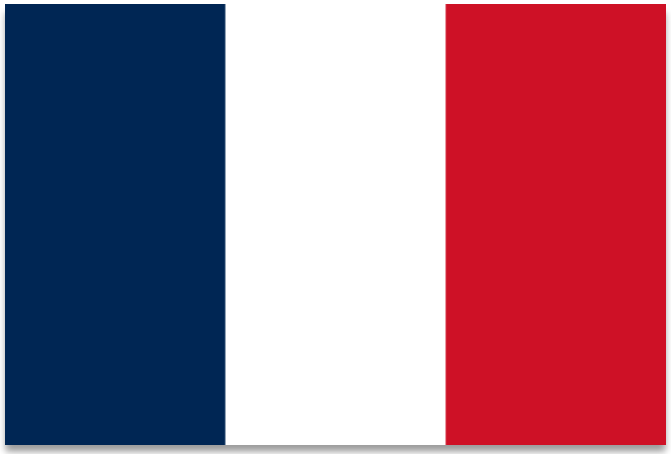


Approval with Runoff

Many people already voted with Plurality with Runoff



84 countries use it to elect their head of state



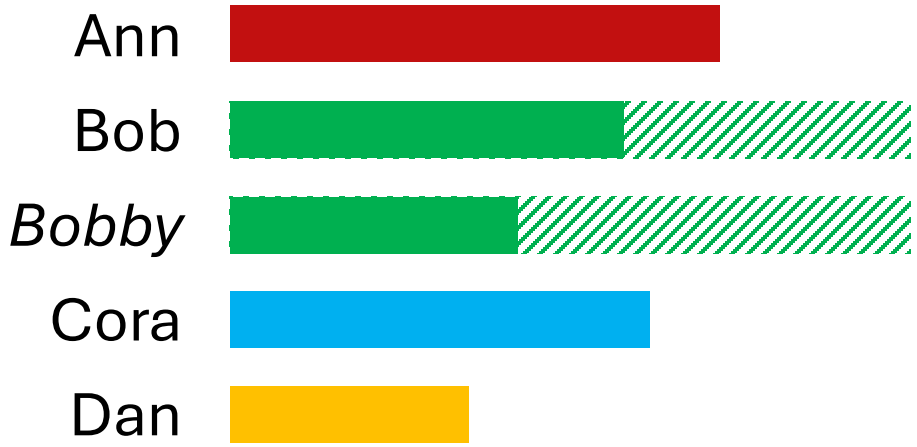
In France, it is used in almost every high-stakes election

Axiom: Independence of clones *(Tideman, 1987)*

Adding a *"clone"* of a candidate should not change significantly the result of the election.

Plurality with Runoff

2nd Round: Ann & Bob



2nd Round: Ann & Cora

The flaws of Plurality with Runoff

Axiom: Independence of clones *(Tideman, 1987)*

Adding a *"clone"* of a candidate should not change significantly the result of the election.

❌ **Plurality with Runoff**

Axiom: Monotonicity *(Fishburn, 1982)*

If some candidate is the winner, and we increase their support, this candidate should still win.

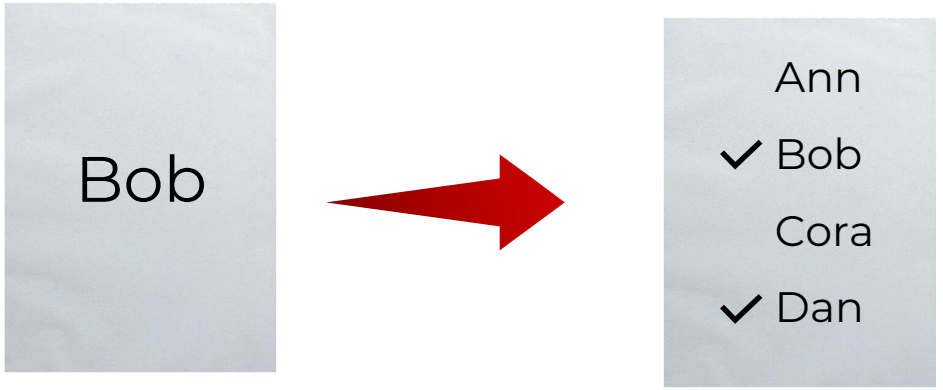
❌ **Plurality with Runoff**

Our proposal: *Approval* with Runoff

1st round: Voters can vote for **several candidates** among the possible ones.

Two finalists are selected based on these votes.

2nd round: Same as before.



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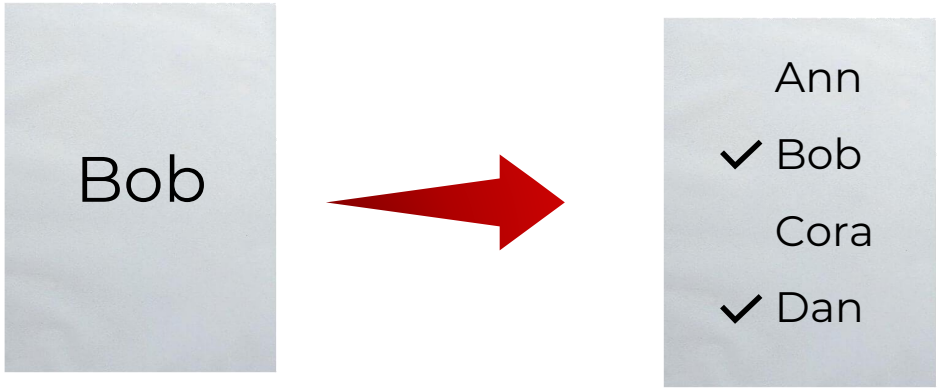


Question: Which candidates should go to the 2nd round?

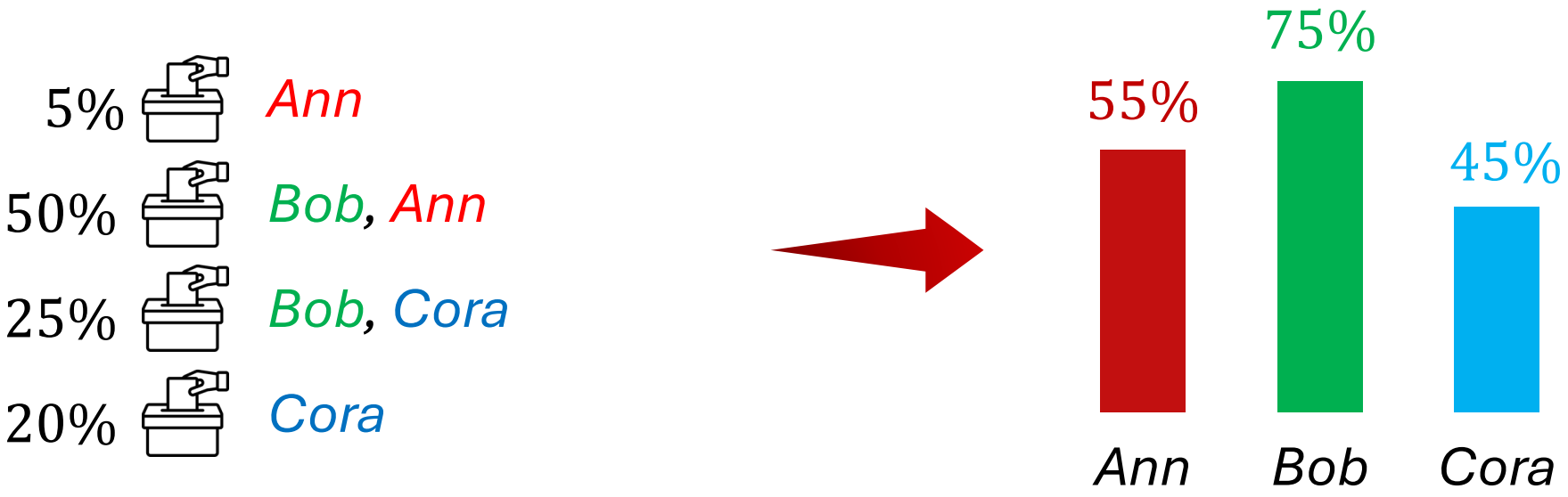
We can use **Approval-Based Committee rules** (ABC rules) to select the two finalists:

- **Rule:** Approval Voting (AV)
- **Rule:** Proportional Approval Voting (PAV)
- **Rule:** Chamberlin-Courant Approval Voting (CCAV)
- **Rule:** Sequential-Phragmén
- **Rule:** Eneström-Phragmén
- **Rule:** Split Approval Voting (SAV)

Rule: Approval Voting (AV)

The two finalists are the candidates with the most votes in the first round.

- ✓ **Monotonicity**
- ✗ **Independence of clones**



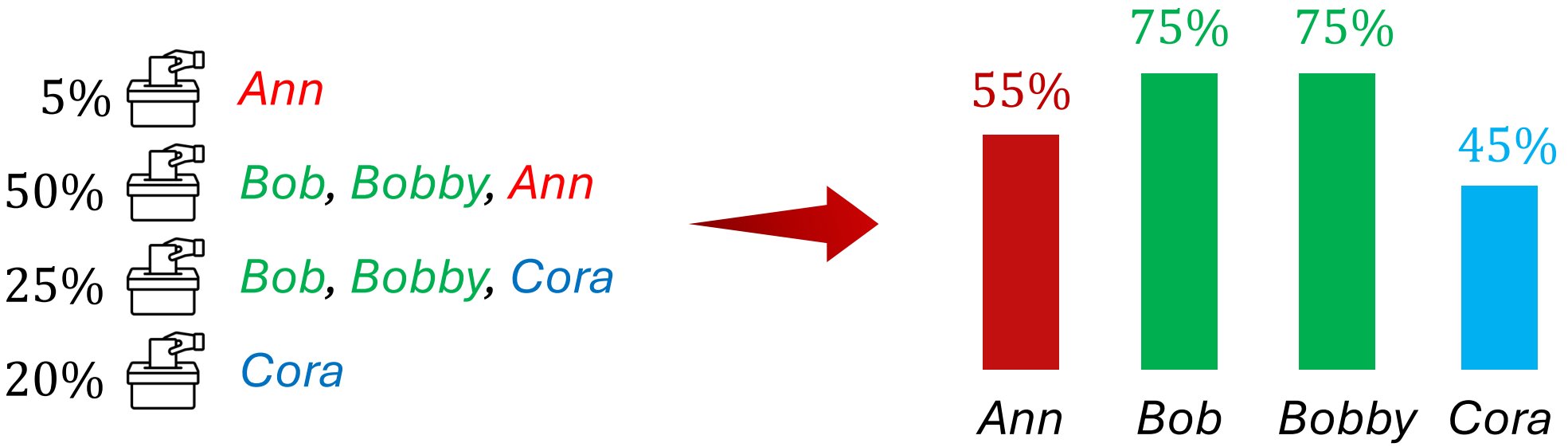
First possibility: the approval voting rule

Rule: Approval Voting (AV)

The two finalists are the candidates with the most votes in the first round.

Monotonicity

Independence of clones

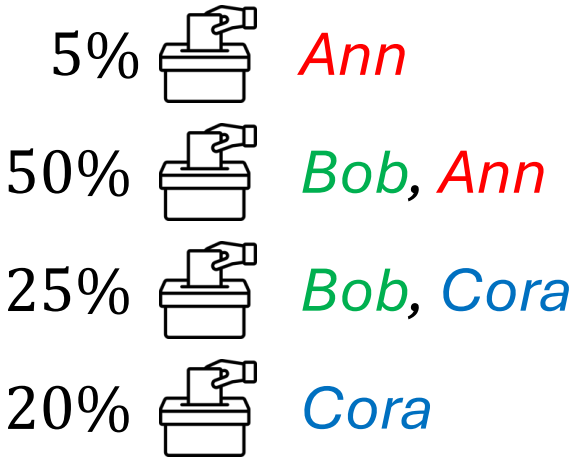


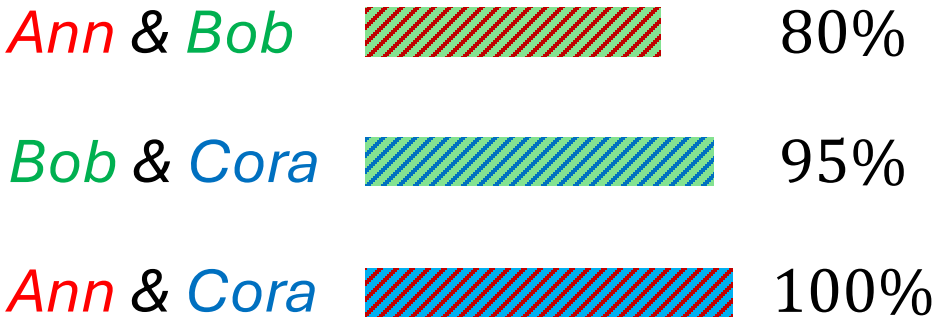
Second possibility: the Chamberlin-Courant rule

Rule: Chamberlin-Courant Approval Voting (CCAV)

The two finalists are the candidates maximizing the number of voters approving at least one of them.

- ✗ **Monotonicity**
- ✓ **Independence of clones**





Second possibility: the Chamberlin-Courant rule

Rule: Chamberlin-Courant Approval Voting (CCAV)

The two finalists are the candidates maximizing the number of voters approving at least one of them.

✗ **Monotonicity**

✓ **Independence of clones**

Impossibility Theorem

No neutral approval with runoff rule satisfies both monotonicity and independence of clones.

Axiom: Favorite consistency

The candidate that received the most votes in the first round should be in the second round.

AV

CCAV

5%	*Ann*
50%	*Bob*, *Ann*
25%	*Bob*, *Cora*
20%	*Cora*

CCAV selects *Ann* and *Cora* but *Bob* receives the most votes.

Third possibility: Sequential Chamberlin-Courant

Rule: Sequential CCAV

The **first** finalist is the candidate that received the most votes.
The **second** finalist is the candidate that received the most votes *among voters who did not vote for the first finalist*.

✓ **Favorite consistency**

5% *Ann*
50% *Bob*, *Ann*
25% *Bob*, *Cora*
20% *Cora*

✓ **Independence of clones**

Ann & *Bob* 80%
Bob & *Cora* 95%
Ann & *Cora* 100%

Summary of the axiomatic analysis

	AV	PAV	CCAV	S-PAV	S-CCAV	EnePhr	S-Phr	SAV	TRIV
Pareto-efficiency	✓	✓	✗	✓	✗	✗	✓	✓	✗
strategy-proofness	✓	✗	✓	✗	✗	✗	✗	✗	✓
strong strategy-proofness	✗	✗	✗	✗	✗	✗	✗	✗	✓
monotonicity	✓	✗	✗	✗	✗	✗	✗	✗	✓
weak ind. of clones	✗	✗	✓	✗	✓	✗	✗	✗	✗
approval-efficiency	✓	✗	✗	✓	✓	✓	✓	✗	✗

Fig. Axioms satisfied by the different rules.

Experimental analysis: *Voter Autrement*

- Statistical analysis in a 1D Euclidean space
- 38 datasets of approval preferences
- 18 of which are political datasets of the ***Voter Autrement*** collection

French presidential elections (since 2002)

Outside polling station, or online

Testing alternative voting methods

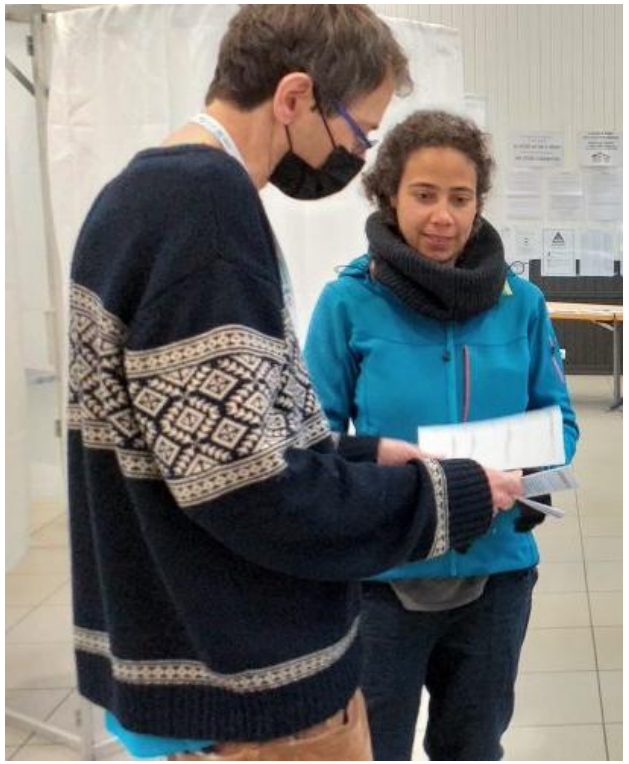
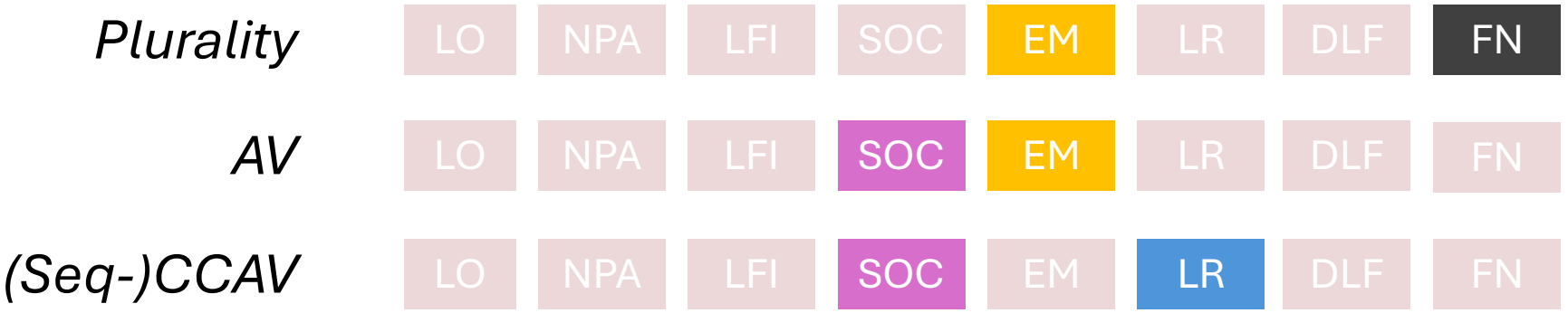


Fig. A researcher explaining how to vote with approval voting to a participant of the experiment.

1. Approval with runoff rules generally return different finalists than plurality with runoff.
2. AV generally selects similar candidates while CCAV selects ideologically distant ones.

Voter Autrement – *Grenoble (2017)*

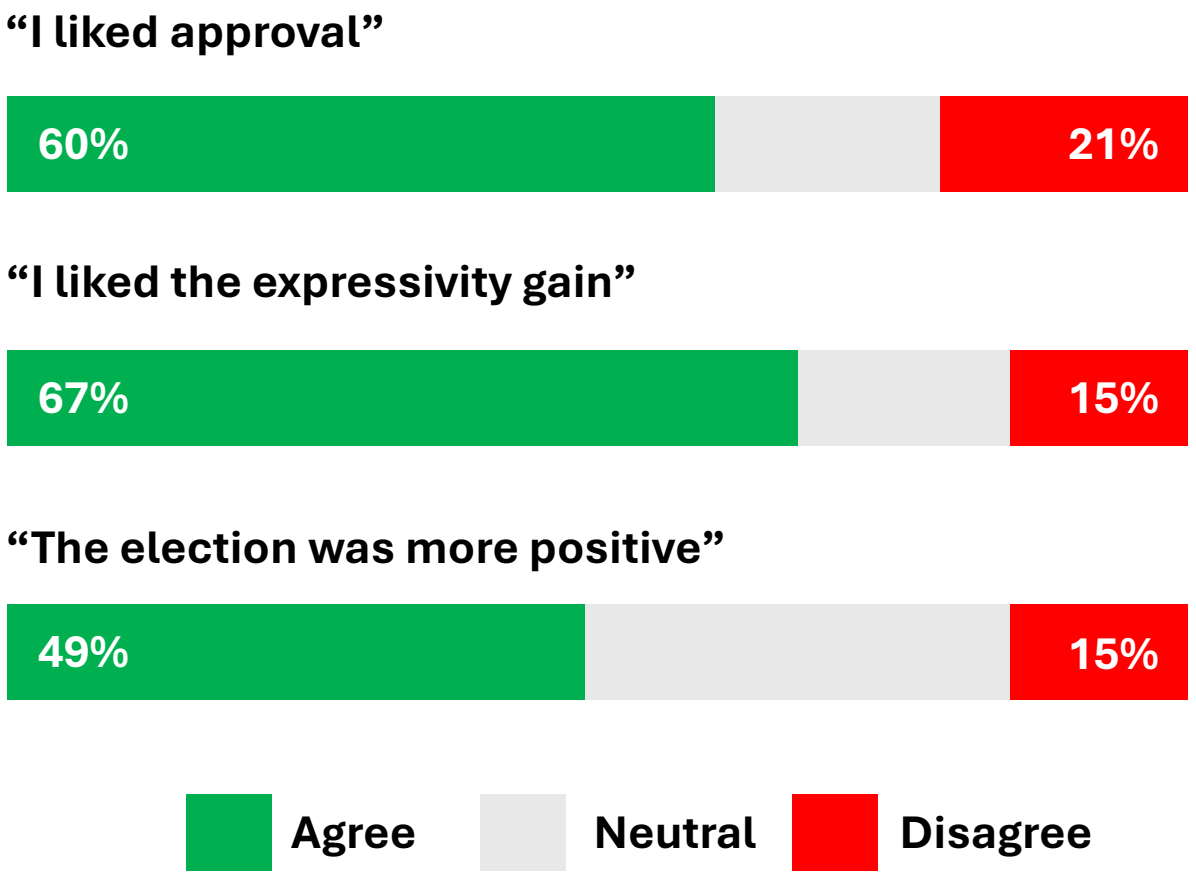
1 069 11



Could this be implemented?

Approval with runoff has been used in *St. Louis, Missouri* since 2021 for city elections.

Fig. St. Louis, Missouri



The axiomatic and experimental approach

We use this approach **for all the problems** considered in this thesis

Model	Rules	Axioms	Experiments
Ballot format: *Approval ballots* **Desired outcome:** *Single winner*	• AV • CCAV • Seq-CCAV • SAV • …	• Independence of clones • Monotonicity • Favorite-consistency • …	**Real data** *Voter Autrement* **Synthetic data** *Euclidean preferences*

- We can enhance plurality with runoff using approval ballots, but there is not a single way to do it.

- Let us now see if we can improve another single-winner voting system: *Instant Runoff Voting (IRV)*.

2. Instant Runoff Voting with Indifferences

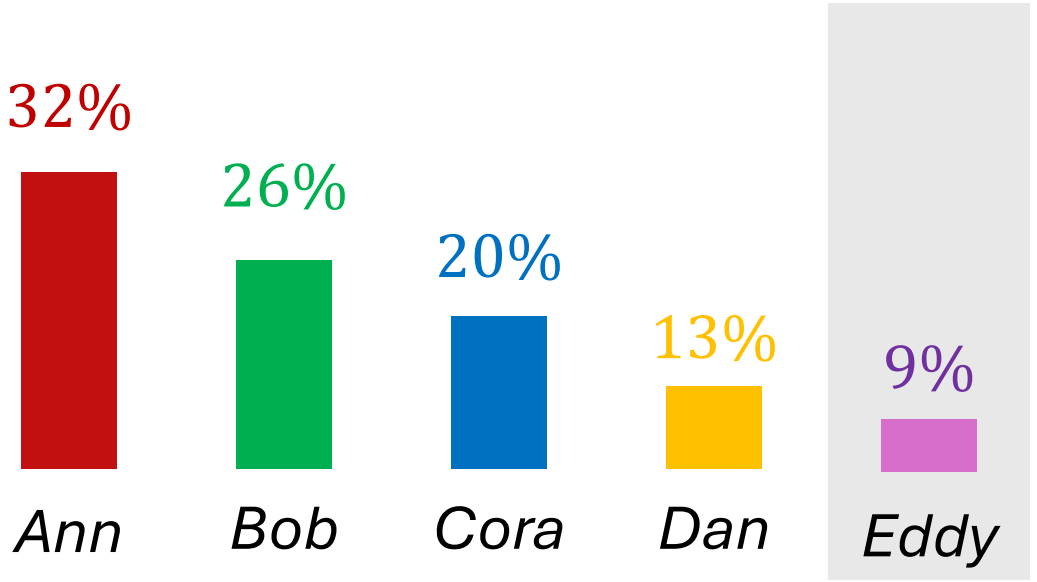
Generalizing Instant Runoff Voting to Allow Indifferences
Théo Delemazure, Dominik Peters
EC-2024

The *Instant Runoff Voting* system (IRV)

Voters provide
a **ranking** of the candidates

1 Eddy
2 Dan
3 Bob
4 Ann
5 Cora

We count the number of **first-place votes** each candidate receives



The candidate with the lowest score is **eliminated**, and their votes are transferred

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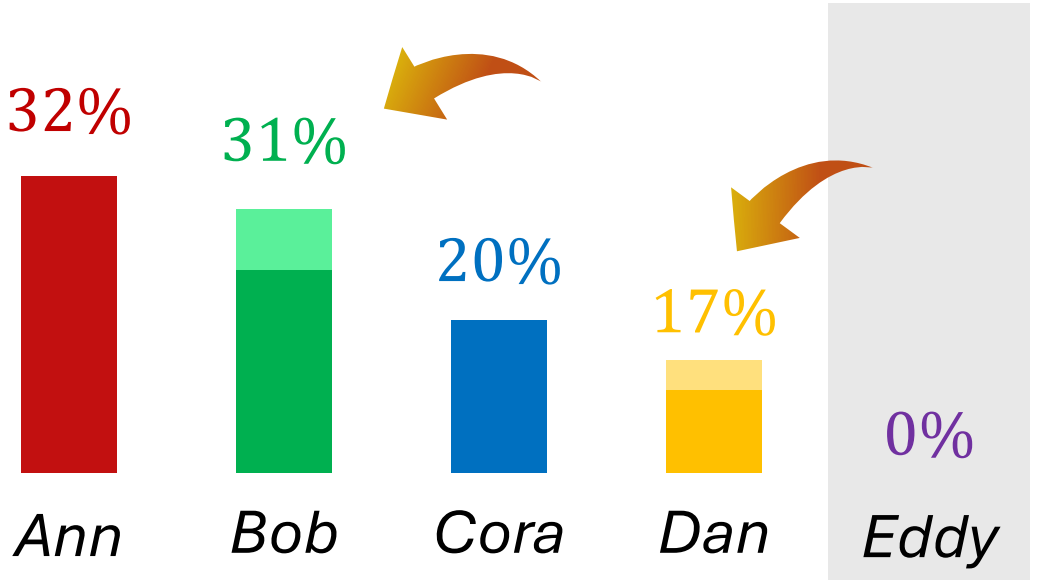

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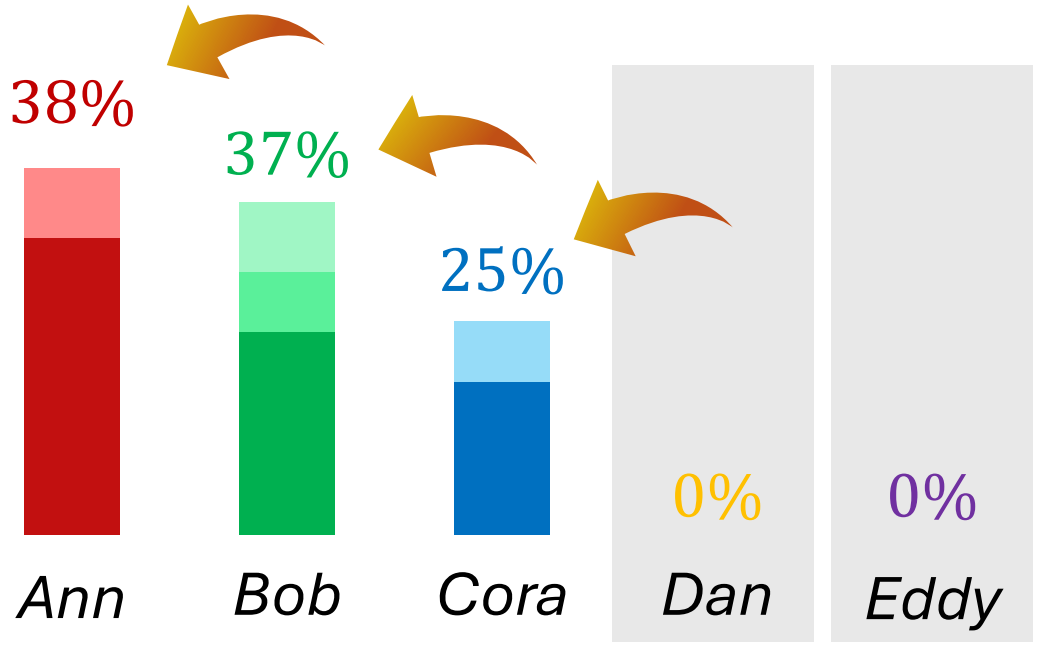

And so on, **until one candidate remains**

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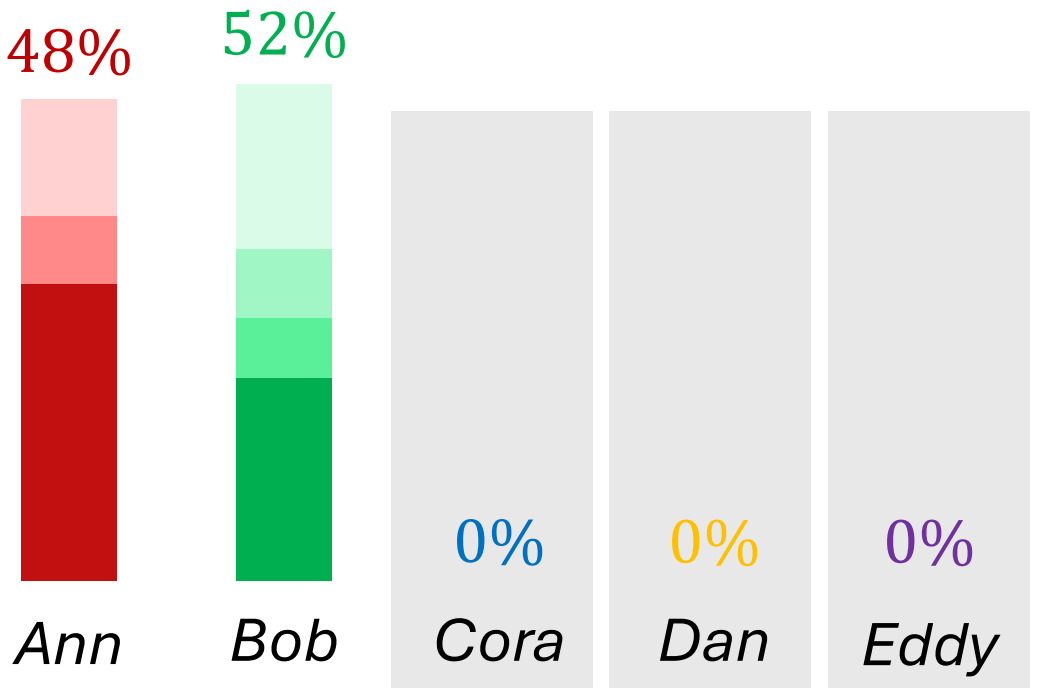

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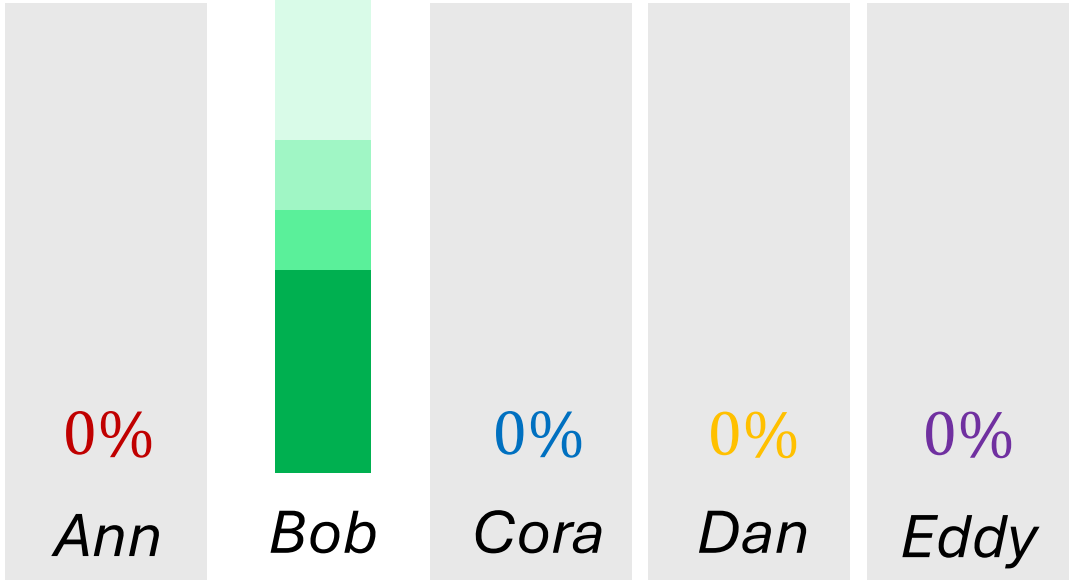
We count the number of **first-place votes** each candidate receives



And so on, **until one candidate remains**

The *Instant Runoff Voting* system (IRV)

Ireland (since 1937)

Australia (since 1918)

Maine, USA (since 2018)

And others...

- ✓ **Independence of clones**
- ✓ **Majority criterion**

> **Axiom:** Majority criterion *(Lepelley, 1992)*
>
> If a majority of voters rank one candidate first, this candidate should be the winner.

- ✗ **Monotonicity**

1	**Ann**	1	**Ann**	1	Bob
2	Bob	2	Dan	2	Cora
3	Cora	3	Cora	3	Ann
4	Dan	4	Bob	4	Dan
31%		20%		49%	

What if we are indifferent?

What if a voter is **indifferent** between several candidates?

1 Eddy
| Dan
2 Bob
| Ann
3 Cora

Voters can cast **weak orders**

Question: how to generalize IRV to weak orders?

Rule: Split-IRV *(Meek and Hill, 1994)*

Each voter gives $1/k$ point to the k candidates that are tied as first among the remaining candidates in their ranking.

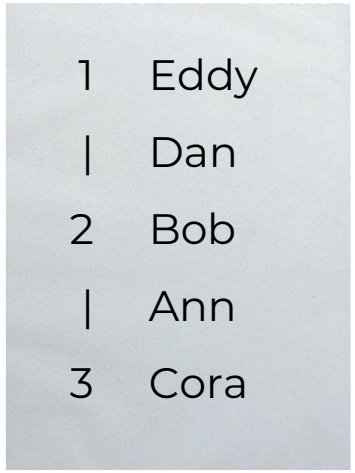
1	Eddy
|	Dan
2	Bob
|	Ann
3	Cora

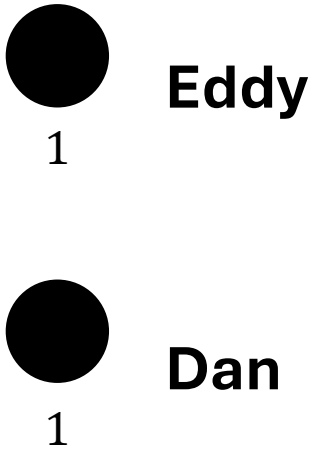
$1/2$ **Eddy**

$1/2$ **Dan**

Rule: Approval-IRV

Each voter gives 1 point to the k candidates that are tied as first among the remaining candidates in their ranking.





Instant Runoff Voting satisfies the following two axioms:

Axiom: Independence of clones *(Tideman, 1987)*

✓ IRV

Axiom: Majority Criterion *(Lepelley, 1992)*

If a majority of voters rank one candidate first, this candidate should be the winner.

1	**Ann**	1	**Ann**	1	Bob
2	Bob	2	Dan	2	Cora
3	Cora	3	Cora	3	Ann
4	Dan	4	Bob	4	Dan
	31%		20%		49%

✓ IRV

We can generalize these axioms to weak orders:

Axiom: Independence of clones *(Tideman, 1987)*

? Split-IRV ? Approval-IRV

Axiom: Respect for cohesive majorities

If a majority of voters rank one candidate first, **the winner should also be ranked first by one of these voters.**

? Split-IRV ? Approval-IRV

1	**Ann**		1	**Ann**		1	Bob	
|	**Bob**		2	Dan		|	Cora	
2	Cora		|	Cora		2	Ann	
3	Dan		3	Bob		|	Dan	
31%			20%			49%		

We can generalize these axioms to weak orders:

Axiom: Independence of clones *(Tideman, 1987)*

✗ Split-IRV ✓ Approval-IRV

Axiom: Respect for cohesive majorities

If a majority of voters rank one candidate first, **the winner should also be ranked first by one of these voters.**

✗ Split-IRV ✓ Approval-IRV

1	**Ann**		1	**Ann**		1	Bob	
|	**Bob**		2	Dan		|	Cora	
2	Cora		|	Cora		2	Ann	
3	Dan		3	Bob		|	Dan	
31%			20%			49%		

First characterization of Approval-IRV

Approval-IRV is the **only** runoff scoring rule for weak orders that satisfies **both** independence of clones and respect for cohesive majorities.

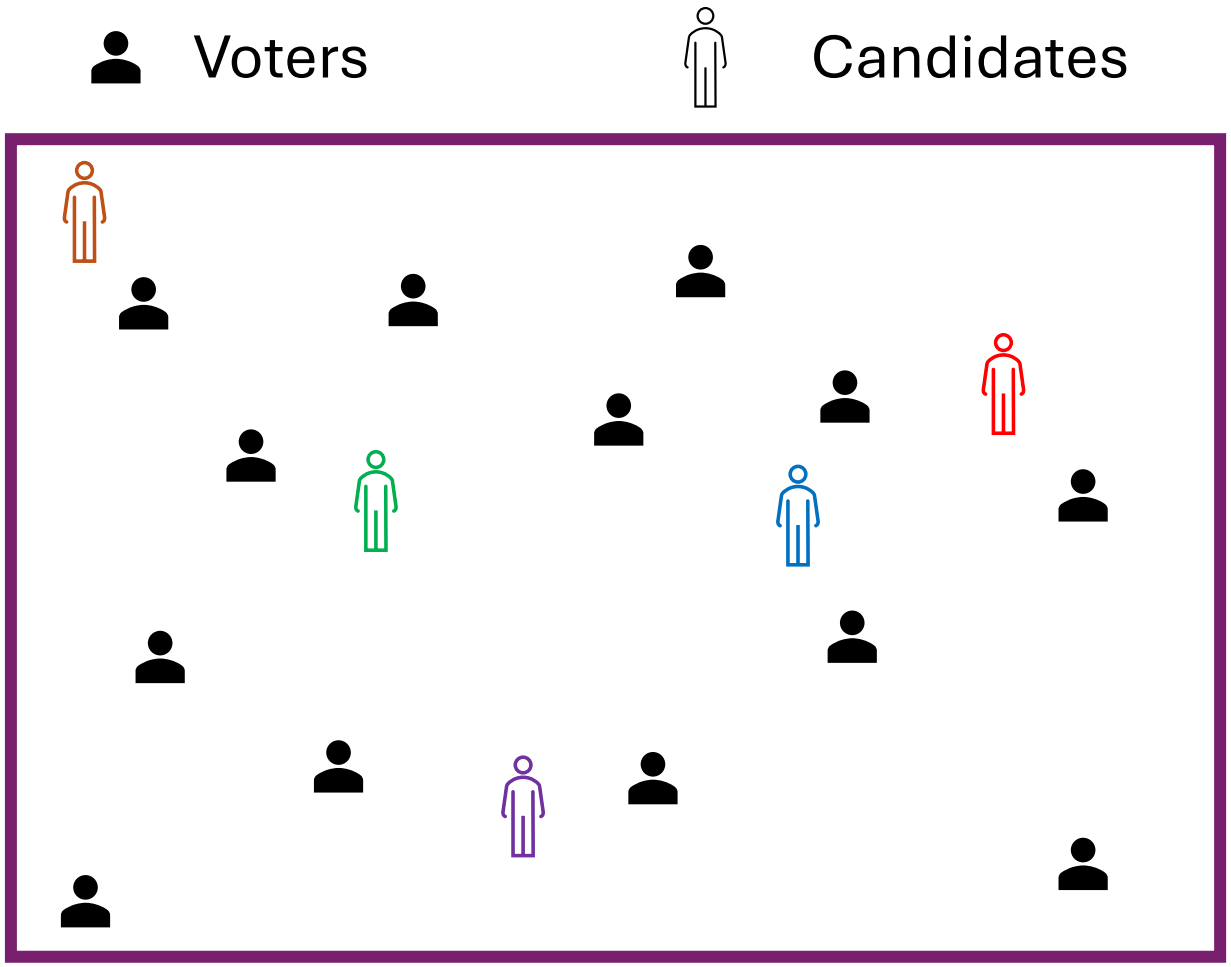
Axiom: Monotonicity *(Fishburn, 1982)* ✗ IRV

Axiom: Weak monotonicity ✗ Split-IRV ✓ Approval-IRV

Second characterization of Approval-IRV

Approval-IRV is the **only** runoff scoring rule for weak orders that generalizes IRV and satisfies weak monotonicity.

Experimental analysis: Euclidean preferences



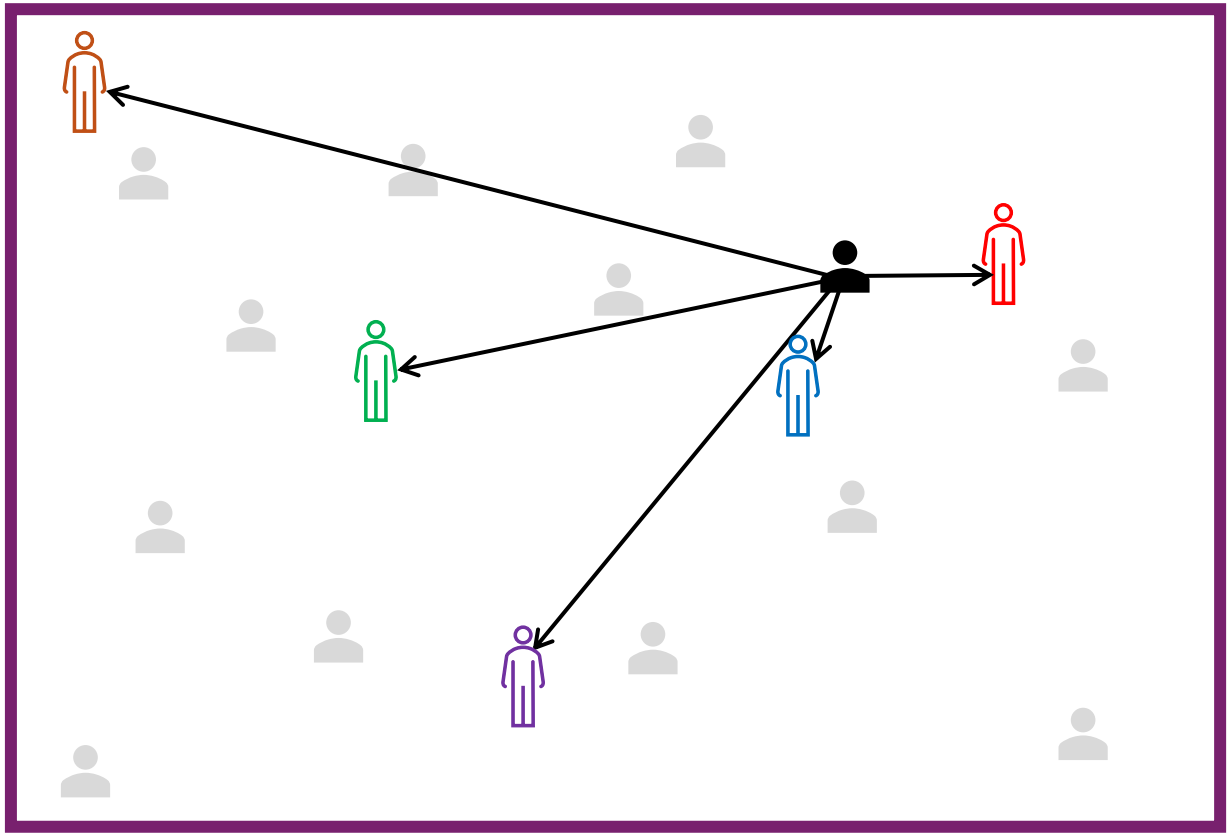
Euclidean Preferences
(Enelow and Hinich, 1984)

» Positions of voters and candidates are sampled randomly in the space.

Experimental analysis: Euclidean preferences

Voters

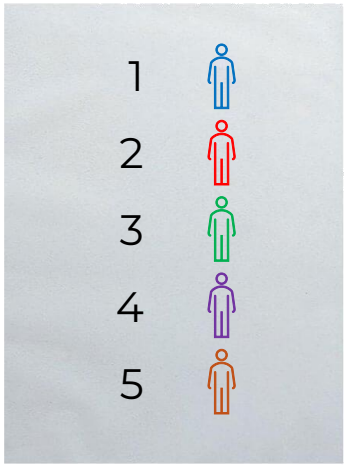
Candidates



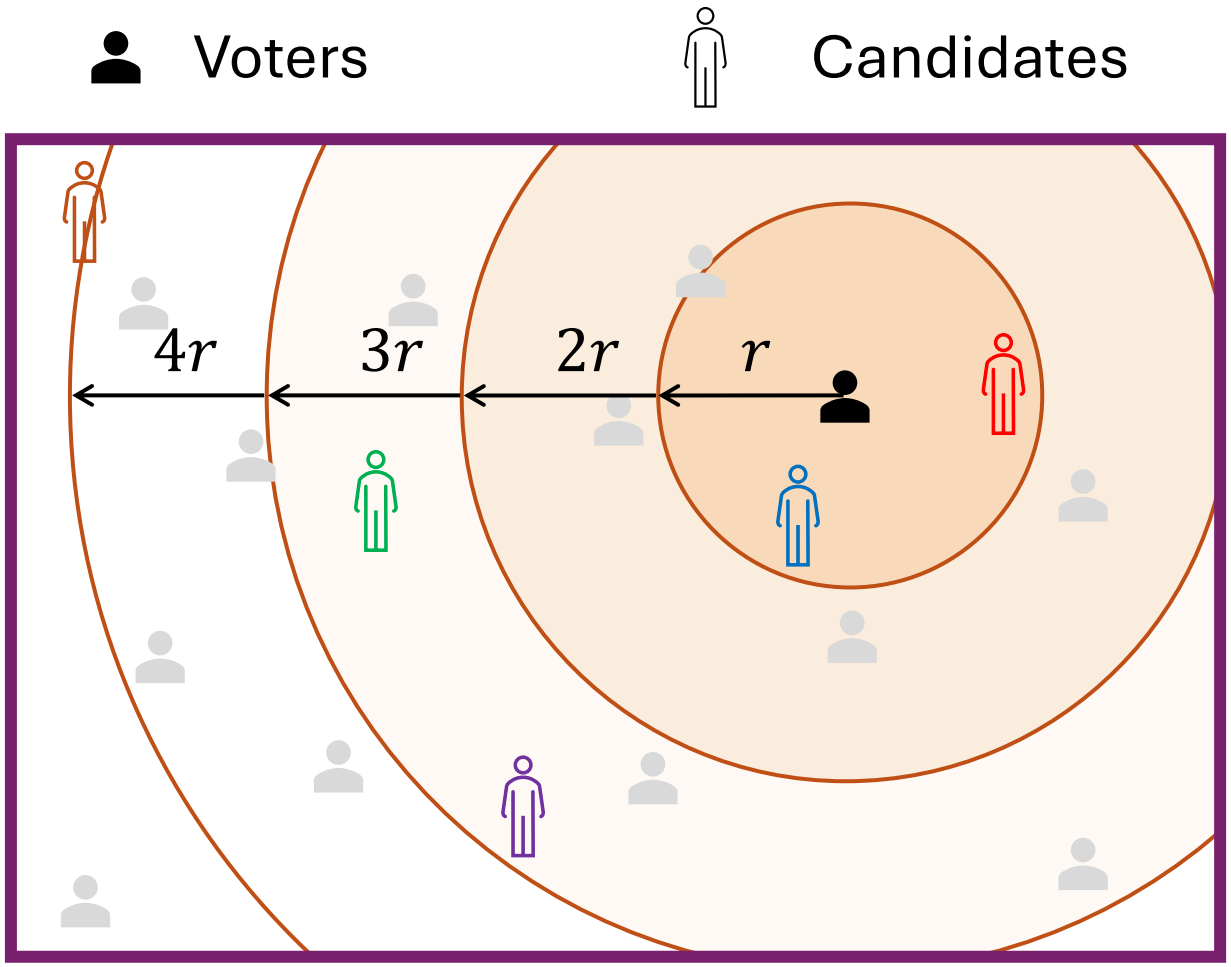
Euclidean Preferences

(Enelow and Hinich, 1984)

» Voters prefer candidates that are closer to them:

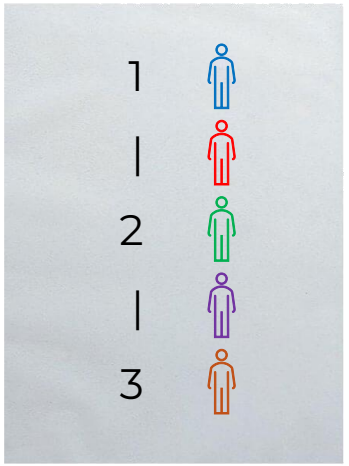


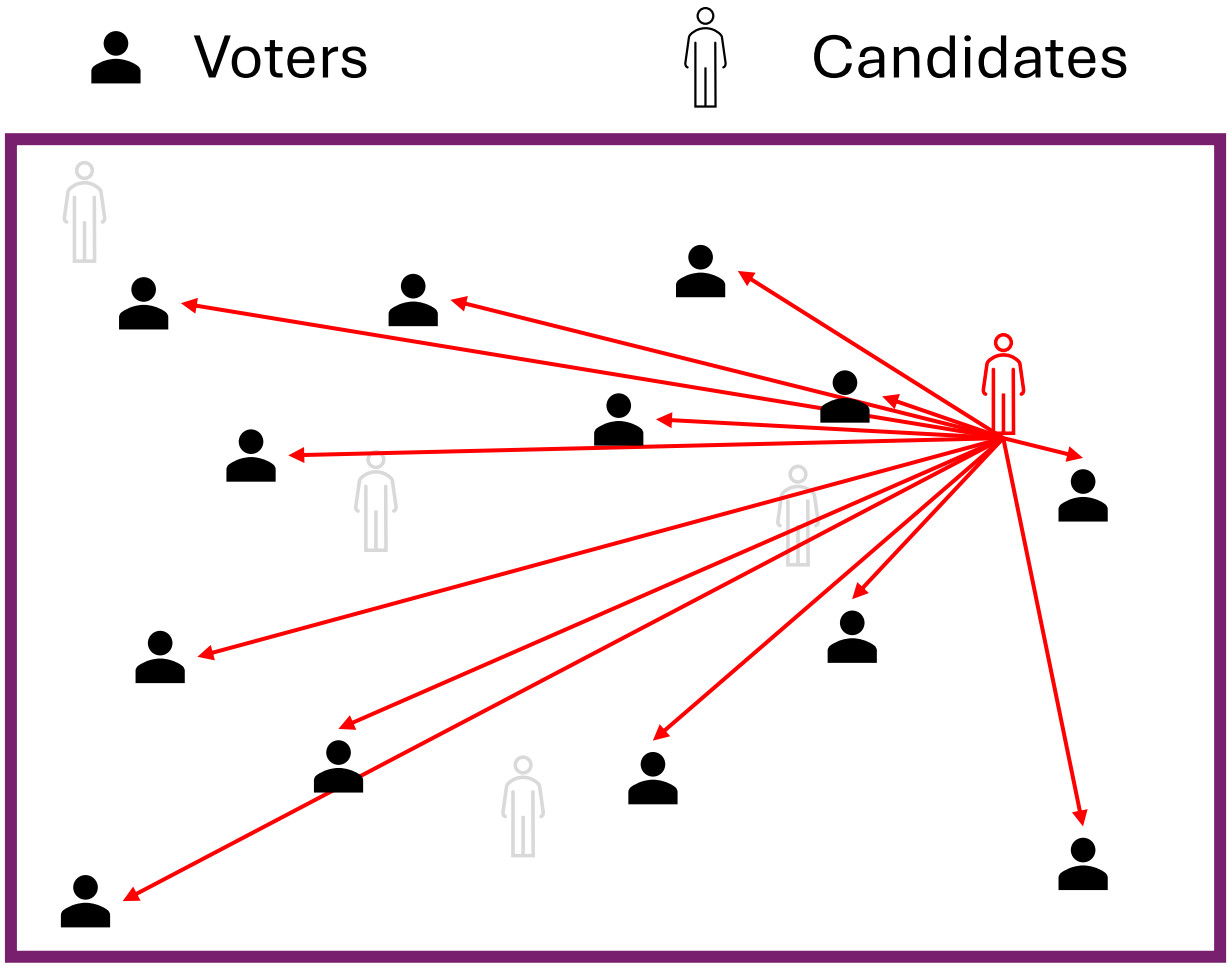
Experimental analysis: Euclidean preferences



Euclidean Preferences
(Enelow and Hinich, 1984)

» We can also obtain weak orders:

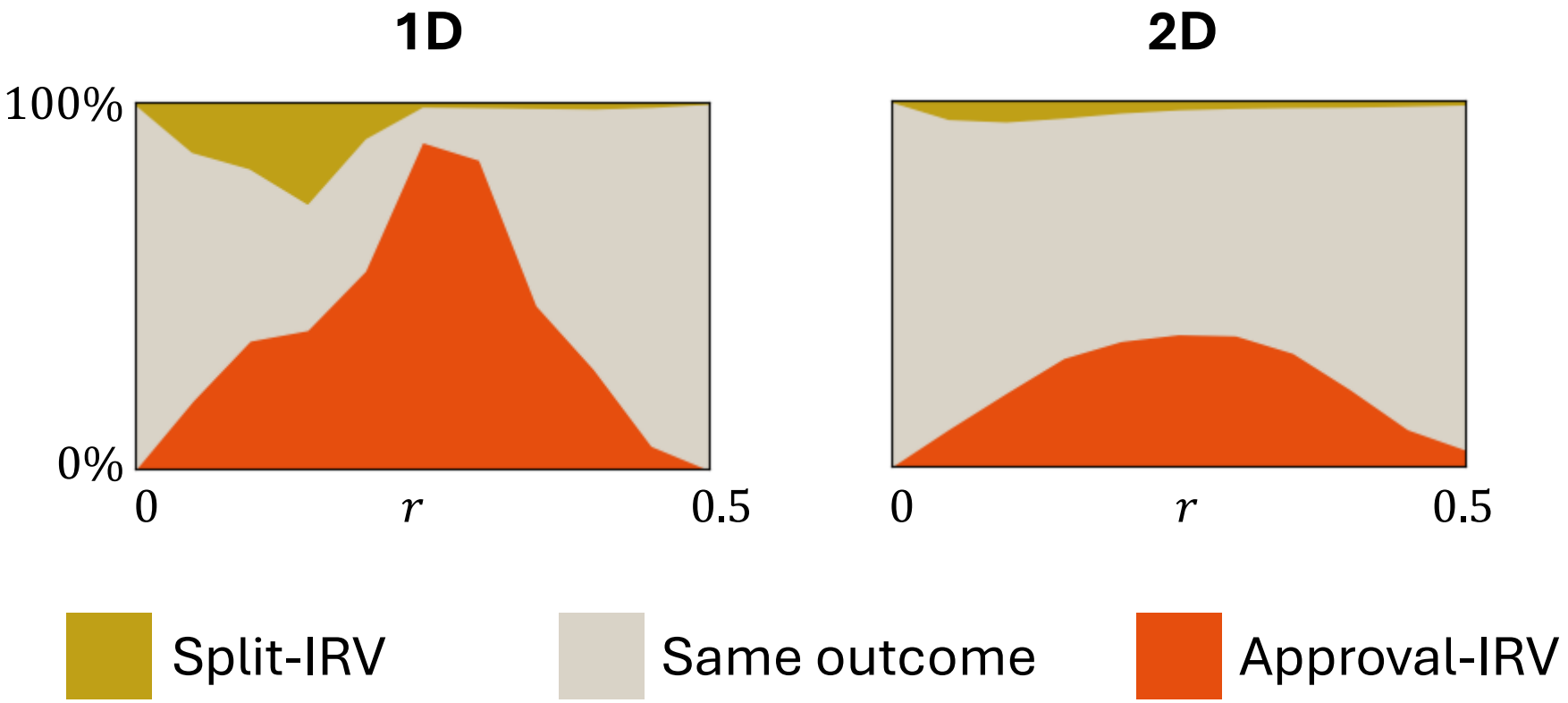




Euclidean Preferences
(Enelow and Hinich, 1984)

- The **cost** of candidates are their average distance to the voters.
- **The lower the cost, the better.**

Which rule returns the candidate **with lowest cost**?
(proportion over 10 000 instances)



- Approval-IRV appears to be a better generalization of IRV to weak orders than Split-IRV.

- Can expressive ballots also improve the way we vote in *parliamentary elections*?

3. Rankings in Parliamentary Elections with Threshold

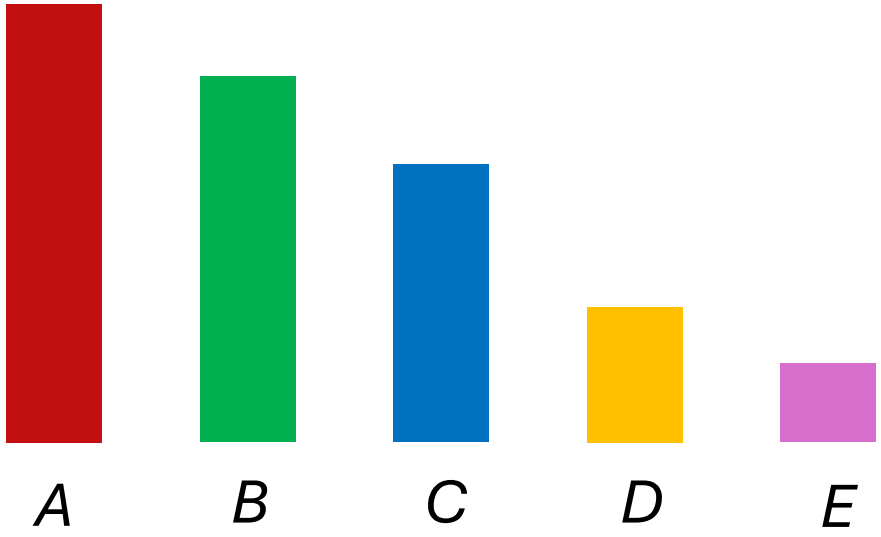
Reallocating Wasted Votes in Proportional Parliamentary Elections with Thresholds
Théo Delemazure, Rupert Freeman, Jérôme Lang, Jean-François Laslier, Dominik Peters
EC-2025

PhD Defense - June 11th, 2025

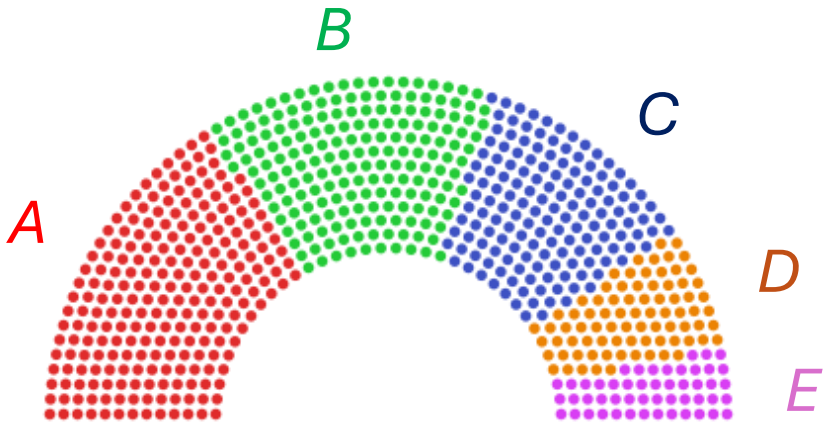
Voting systems for parliamentary elections

Proportional representation

Voters vote for one of the parties.

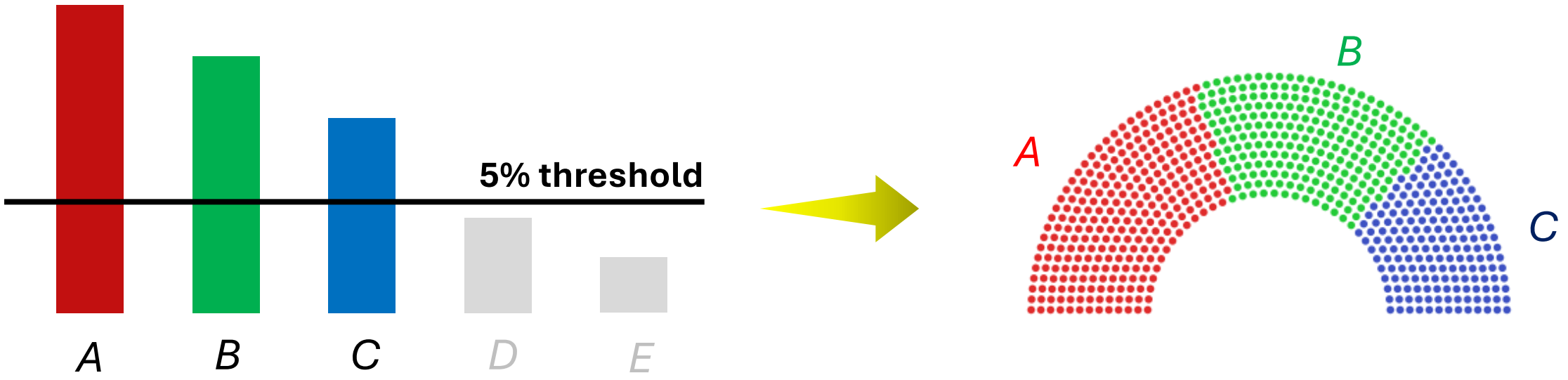


Seats are allocated to the parties **proportionally to their scores**.

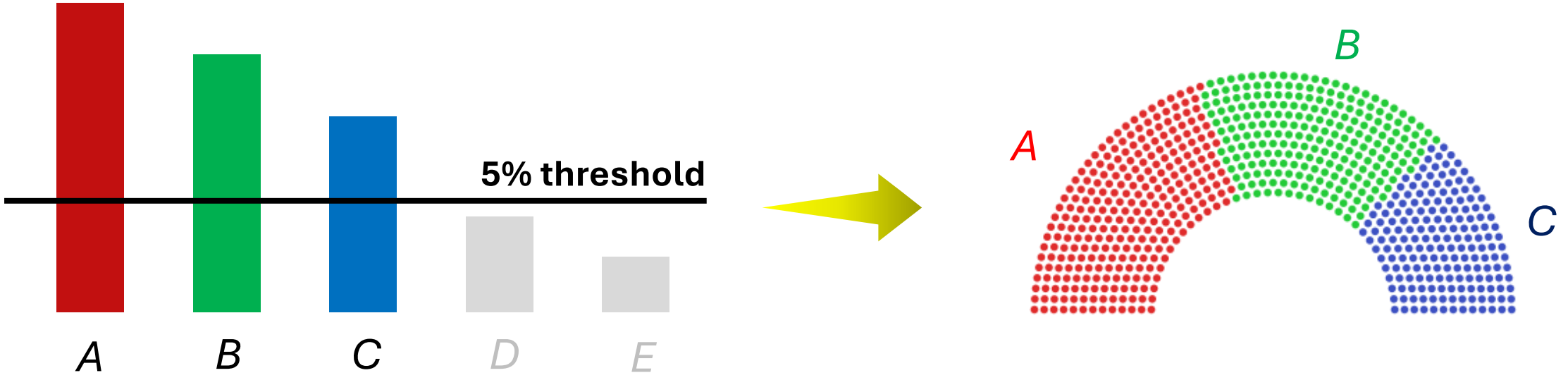


Proportional representation *with threshold*

Many countries impose and **electoral threshold** to reduce political fragmentation.



Proportional representation *with threshold*

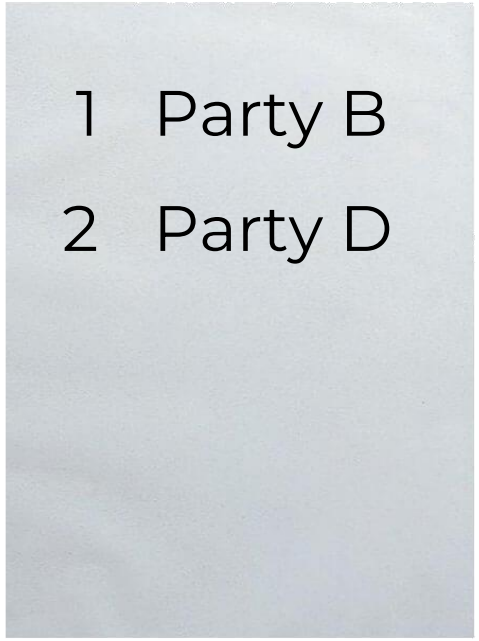


- **Some votes are "lost":** *D* and *E* supporters have no influence on the seat distribution.
- This incentivizes forms of **tactical voting.**

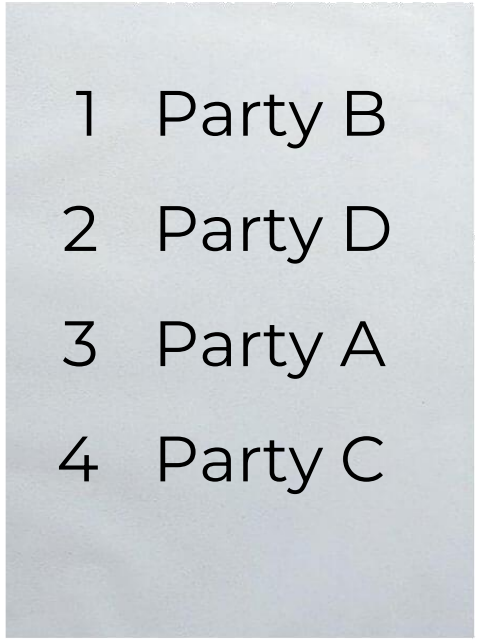
Proportional representation *with threshold*

	Threshold	"Lost" votes
2019 election of the French *representative to the EU Parliament*.	5%	20%
2025 election of the *Bundestag* members.	5%	14%
2002 election of the *Turkish Parliament* members.	10%	46%

We could ask voters **to vote for two parties**, and we use the second vote only if the first one does not reach the threshold.



We could ask voters **to vote for two parties**, and we use the second vote only if the first one does not reach the threshold.



We can even ask for a **truncated ranking**

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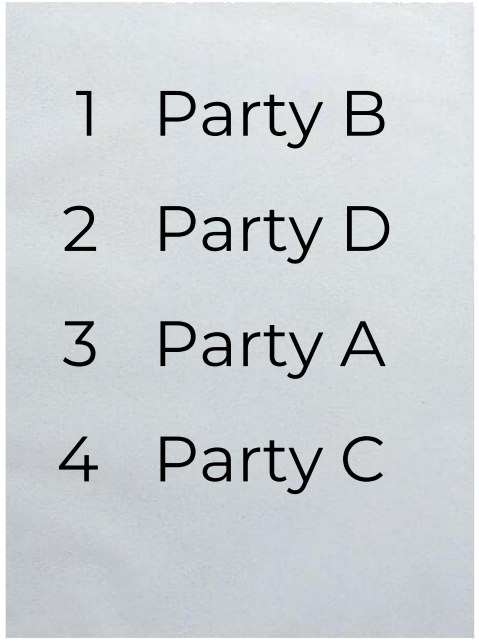


We can even ask for a **truncated ranking**

Question: how to select the parties that are above the threshold?

Five different rules

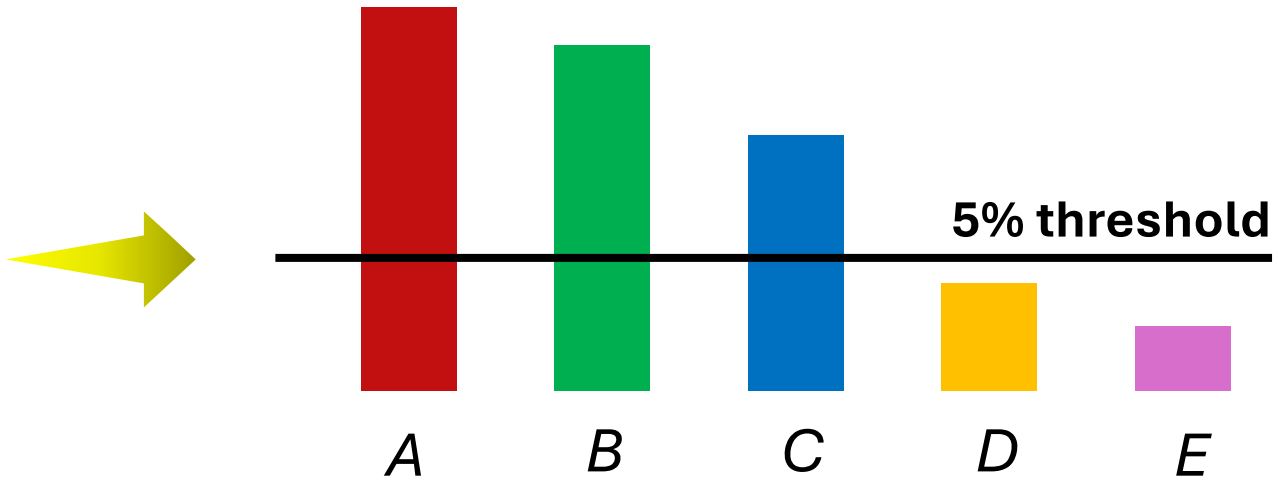
- **Rule:** Direct Winners Only (DO)
- **Rule:** Single Transferable Vote (STV)
- **Rule:** Greedy Plurality (GP)
- **Rule:** MaxPlurality (MaxP)
- **Rule:** MaxRepresentation (MaxR)

First possibility: *Direct Winners Only* (DO)

Rule: Direct Winners Only (DO)

The selected parties are all those which receive more first-place votes than required by the threshold.

1. A 2. B	1. B	1. C 2. D 3. A
34%	32%	15%
1. C 2. B	1. D 2. C	1. E 2. D 3. B
12%	4%	3%



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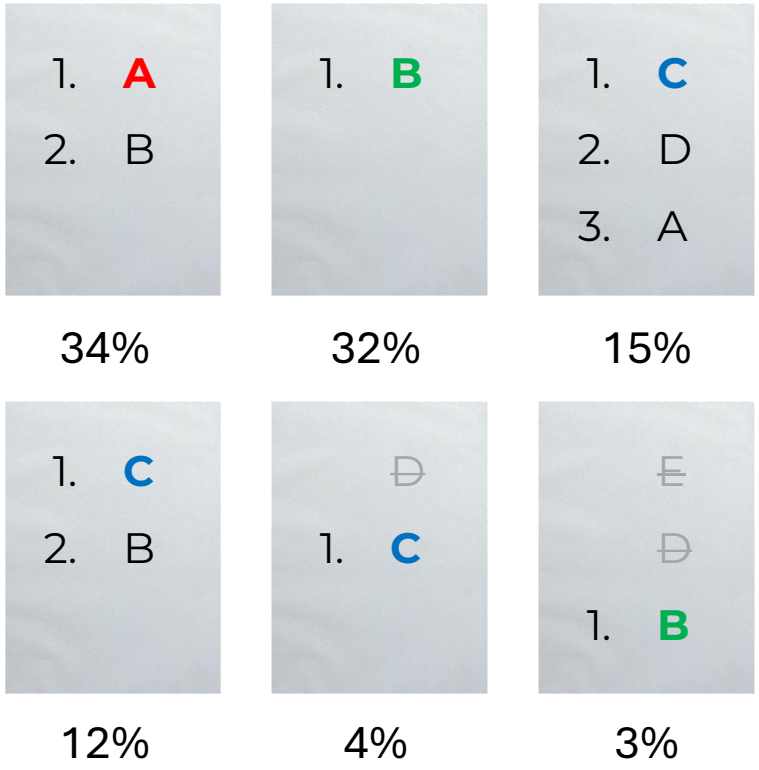

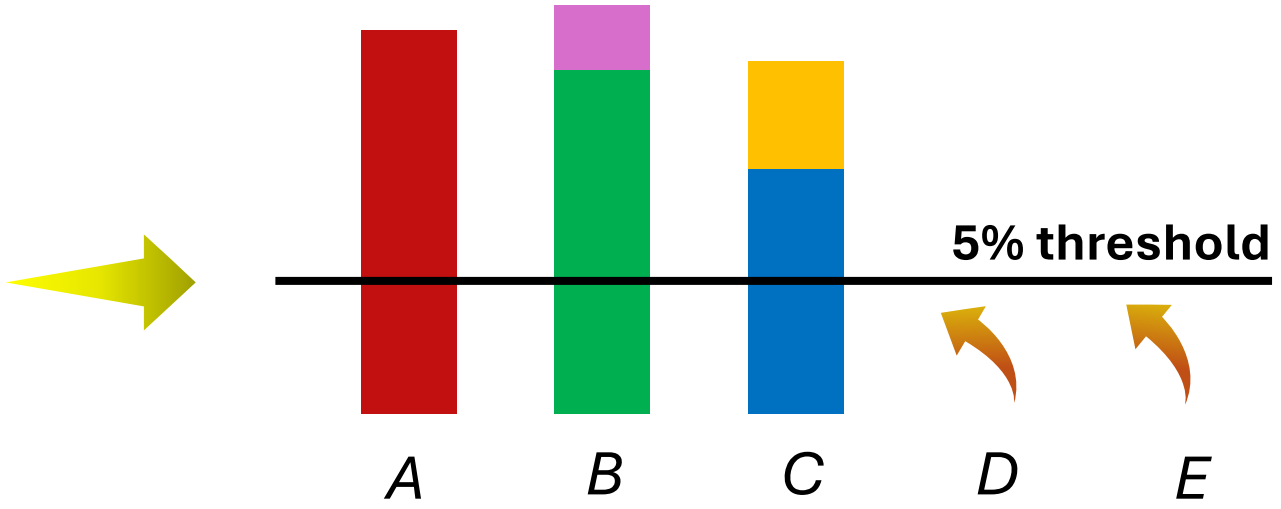

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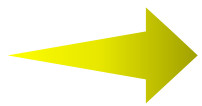
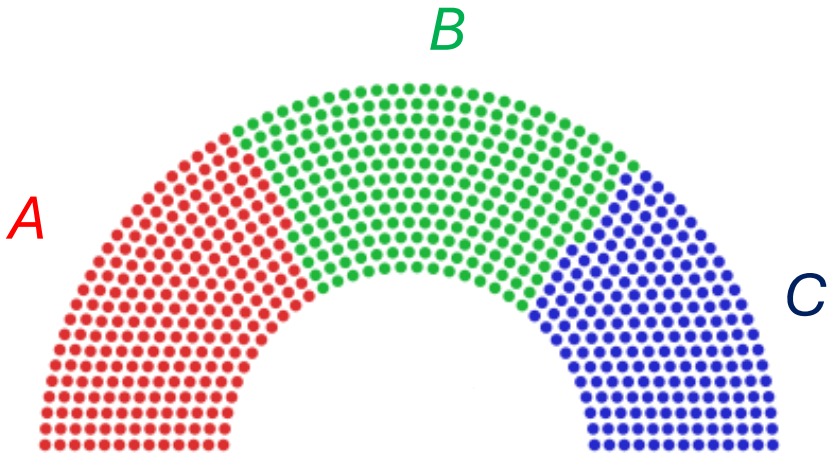
34%	32%	15%
1. C 2. B	~~D~~ 1. C	~~E~~ ~~D~~ 1. B
12%	4%	3%



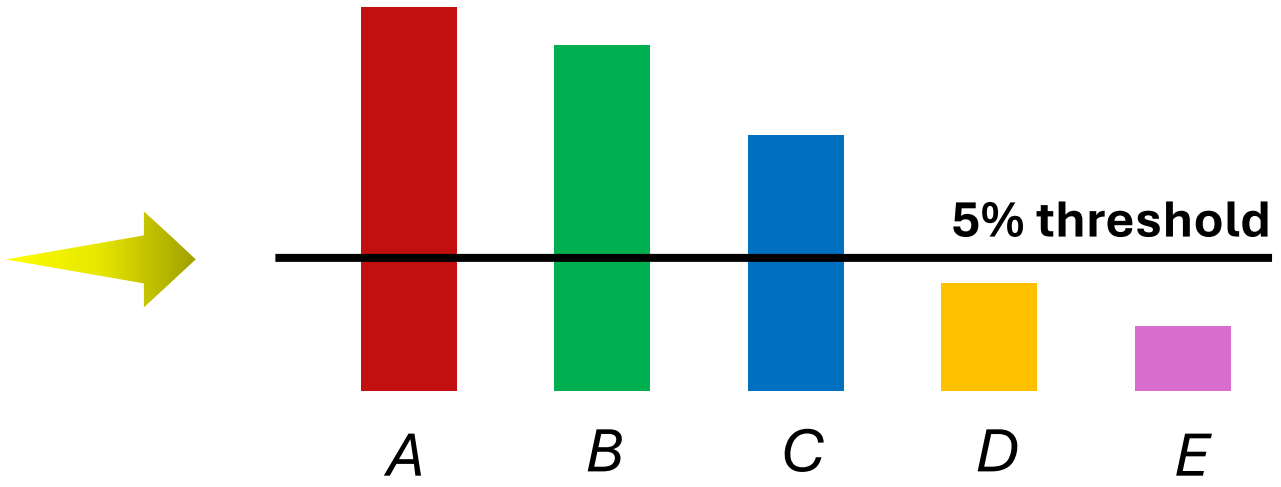


Second possibility: *Single Transferable Vote (STV)*

Rule: Single Transferable Vote (STV)

Parties that receive the fewest votes are successively eliminated until all parties receive more votes than required by the threshold.

1. A 2. B	1. B	1. C 2. D 3. A
34%	32%	15%
1. C 2. B	1. D 2. C	1. E 2. D 3. B
12%	4%	3%



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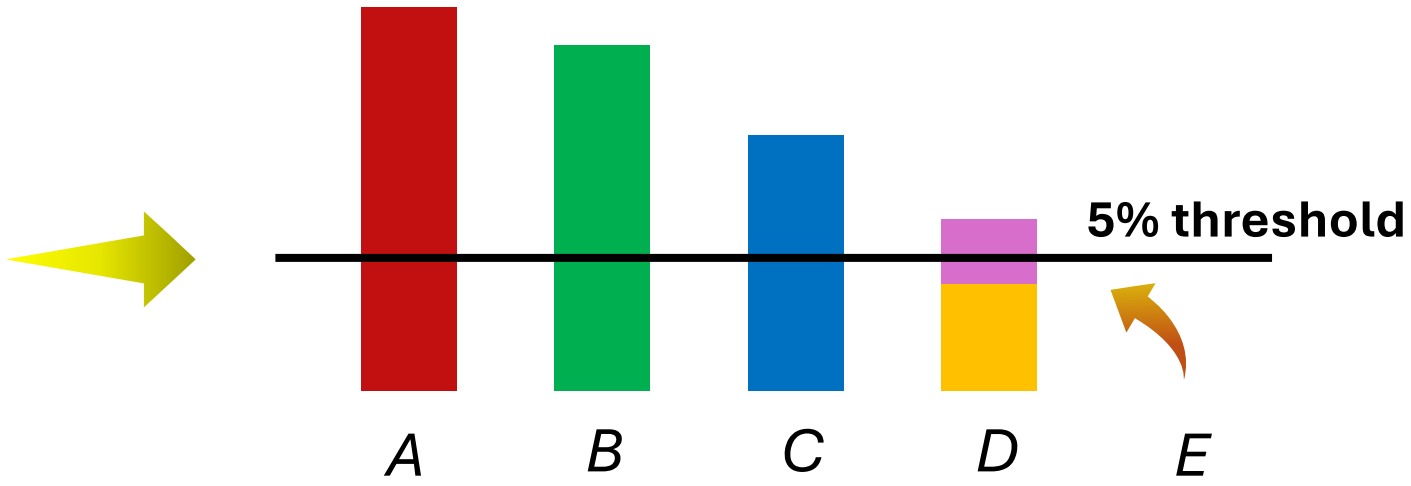

Second possibility: *Single Transferable Vote (STV)*

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Parties that receive the fewest votes are successively eliminated until all parties receive more votes than required by the threshold.

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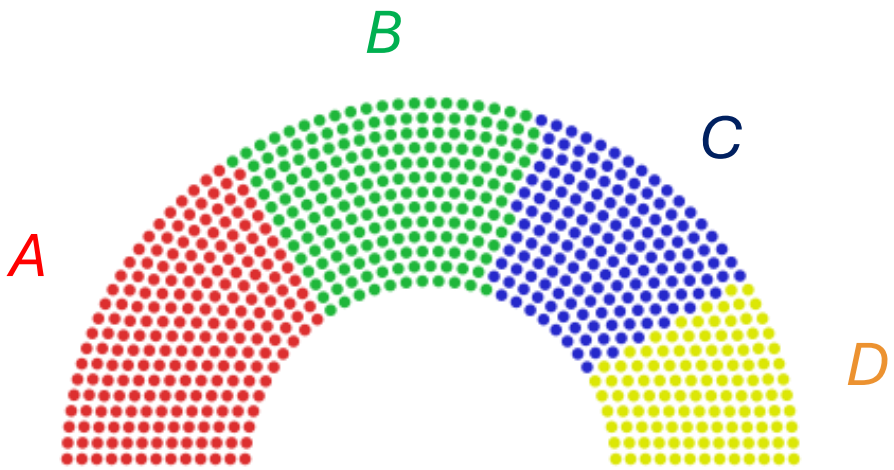
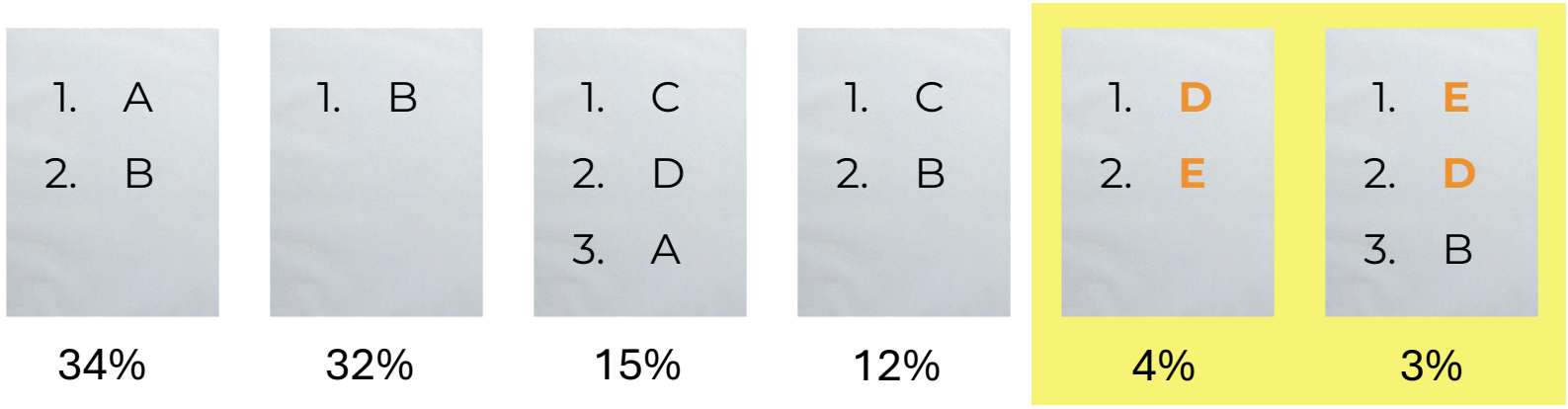


Axiom: Representation of solid coalitions

DO ✗ STV ✓

If more voters than required by the threshold all rank a set of parties *S* **on top of their rankings**, at least one of these parties should be selected.



Axiomatic analysis

Axiom: Representation of solid coalitions	✗ DO	✓ STV
Axiom: Independence of clones	✗ DO	✓ STV
Axiom: Monotonicity	✓ DO	✗ STV

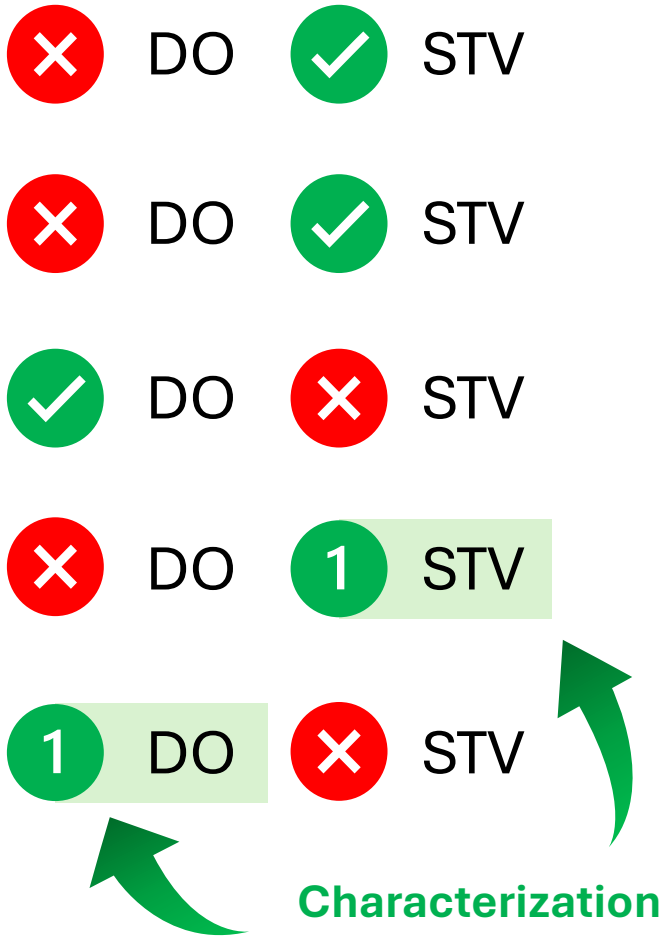
Axiom: Representation of solid coalitions

Axiom: Independence of clones

Axiom: Monotonicity

Axiom: Independence of definitely losing parties

Axiom: Reinforcement for winning parties



The rules satisfy different sets of axioms.

Axiomatic analysis

	DO	STV	GP	MaxP	MaxR
Set-maximal	✗	✗	✓	✓	✓
Inclusion of direct winners	✓	✓	✓	✗	✗
Representation of solid coalitions	✗	✓	✗	✗	✗
Representation of unrepresented voters	✗	✗	✗	✗	✗
Threshold monotonicity	✓	✓	✗	✗	✗
Independence of definitely losing parties	✗	✓	✗	✗	✗
Independence of clones	✗	✓	✗	✗	✓
Reinforcement for winning parties	✓	✗	✗	✗	✗
Monotonicity	✓	✗	✗	✗	✗
Representative-strategyproof (one risky party)	✗	✗	✓	✓	✓
Share-strategyproof (safe first or second)	✓	✗	✗	✗	✗
Share-strategyproof (representative ranked first)	✓	✗	✓	✗	✗

Fig. Axioms satisfied by the different rules.

1. Explanation of the issues caused by the threshold.
2. Presentation of the candidate lists.
3. **Vote with alternative voting methods.**
4. Questionnaire.

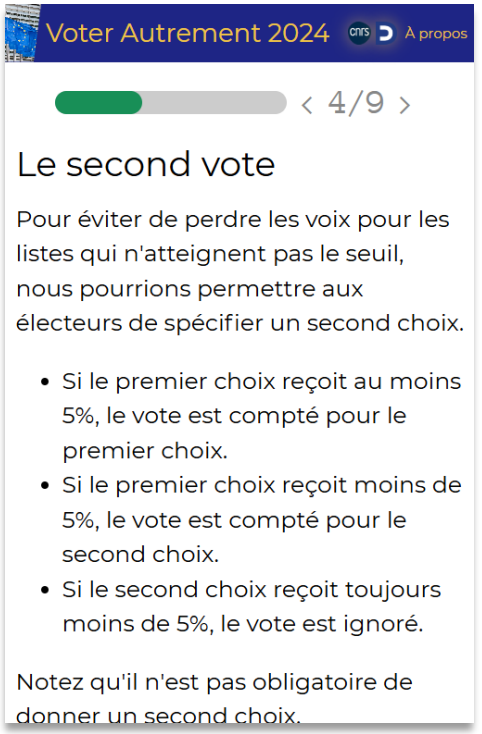


Fig. Screenshot of the website of the experiment conducted during the 2024 election of the French representative to the EU Parliament.

Two samples of participants

1 *Self-selected* sample

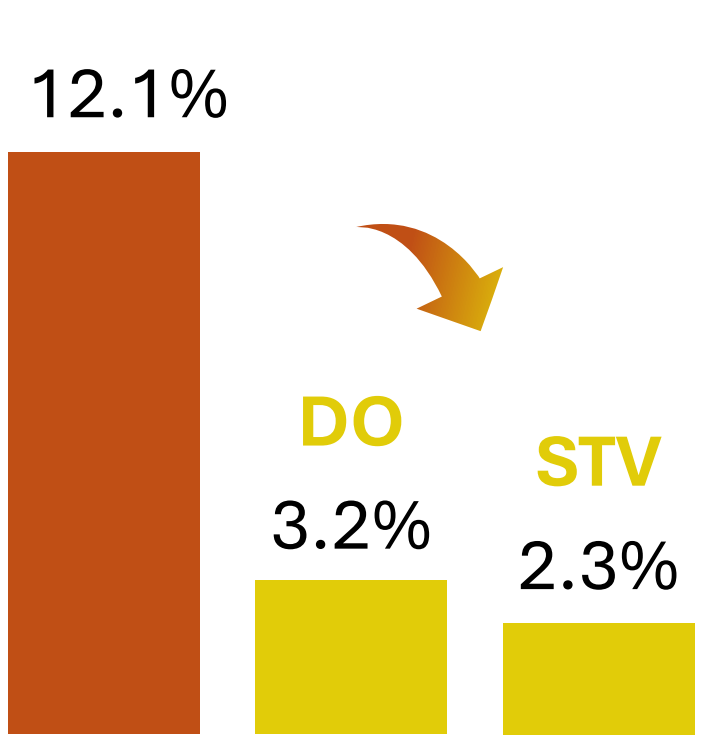
- 3046 participants in a week.
- Recruited through social media.
- Overrepresentation of left-wing, young and educated people.

2 *Representative* sample

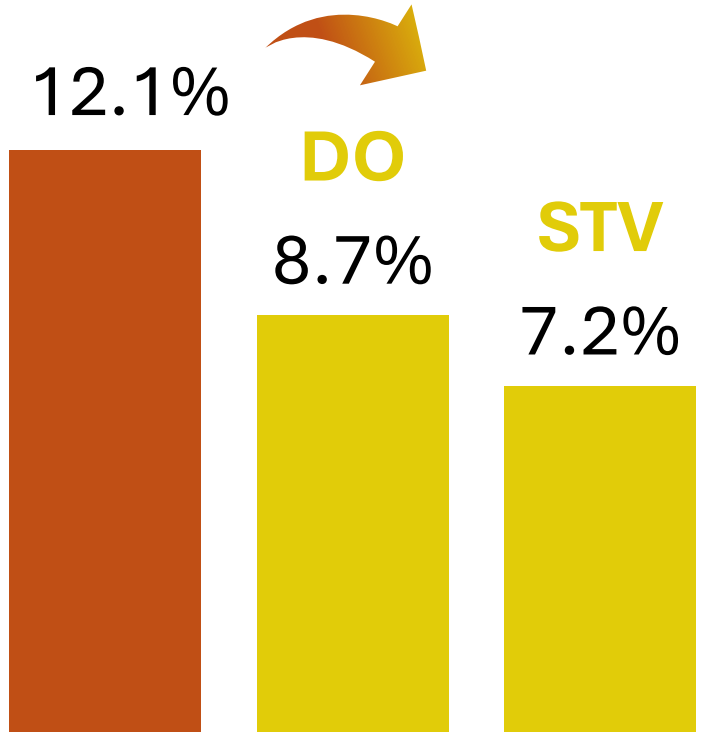
- 1000 participants.
- Recruited via a polling institute and paid a fixed amount to participate.
- Representative of the French population.

Observation 1: less votes are “lost”

» Less votes are “lost” and not taken into account



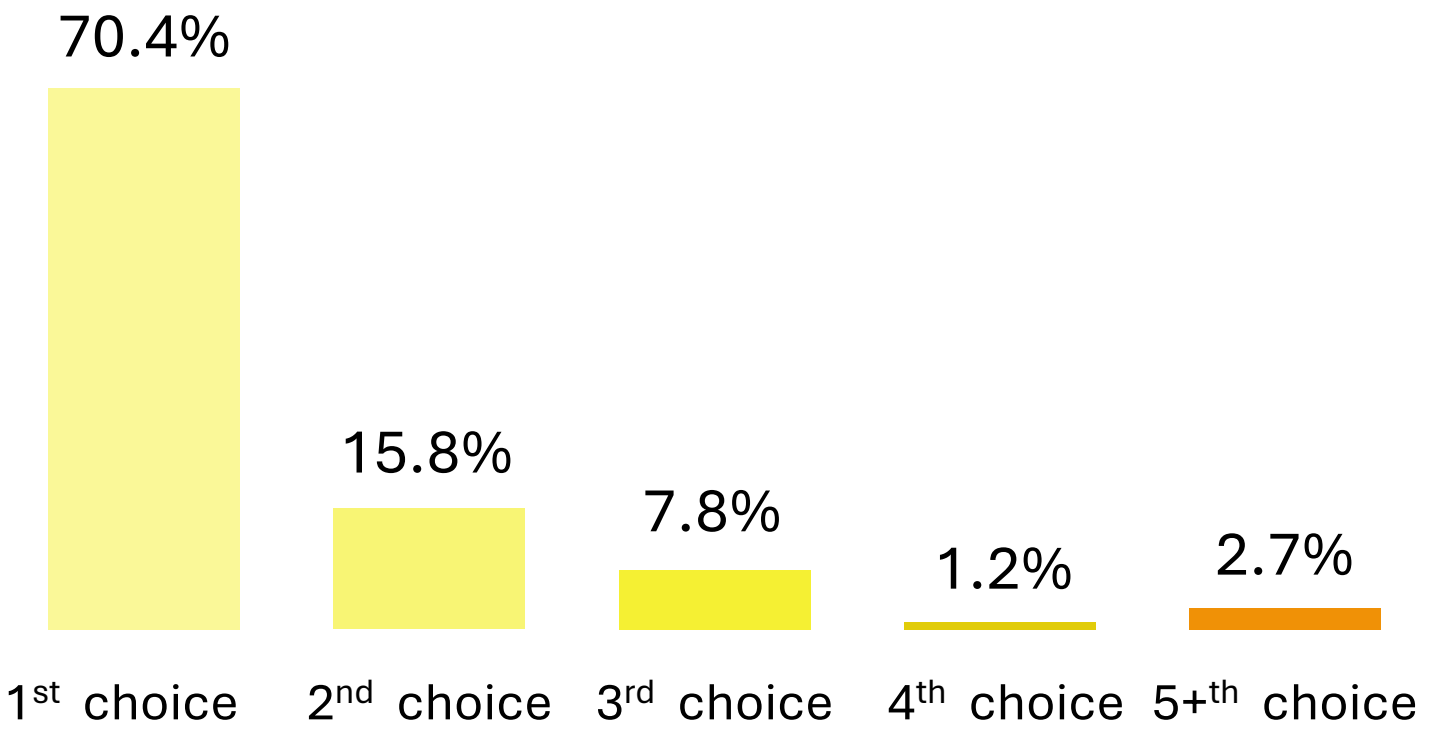
1 *Self-selected* sample



2 *Representative* sample

Observation 2: we can ask for short rankings

» We still reduce the number of lost votes if we impose short rankings



● *Self-selected* sample with the ***STV*** rule

- We can heavily reduce the number of “lost votes” using rankings instead of uninominal votes in parliamentary elections.
- Expressive ballots also inform us about the structure of the electorate, and the candidate set.

4. Learning Candidate Axes from Approval Data

Comparing Ways of Obtaining Candidate Orderings from Approval Ballots
Théo Delemazure, Chris Dong, Dominik Peters, Magdalena Tydrichova
IJCAI-2024

PhD Defense - June 11th, 2025

The political left-right axis

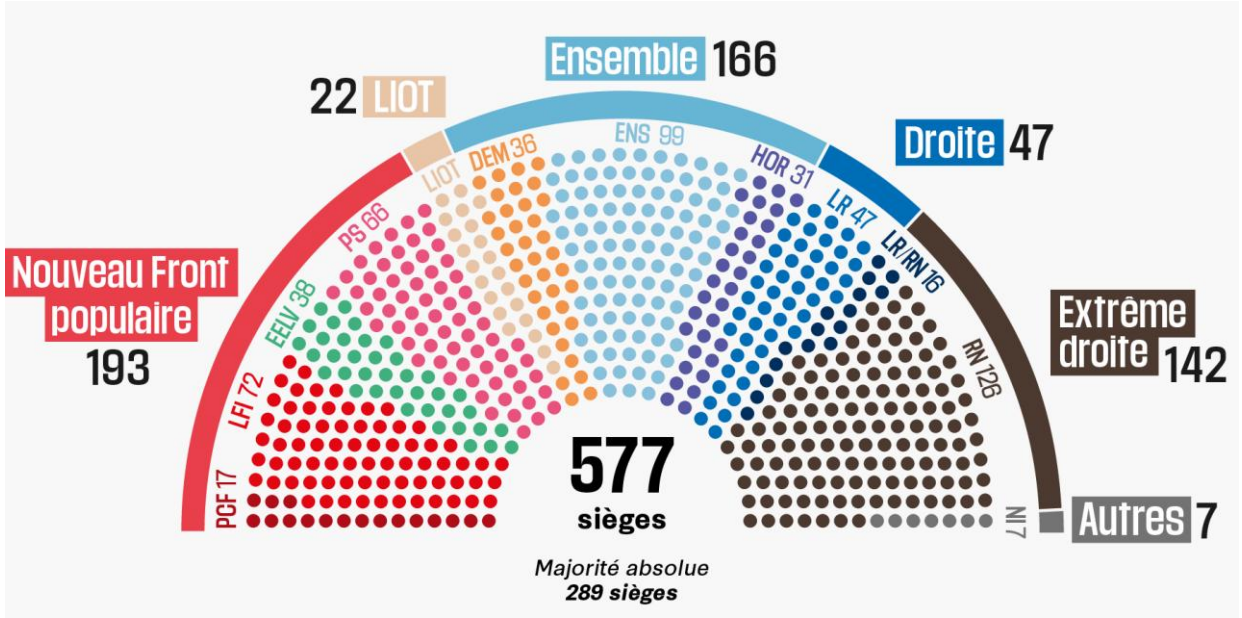
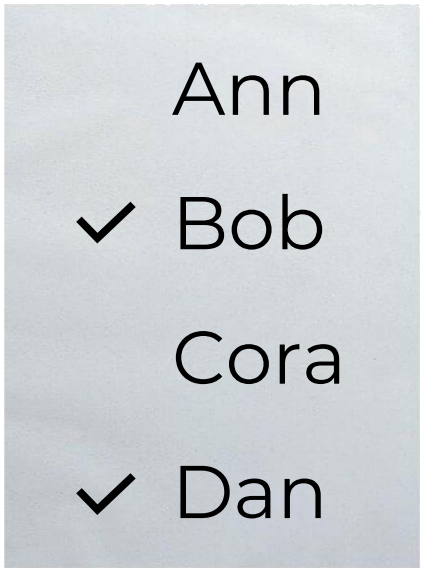
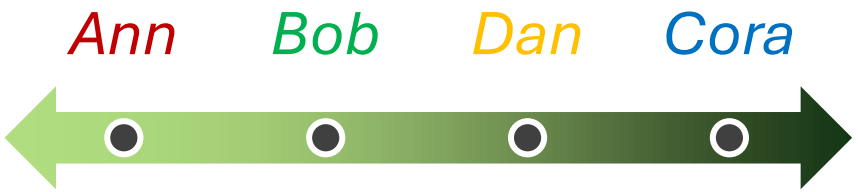


Fig. Picture of the seat distribution in the French National Assembly used by the newspaper *Libération* after the 2024 election.

We know voters' **approval ballots**

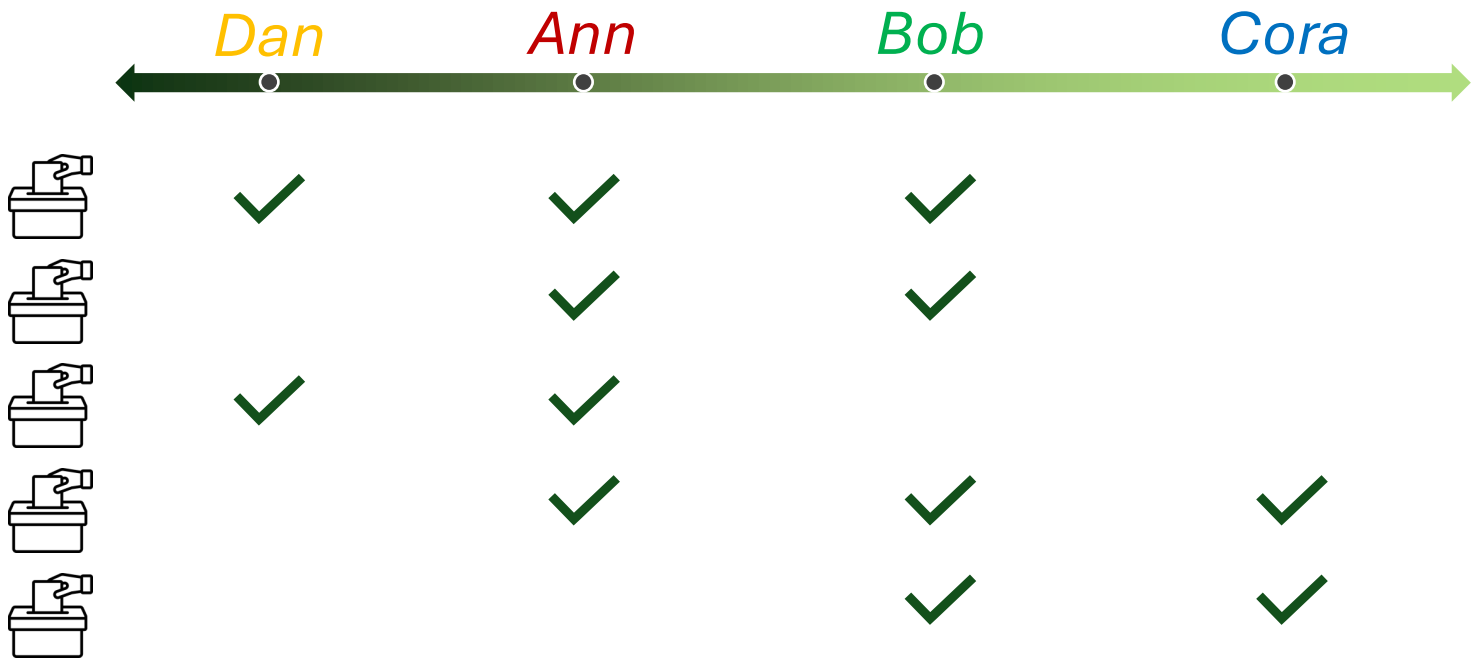


We want to find an **ordering** of the candidates



Candidate Interval Property *(Elkind & Lackner, 2015)*

Every approval ballot is an “interval” of the axis.



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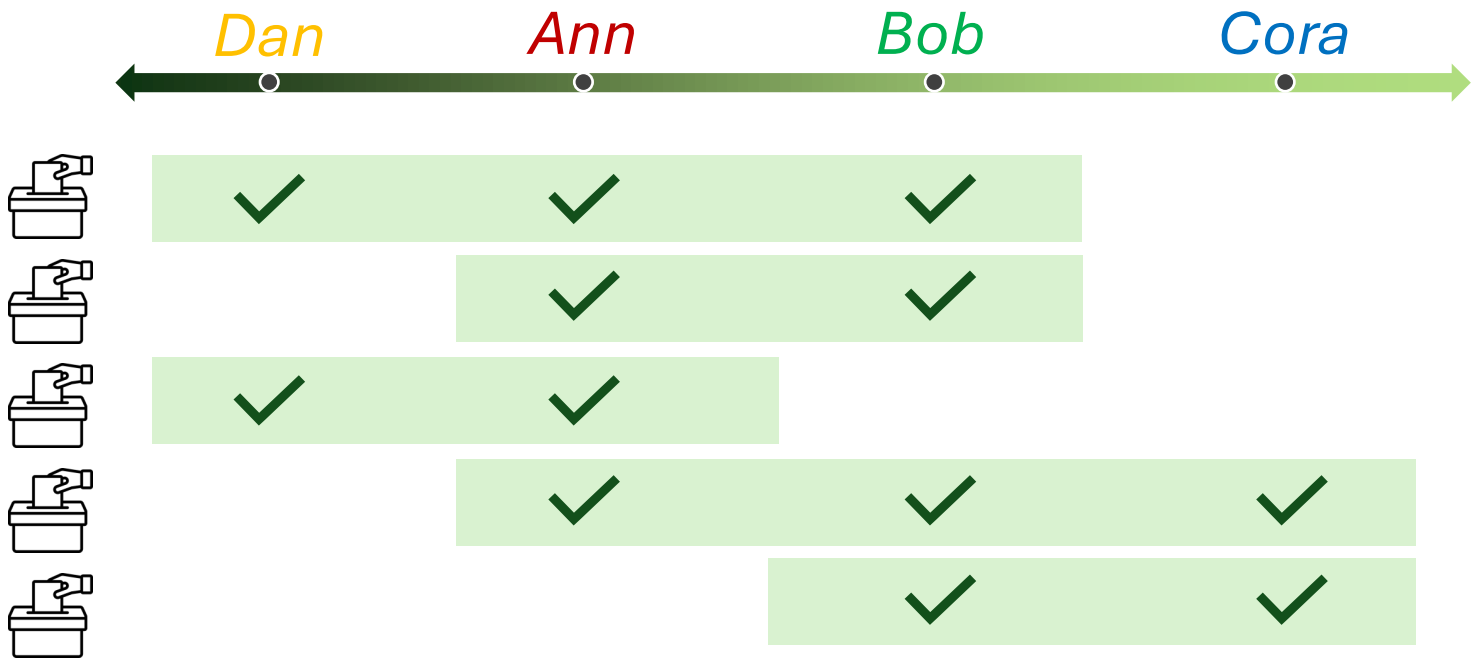
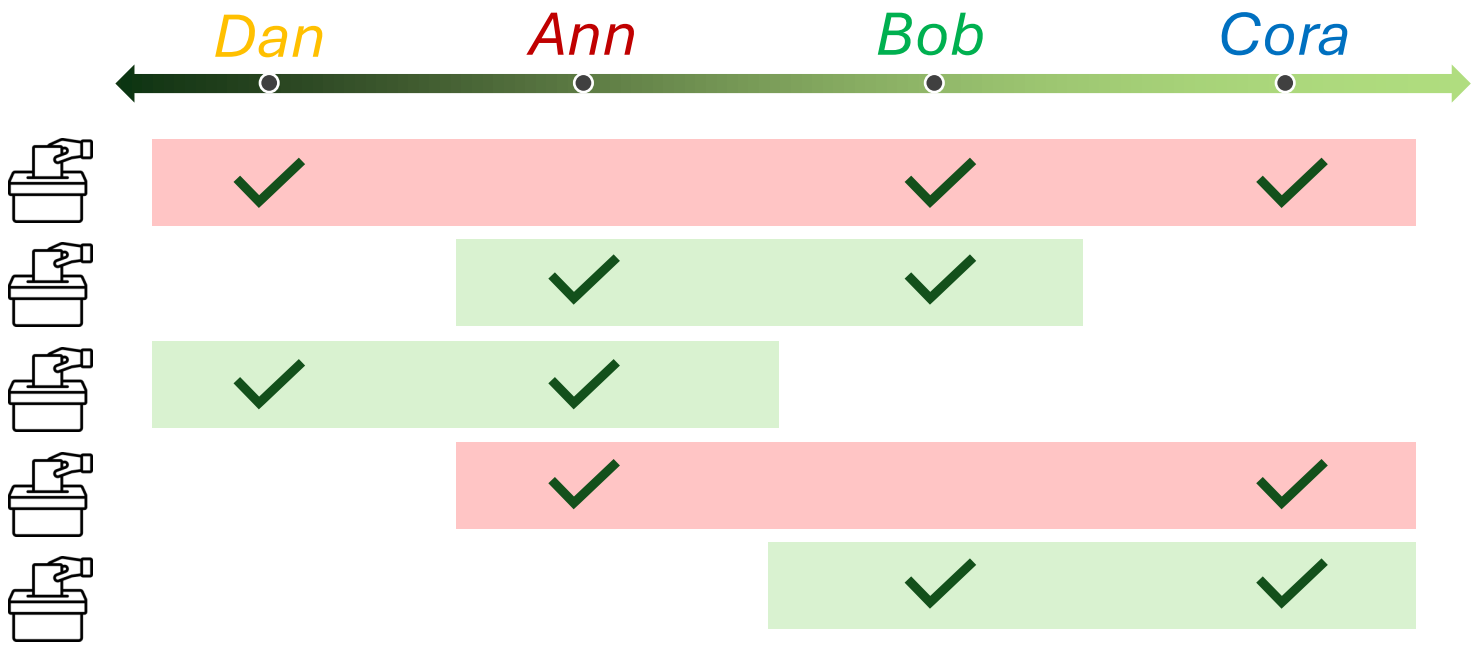


In real life, there generally is no “*perfect axis*”



Question: how to select the axis that fits the ballots the best?

- **Rule:** Voter Deletion (VD)
- **Rule:** Minimum Flips (MF)
- **Rule:** Ballot Completion (BC) *(Lebon et al, 2017)*
- **Rule:** Minimum Swaps (MS)
- **Rule:** Forbidden Triples (FT)

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Axiom: Clone proximity

Clones should be next to each other on the axis (or separated by other clones).

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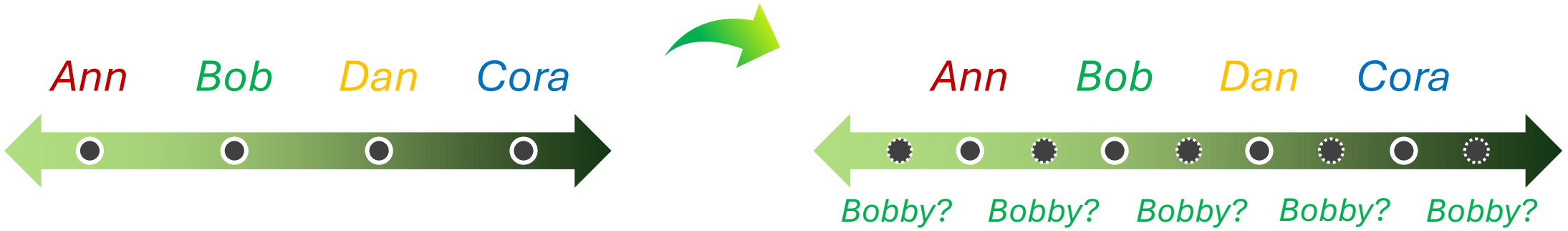
Axiom: Independence of clones

If we add clones, the relative order of other candidates should not change.



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Impossibility Theorem

No neutral axis scoring rule satisfies clone proximity, independence of clones and consistency with linearity.

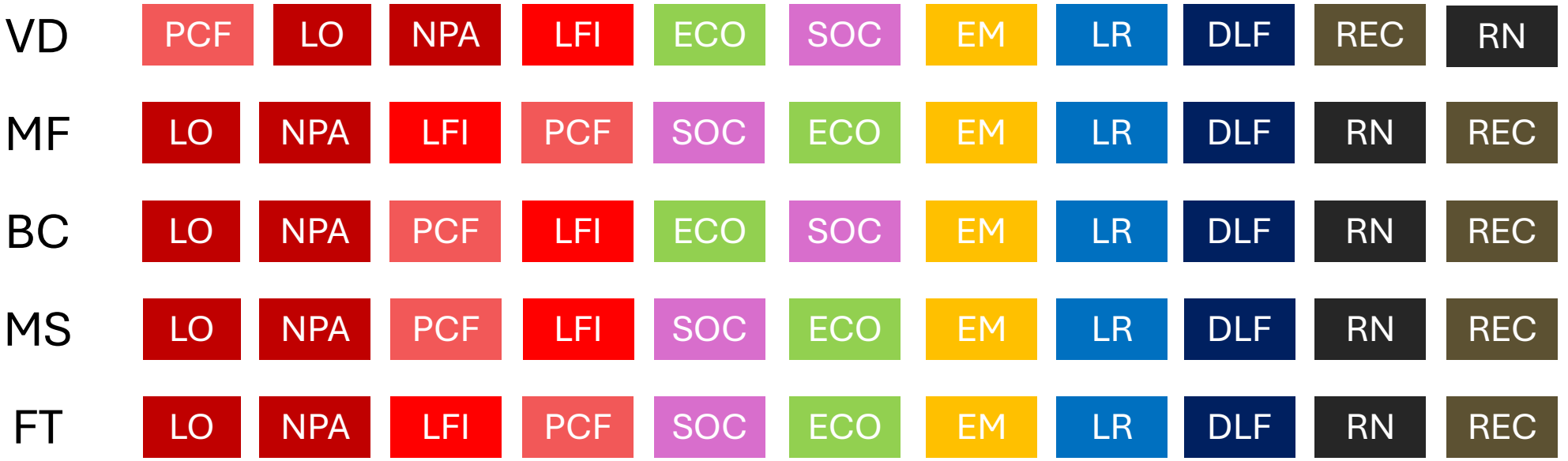
Axiomatic analysis

	VD	MF	BC	MS	FT
Consistency with linearity	✓	✓	✓	✓	✓
Independence of clones	✓	✗	✗	✗	✗
Stability	✓	✗	✗	✗	✗
Ballot monotonicity	✓	✗	✓	✗	✗
Clearance	✗	✗	✓	✓	✓
Partition consistency	✗	✗	✓	✓	✓
Veto-winner centrism	✗	✗	✗	✓	✓
Clone-proximity	✗	✗	✗	✗	✓

Fig. Axioms satisfied by the different rules.

Voter Autrement – *Online (2022)*

1 379 12



Supreme Court Of The US *(2021 term)*

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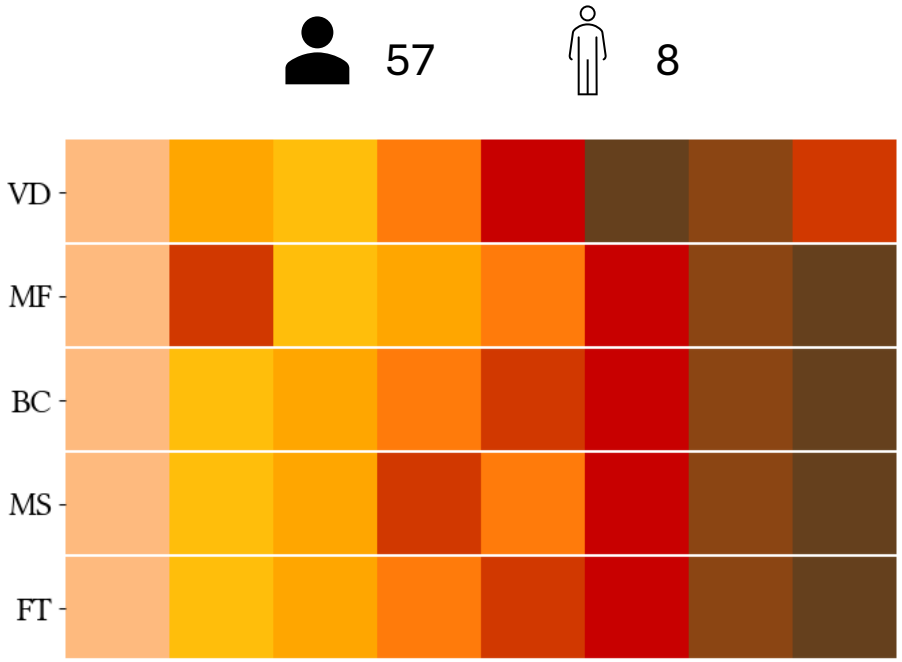
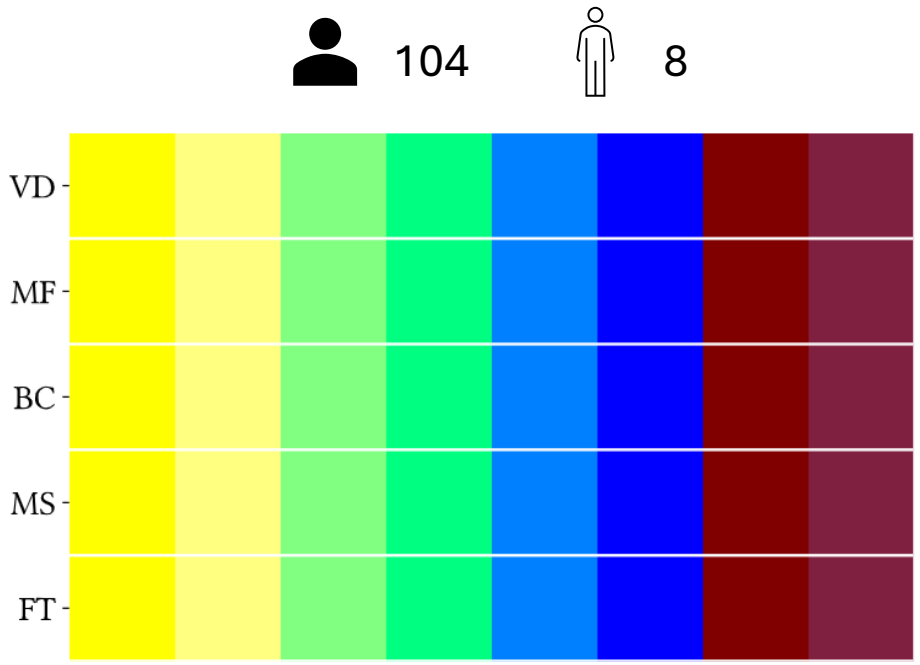
Forbidden Triples

Preferences on colors





- Approval ballots can be used to construct a unidimensional axis of the candidates (or the voters).

- Can expressive preferences also tell us which candidates are the inducing the most *conflict* among voters?

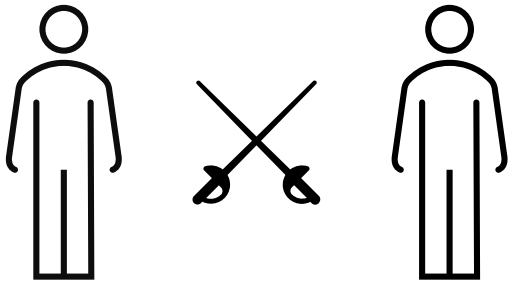
5. Identify Conflicting Pairs of Candidates via Rankings

Selecting the Most Conflicting Pair of Candidates
Théo Delemazure, Łukasz Janeczko, Andrzej Kaczmarczyk and Stanisław Szufa
IJCAI-2024

We know voters' **rankings** of the candidates

1 Eddy
2 Dan
3 Bob
4 Ann
5 Cora

We want to find the **pair of candidates** inducing the most conflict



Question: how to find the most conflictual pair of candidates?

There is **a trade-off** between different aspects of conflict

1 *Ann*	1 *Ann*	1 *Bob*
2 *Cora*	2 *Dan*	2 *Dan*
3 *Dan*	3 *Cora*	3 *Cora*
4 *Bob*	4 *Bob*	4 *Ann*
50%	25%	25%

Ann & Bob maximizes conflict **intensity**

Cora & Dan maximizes conflict **partitioning**

	MaxSum	MaxNash	MaxSwap	MaxPolar
Reverse Stability	✓	✓	✓	✓
Conflict Consistency	✓	✓	✓	✓
Conflict Monotonicity	✗	✗	✗	✗
Antagonization Consistency	✓	✓	✓	✓
Matching Domination	✓	✓	✗	✓
Balance Preference	✗	✓	✓	✗

Fig. Conflict rules, and the axioms they satisfy.

Experimental analysis: Voter Autrement

Voter Autrement – *Online (2022)*

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MaxSwap	LO	NPA	LFI	PCF	SOC	ECO	EM	LR	DLF	RN	REC
MaxSum	LO	NPA	LFI	PCF	SOC	ECO	EM	LR	DLF	RN	REC
MaxNash	LO	NPA	LFI	PCF	SOC	ECO	EM	LR	DLF	RN	REC

Concluding Remarks

- Inform **the policy makers** on alternative voting methods (via think tanks or directly talking to them).
- Make the **non-academic public** aware of our works and convince them of the value of our methods (events, blog posts, social media).
- Make **datasets of expressive preferences** available for researchers and other interested people to use.
- Use the axiomatic and experimental approaches to analyze **other forms** of collective decision processes, like *citizens' assemblies*?

Conclusion and further works

1. Approval with Runoff

2. Instant Runoff Voting with Indifferences

3. Reducing lost votes in Parliamentary election with thresholds

4. Learning candidate axes from approval data

5. Identify conflicting pairs of candidates via rankings

+. Omitted works:

- *Liquid Democracy with Ranked Delegations* (Brill et al) AAAI-2022
- *Strategy-Proofness and Proportionality in Party-Approval Multi-Winner Elections* (Delemazure et al) AAAI-2023
- *Measuring a Priori Voting Power -- Taking Delegations Seriously* (Colley et al) IJCAI-2023
- *Aggregating Correlated Estimations with (Almost) no Training* (Delemazure et al) ECAI-2023
- *Independence of Irrelevant Alternatives under the Lens of Pairwise Distortion* (Delemazure et al) AAAI-2024
- *Are Alternative Voting Methods Ideologically Biased? Insights from the 2022 Italian Election* (Marsilio & Delemazure) Working Paper
- *Do Grades Have Absolute Meaning? An Experiment on Majority Judgment* (Baujard et al) Working Paper

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...and to my **co-authors** !

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