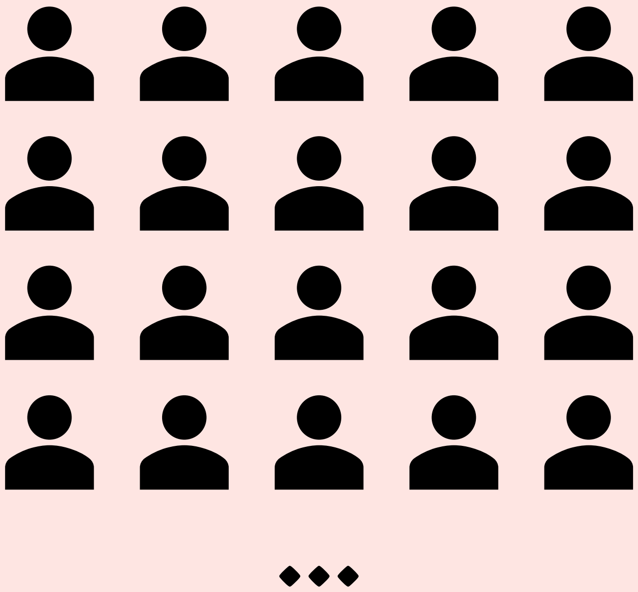
Friends or Foes?
Inferring Latent Candidate Structures from Preference Rankings

Théo Delemazure – ILLC, University of Amsterdam

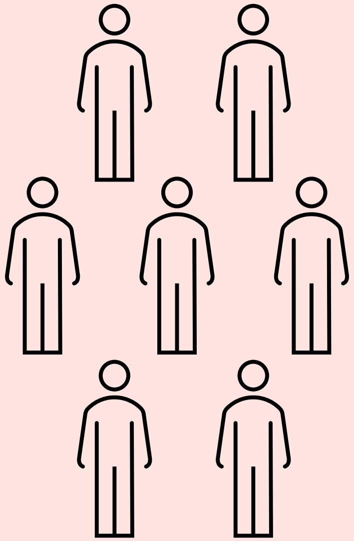
- 0 -
Learning From Preferences

Computational Social Choice Setting

Voters

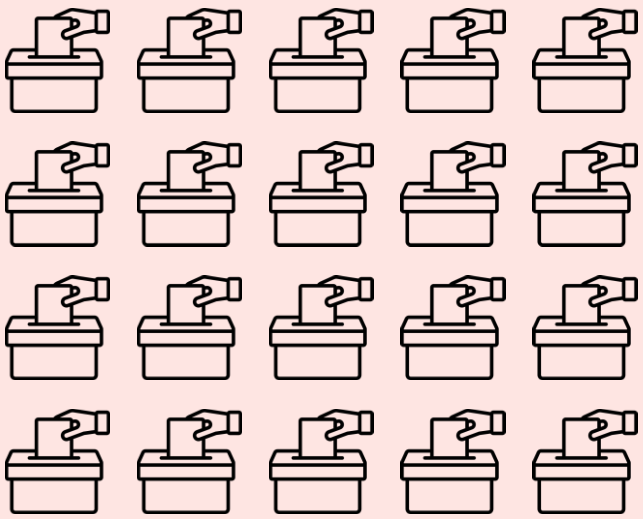


Candidates



Computational Social Choice Setting

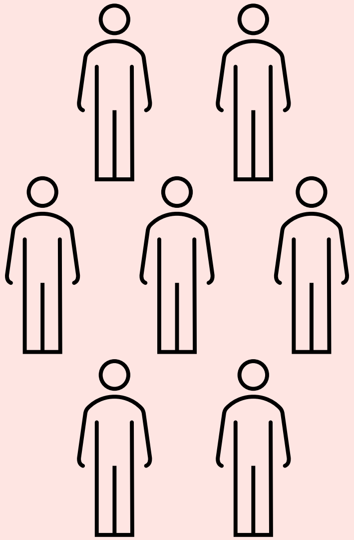
Voters give their preferences over candidates



…

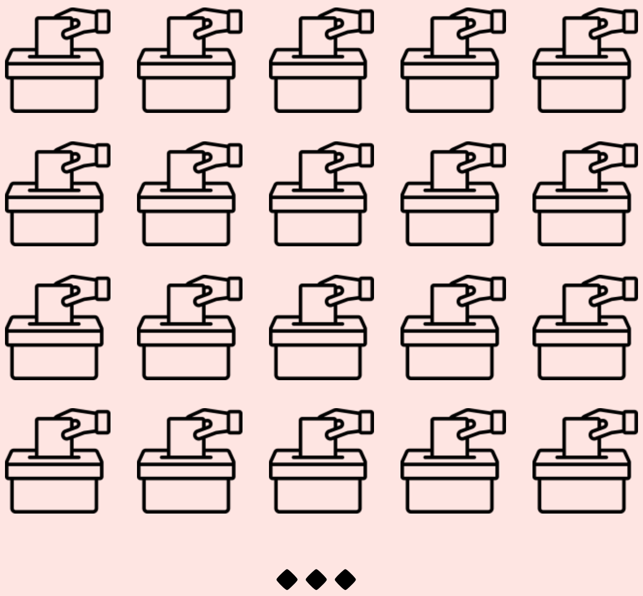
$A \succ B \succ C \succ D \succ E$

Candidates



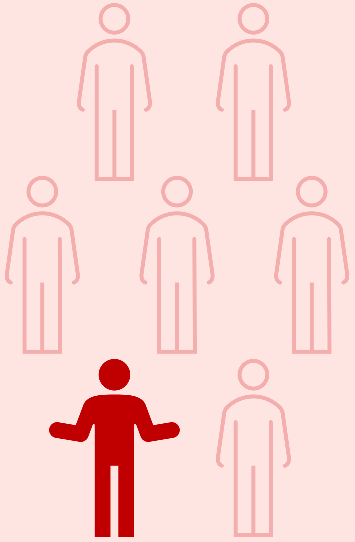
Computational Social Choice Setting

Voters give their preferences over candidates



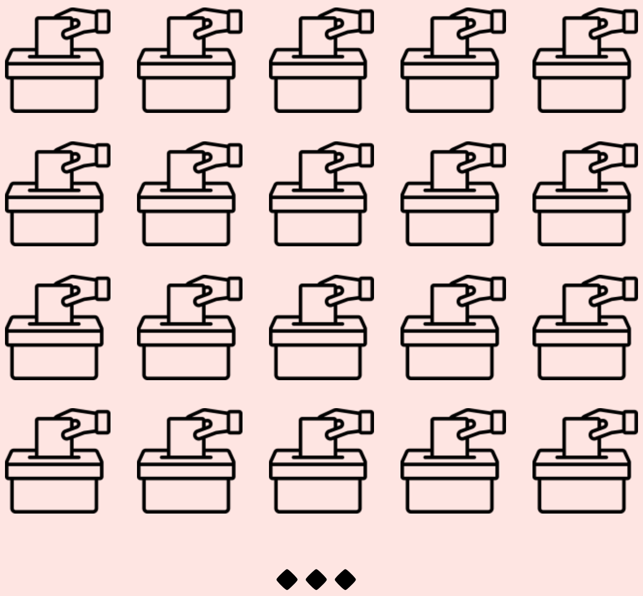
Voting rule

A **winner** is selected

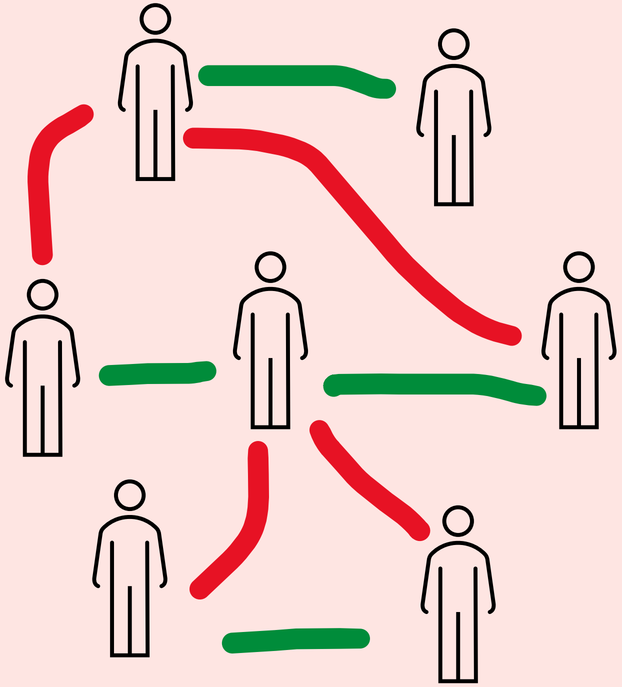


Learning Structures from Preferences

Voters give their preferences over candidates



Structure of the electorate/candidate set



Different **Levels** of Structure

At the **full profile** level:

50% $A \succ B \succ C \succ D \succ E$

50% $E \succ D \succ C \succ B \succ A$

“This society is polarized as a whole”

[Can et al, 2015] Measuring polarization in preferences. *Mathematical Social Sciences.*
[Faliszewski et al, 2023] Diversity, agreement, and polarization in elections. *IJCAI 2023.*
[Hashemi and Endriss, 2014] Measuring diversity of preferences in a group. *ECAI 2014.*

Different **Levels** of Structure

At the **candidate/proposal** level:

50% $A \succ B \succ C \succ D \succ E$

50% $B \succ C \succ D \succ E \succ A$

*"**A** is very polarizing in the society"*

[Colley et al, 2023] Measuring and controlling divisiveness in rank aggregation. *IJCAI 2023.*
[Endriss, 2025] On the Difficulty of Measuring Divisiveness of Proposals under Ranked Preferences. *Working paper.*

Different Levels of Structure

At the **candidate pairs** level:

50% $A \succ C \succ D \succ E \succ B$

50% $B \succ C \succ D \succ E \succ A$

*“**A** and **B** are inducing a lot of conflict in the society”*

[Delemazure et al., 2024] Selecting the Most Conflicting Pair of Candidates. *IJCAI 2024.*

Types of Interactions

$A \succ C \succ D \succ E \succ B$

$A \succ D \succ C \succ E \succ B$

$A \succ E \succ D \succ C \succ B$

$B \succ E \succ D \succ C \succ A$

$B \succ D \succ C \succ E \succ A$

$B \succ C \succ D \succ E \succ A$

Conflict

$A \succ B \succ C \succ D \succ E$

$D \succ B \succ A \succ C \succ E$

$E \succ D \succ B \succ A \succ C$

$B \succ A \succ E \succ D \succ C$

$D \succ C \succ E \succ B \succ A$

$C \succ B \succ A \succ D \succ E$

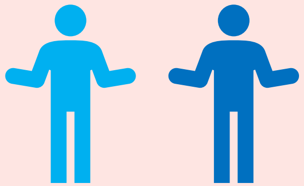
Proximity

Question: how to **measure** how conflicting/similar two candidates are based on ranked preferences of voters?

Question: how to **find the pairs** of candidates that causes the most conflict/that are the most similar?

Methodology

We define measures that assign **a conflict/similarity score** to pairs of candidates



$S(A, B) = 0.8$

How to compare these measures and decides which one should be used?

☛ **Axiomatic Analysis**

Which measures satisfy which desirable normative properties?

☛ **Experimental Analysis**

How do the measures behave in practice on actual preference data?

Outline of the Talk

Selecting the Most Conflicting Pair of Candidates *(IJCAI 2024)*
Théo Delemazure, Łukasz Janeczko, Andrzej Kaczmarczyk, and Stanisław Szufa

0 **Introduction:** Learning from Preferences

1 Finding the **Most Divisive** Pair of Candidates

- Measures
- Rules
- Axioms
- Experiments

2 Finding **Approximate Clones**

- Measures
- Axioms
- Experiments
- Link to Independence of Clones

Independence of Approximate Clones *(MPREF-2025)*
Théo Delemazure

- 1 -
What does it mean for two candidates to induce conflict?

Formal model

We have:

- Set of **voters** $\boldsymbol{V} = \{\mathbf{1}, \mathbf{2}, \dots, \boldsymbol{n}\}$ and **candidates** $\boldsymbol{C} = \{\boldsymbol{c_1}, \dots, \boldsymbol{c_m}\}$.
- A preference **profile** $\boldsymbol{P} = (\succ_{\mathbf{1}}, \dots, \succ_{\boldsymbol{n}})$ of rankings of voters over candidates.

We want:

- **Conflict measures** σ that given a profile P associate to each pair of candidates $a, b \in C$ a conflict value $\sigma_P(a, b) \in [0,1]$.
- **Conflict rules** f that give for each profile P the most conflicting pair(s) of candidates.

Extra notations

Rank of candidate A in ranking of voter i: $\mathrm{rank}_i(A)$.
"Distance" between A and B in ranking of voter i:

$$d_i(A,B) = |\mathrm{rank}_i(A) - \mathrm{rank}_i(B)|$$

Example

$$A \succ_i B \succ_i C \succ_i D \succ_i E \succ_i F$$

$\mathrm{rank}_i(A) = 1$ $\mathrm{rank}_i(D) = 4$ $d_i(A,D) = 3$

Notions of Conflict

$$A \succ B \succ C \succ D$$
$$A \succ B \succ C \succ D$$
$$A \succ B \succ C \succ D$$
$$A \succ B \succ C \succ D$$
$$D \succ C \succ B \succ A$$
$$D \succ C \succ B \succ A$$
$$D \succ C \succ B \succ A$$
$$D \succ C \succ B \succ A$$

Conflict Intensity: A and **D** are inducing more conflict if they are distant in voters' preferences.

Conflict Partitioning: A and **D** are inducing more conflict if they split the voters more evenly.

Notions of Conflict

Conflict intensity

$$\beta_P(A, B) = \frac{1}{n(m-1)} \sum_{i \in V} d_i(A, B) \in [0,1]$$

Conflict partitioning

Number of voters preferring A to B.

$$\alpha_P(A, B) = \frac{2}{n} \min(N^{A \succ B}, N^{B \succ A}) \in [0,1]$$

Trade-off between them

$$A \succ C \succ D \succ B$$

$$A \succ C \succ D \succ B$$

$$A \succ D \succ C \succ B$$

$$B \succ D \succ C \succ A$$

A and **B** have maximal conflict intensity

C and **D** have maximal conflict partitioning

A third notion: Conflict balance

$$A \succ C \succ E \succ F \succ G \succ D \succ B$$

$$A \succ C \succ E \succ F \succ G \succ D \succ B$$

$$E \succ D \succ B \succ F \succ A \succ C \succ G$$

$$E \succ D \succ B \succ F \succ A \succ C \succ G$$

(A,B) and (C,D) have **the same value** of conflict intensity and conflict partitioning. However, (C,D) is more balanced.

A third notion: Conflict balance

Conflict balance

$$\gamma_P(A, B) \in [0,1]$$

$$\phi_P(A, B) \in [0,1]$$

- 2 -
Finding the most conflicting pair of candidates

Conflict Rules from Measures

A simply way to create conflict rules is to select the pair that maximizes **some increasing function** of the conflict measures:

$$\text{argmax}_{A,B \in C^2} f(\alpha_P, \beta_P, \gamma_P, \phi_P)$$

Rule: p-MaxPolarization Rule

The p-MaxPolar rule with $p > 0$ selects the pairs maximizing $\alpha_P \cdot (\beta_P)^p$.

Conflict Rules from Swap Distance

How many swaps of adjacent candidates need to be done to have a profile in which all voters agree on the ordering of **A** and **D**?

$$\boldsymbol{D} \succ C \succ B \succ \boldsymbol{A}$$
$$\boldsymbol{A} \succ B \succ C \succ \boldsymbol{D}$$
$$B \succ C \succ \boldsymbol{D} \succ \boldsymbol{A}$$
$$B \succ \boldsymbol{A} \succ C \succ \boldsymbol{D}$$

Conflict Rules from Swap Distance

How many swaps of adjacent candidates need to be done to have a profile in which all voters agree on the ordering of **A** and **D**?

$$D \succ C \succ B \succ A$$

$$A \succ B \succ C \succ D$$

$$B \succ C \succ D \succ A$$

$$B \succ A \succ C \succ D$$

1 2 3

4

Conflict Rules from Swap Distance

Rule: MaxSwapConflict

This rule selects the pairs **maximizing the minimal number of swaps** required to have all voters agree on the ordering of the pair.

Conflict Rules

- **Rule:** p-MaxPolarization
- **Rule:** MaxSumConflict
- **Rule:** MaxNashConflict
- **Rule:** MaxSwapConflict

Question: What are the behavioral differences between these rules? Which one should we use?

Fundamental Axioms

Axiom: Reverse Stability

The outcome should not change if all rankings are reversed.

Satisfy

All

$$D \succ B \succ C \succ A \qquad D \prec B \prec C \prec A$$
$$A \succ B \succ C \succ D \quad = \quad A \prec B \prec C \prec D$$
$$B \succ C \succ D \succ A \qquad B \prec C \prec D \prec A$$
$$B \succ A \succ C \succ D \qquad B \prec A \prec C \prec D$$

Matching Domination

Axiom: Matching Domination

If a pair (a, b) matching-dominates a pair (x, y), then the pair (x, y) should **not** be selected.

✓ Satisfy	✗ Break
Polar	**Swap**
Sum	
Nash	

$$A \succ X \succ Y \succ B$$
$$A \succ X \succ Y \succ B$$
$$A \succ X \succ Y \succ B$$
$$Y \succ B \succ X \succ A$$
$$B \succ Y \succ A \succ X$$

→

(A,B) dominates (X,Y). Thus, (X,Y) should not be selected.

The Paradox of Monotonicity

Break

All

Axiom: Conflict Monotonicity

If a pair (a, b) is selected in a profile of rankings, it should still be selected if we increase the conflict between a and b in the profile.

$$A \succ B \succ C \succ D$$
$$B \succ A \succ D \succ C$$

(A,B) is selected.

→

$$A \succ B \succ C \succ D$$
$$B \succ D \succ C \succ A$$

(A,B) **should be** selected.

The Paradox of Monotonicity

Break

All

Axiom: Conflict Monotonicity

If a pair (a, b) is selected in a profile of rankings, it should still be selected if we increase the conflict between a and b in the profile.

$$A \succ B \succ C \succ D$$
$$B \succ A \succ D \succ C$$

(A,B) is selected.

$$A \succ B \succ C \succ D$$
$$B \succ D \succ C \succ A$$

(A,D) is selected.

The Paradox of Monotonicity

Impossibility Theorem: No rule can satisfy (1) conflict consistency, (2) matching domination and (3) conflict monotonicity.

Summary of the Axiomatic Analysis

	Polar	Sum	Nash	Swap
Conflict Consistency	✔	✔	✔	✔
Reverse Stability	✔	✔	✔	✔
Conflict Monotonicity				
Matching Domination	✔	✔	✔	
Balance Preference			✔	✔
Uniform Reinforcement				✔

- 3 -
Experiments

Experiments: Voter Autrement

- French presidential elections (2017, 2022)
- Online survey
- Testing alternative voting methods

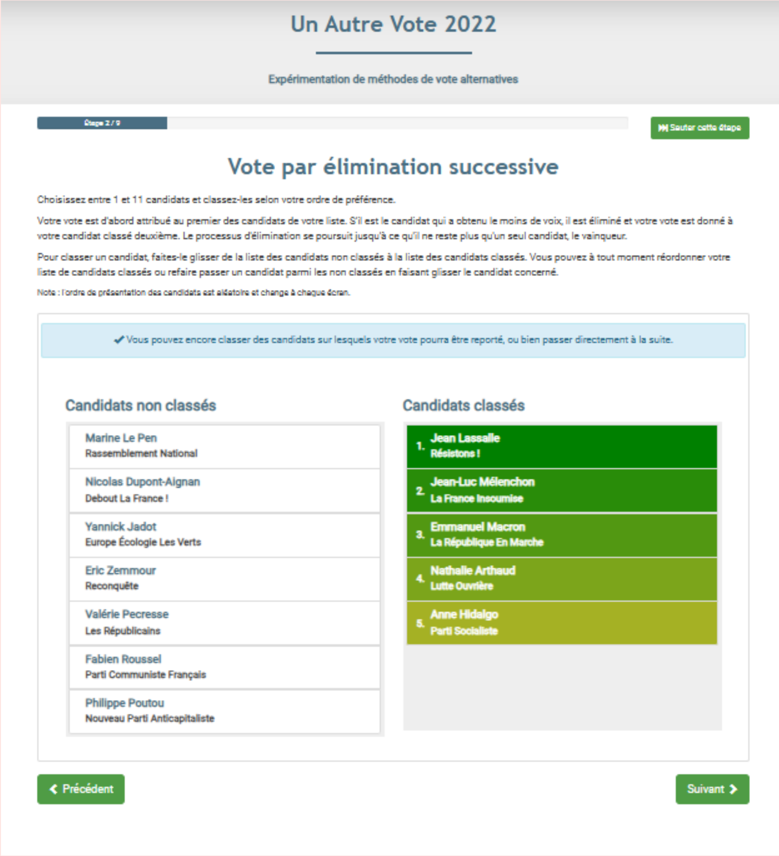
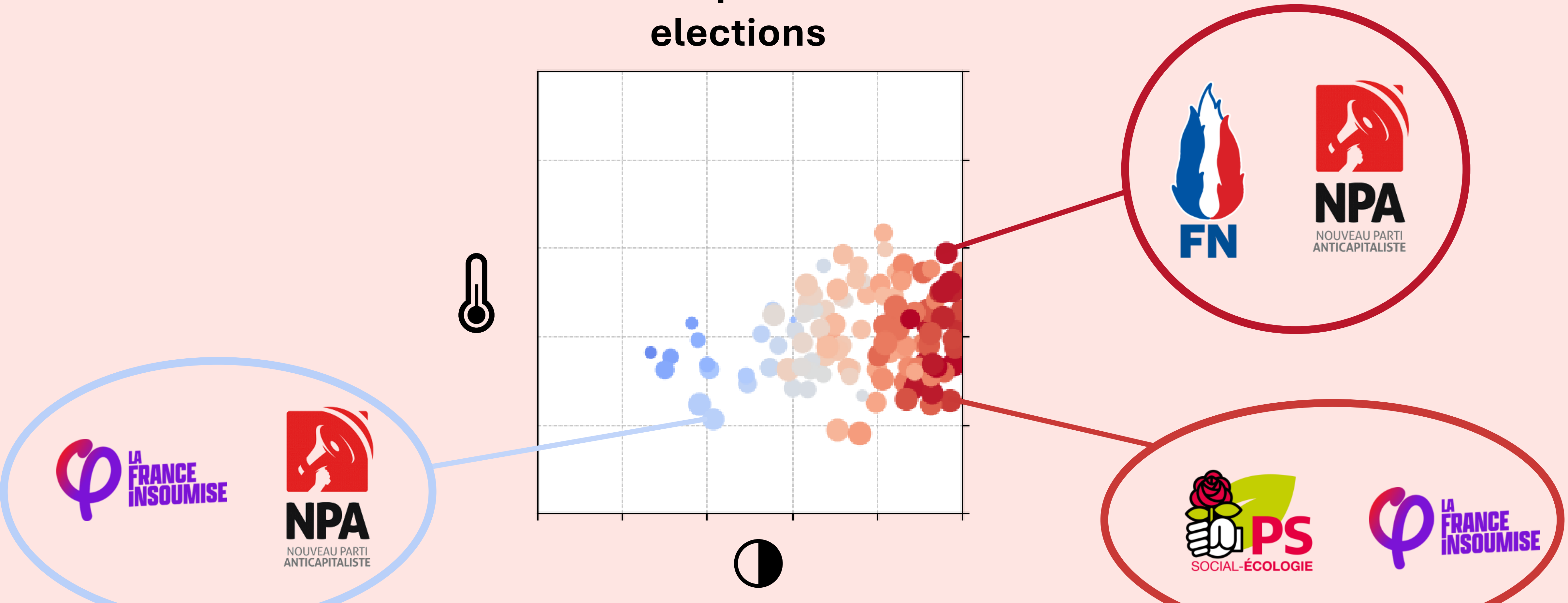


Fig. Website vote.imag.fr

Experiments: Voter Autrement

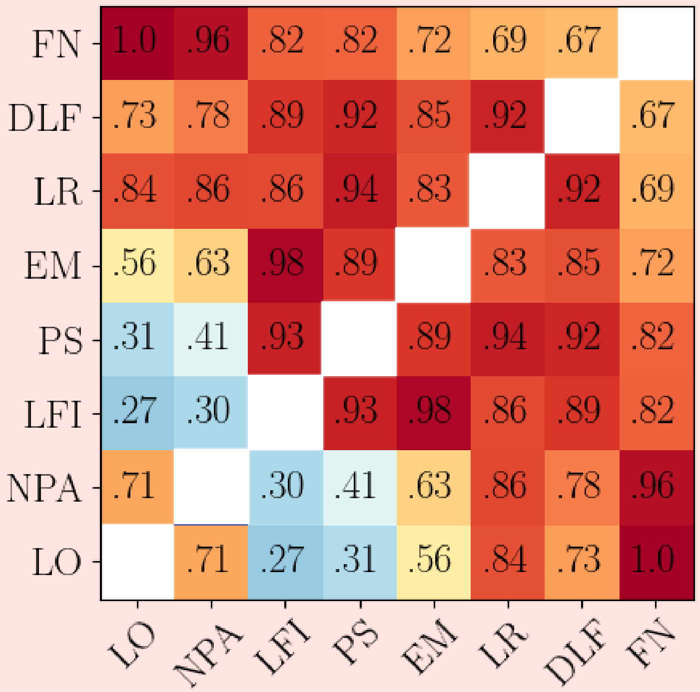
Preferences over candidates
In **French presidential elections**

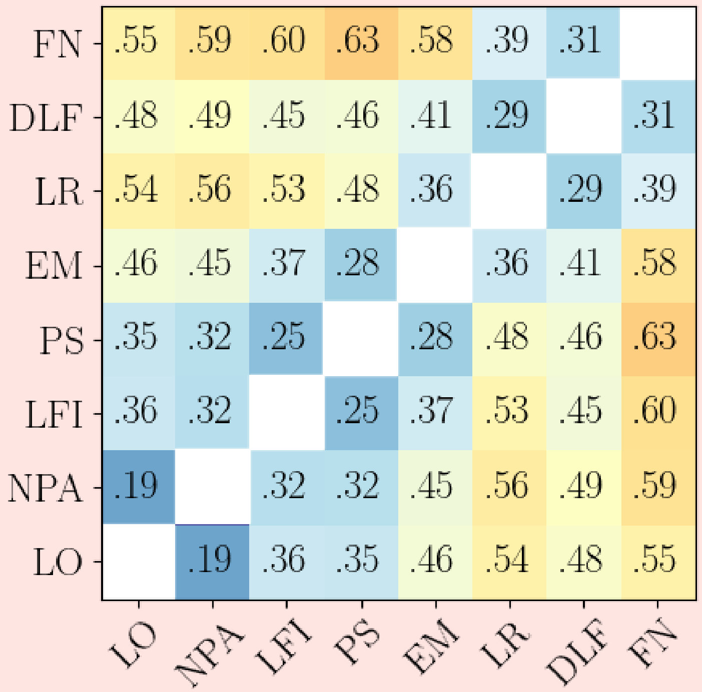


Experiments: Voter Autrement

Left-right axis

LO | NPA | LFI | PCF | SOC | ECO | EM | LR | DLF | RN | REC





Experiments: Voter Autrement

	LO	NPA	LFI	PCF	SOC	ECO	EM	LR	DLF	RN	REC
2-MaxPolar	LO	**NPA**	LFI	PCF	SOC	ECO	EM	LR	DLF	**RN**	REC
MaxSwap	LO	**NPA**	LFI	PCF	SOC	ECO	EM	LR	DLF	RN	**REC**
MaxSum	LO	**NPA**	LFI	PCF	SOC	ECO	EM	LR	DLF	**RN**	REC
MaxNash	LO	NPA	**LFI**	PCF	SOC	ECO	EM	LR	DLF	**RN**	REC

- 4 -
Finding Approximate Clones

Formal model

We have:

- Set of **voters** $\boldsymbol{V} = \{\boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{n}\}$ and **candidates** $\boldsymbol{C} = \{\boldsymbol{c_1}, \dots, \boldsymbol{c_m}\}$.
- A preference **profile** $\boldsymbol{P} = (\succ_{\boldsymbol{1}}, \dots, \succ_{\boldsymbol{n}})$ of rankings of voters over candidates.

We want:

- **Closeness measures** σ that given a profile P associate to each pair of candidates $a, b \in C$ a closeness value $\sigma_P(a, b) \geq 0$.

Perfect Clones

$A \succ B \succ C \succ D \succ E$

$D \succ B \succ A \succ C \succ E$

$E \succ D \succ B \succ A \succ C$

$B \succ A \succ E \succ D \succ C$

$D \succ C \succ E \succ B \succ A$

$C \succ B \succ A \succ D \succ E$

In this preference profile, A and B are perfect clones: they are always ranked next to each other.

Approximate Clones

$A \succ B \succ C \succ D \succ E$

$D \succ B \succ A \succ C \succ E$

$E \succ D \succ B \succ A \succ C$

✕ $B \succ E \succ A \succ D \succ C$

✕ $D \succ B \succ C \succ E \succ A$

$C \succ B \succ A \succ D \succ E$

In practice, perfect clones **are very rare**. How to still quantify the closeness of a pair of candidates to being clones?

Approximate Clones: the simplest measure

α-deletion clones

Two candidates are ***α*-deletion clones** if we can remove a α portion of the profile such that they become perfect clones.

$A \succ B \succ C \succ D \succ E$

$D \succ B \succ A \succ C \succ E$

$E \succ D \succ B \succ A \succ C$

✖ $B \succ E \succ A \succ D \succ C$

✖ $D \succ B \succ C \succ E \succ A$

$C \succ B \succ A \succ D \succ E$

$$\alpha = \frac{1}{3}$$

[Janeczko et al., 2024] Discovering Consistent Subelections. *AAMAS 2024.*
[Faliszewski et al, 2026] Identifying Imperfect Clones in Elections. *AAAI 2026.*
[Delemazure, 2026] Independence of Approximate Clones. *Working Paper 2026.*

Candidates Closeness

$$A \succ B \succ C \succ D \succ E \succ F \succ G$$
$$A \succ B \succ C \succ D \succ E \succ G \succ F$$
$$A \succ B \succ C \succ D \succ F \succ G \succ E$$
$$A \succ B \succ C \succ D \succ F \succ E \succ G$$
$$A \succ D \succ E \succ C \succ F \succ G \succ B$$
$$A \succ D \succ E \succ C \succ F \succ G \succ B$$

Pairs (**A**,**B**) and (**C**,**D**) have the same value of α.

Another approximate clone measure

$\boldsymbol{\beta}$-swap clones

Two candidates are **$\boldsymbol{\beta}$-swap clones** if we can perform an average* of β swaps per voter such that they become perfect clones.

$A \succ B \succ C \succ D \succ E$

$D \succ B \succ A \succ C \succ E$

$E \succ D \succ B \succ A \succ C$

⊗ $B \succ E \succ A \succ D \succ C$

⊗ $D \succ B \succ C \succ E \succ A$

$C \succ B \succ A \succ D \succ E$

$$\beta = \frac{3}{6}$$

[Faliszewski et al, 2026] alternatively consider the ***maximum*** instead of the ***average***.

Link with conflict intensity

β-swap clones

$$\beta = \frac{1}{n}\sum_{i \in V}(d_i(a,b) - 1)$$

Conflict intensity

$$\beta = \frac{1}{n(m-1)}\sum_{i \in V} d_i(a,b)$$

Link with conflict intensity

β-swap clones

$$\beta = \frac{1}{n}\sum_{i \in V}(d_i(a,b) - 1)$$

Conflict intensity

$$\beta = \frac{1}{n(m-1)}\sum_{i \in V} d_i(a,b)$$

What about partitioning and balance?

Axioms: Monotonicity

Axiom: Closeness Monotonicity

If a pair (a, b) is the closest to being clones in a profile of rankings, it should still be the closest if we make them closer in the profile

Satisfy

All

$$A \succ C \succ D \succ B \succ E \quad \rightarrow \quad A \succ B \succ C \succ D \succ E$$

Axioms: Matching-domination

Axiom: Matching Domination

If a pair (a, b) matching-dominates a pair (x, y), then the pair (x, y) should **not** be selected.

Satisfy	Break
Swap	**Delet**
	Max

$$A \succ B \succ X \succ Y \succ E$$
$$A \succ B \succ X \succ Y \succ E$$
$$A \succ B \succ X \succ Y \succ E$$
$$Y \succ B \succ E \succ A \succ X$$

(**A**,**B**) dominates (**X**,**Y**). Thus, (**X**,**Y**) should not be selected.

In real datasets: Scottish elections

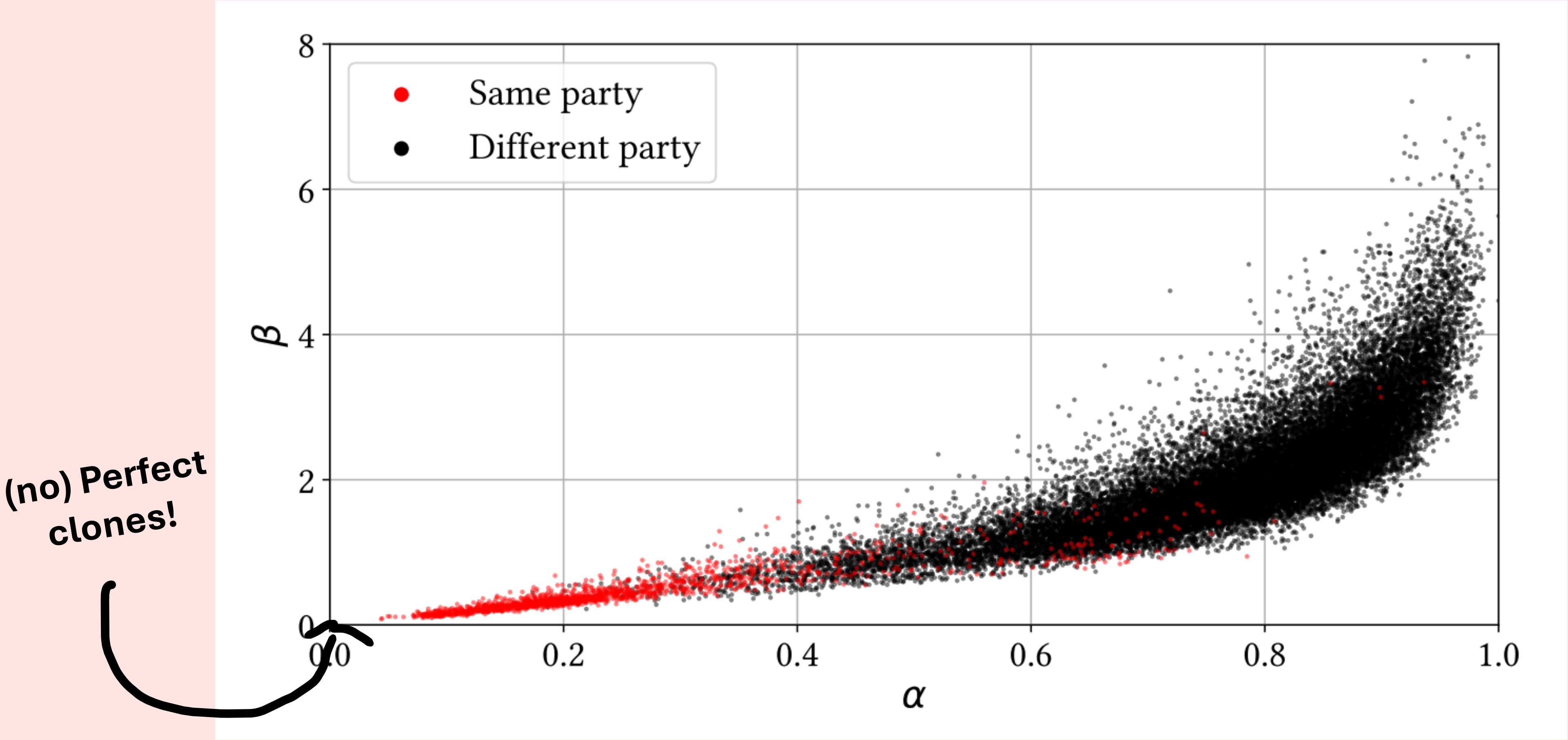


Fig. Values of closeness measures of the pairs in the Scottish elections dataset.

- / -
Further works

Related and Further works

Bigger sets instead of pairs.

Computational complexity of finding these sets of candidates.

[Faliszewski et al, 2026] Identifying Imperfect Clones in Elections. *AAAI 2026*.

Other settings (approval ballots, budget allocation…)

[Delemazure et al, 2026] Detecting Approximate Clones under Approval Voting. *AAMAS 2026*.

Going from perfect clones to approximate clones

[Delemazure, 2025] Independence of Approximate Clones. *MPREF 2025*.

Thanks for your attention!

Questions?

Théo Delemazure – ILLC, University of Amsterdam

ADDI