

Reallocating Wasted Votes in Proportional Parliamentary Elections with Thresholds

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Paris School of Economics

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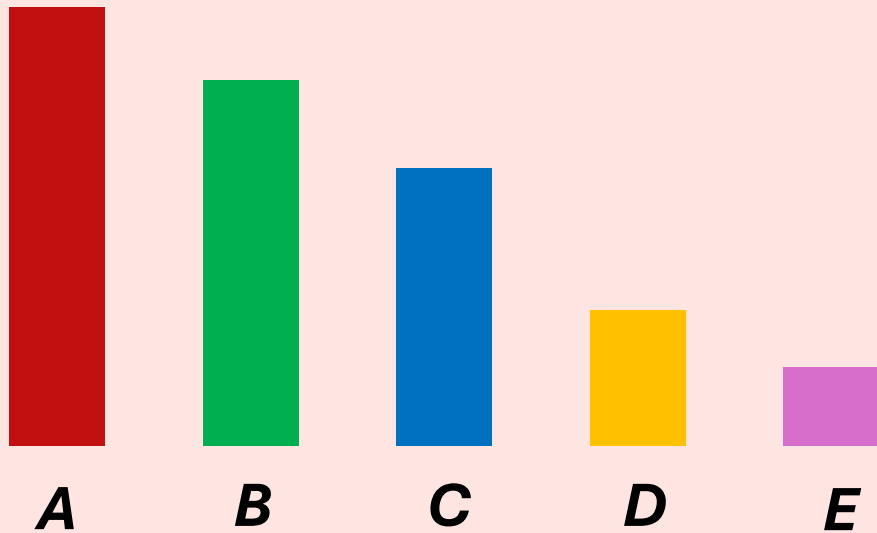
Introduction and motivation

Electing a **parliament**

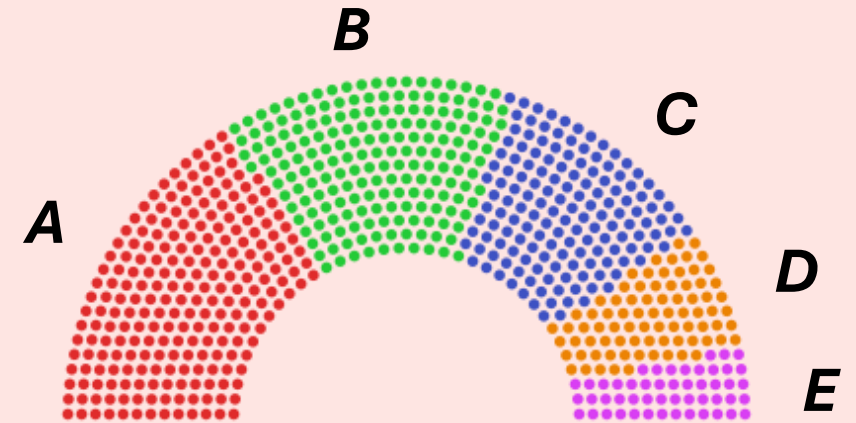


Proportional representation

Voters vote for **one** party.

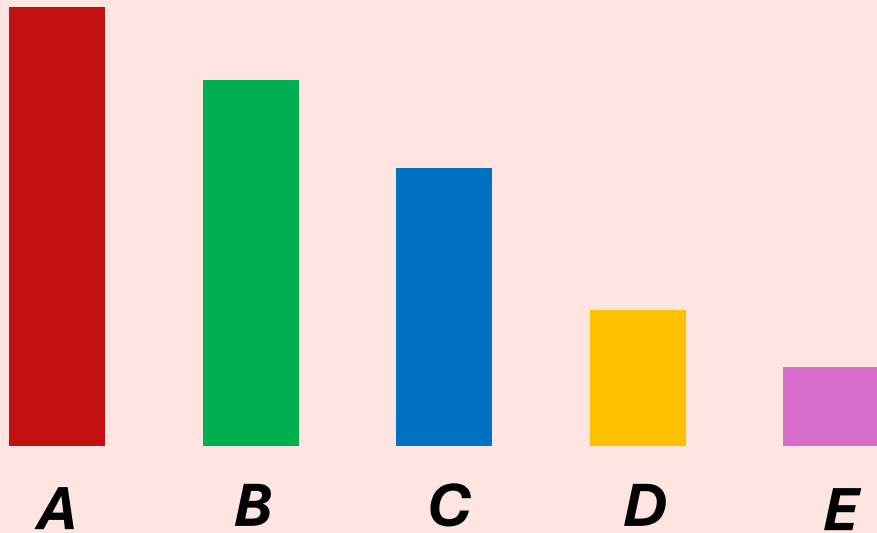


Seats are allocated to parties **proportionally to their scores.**

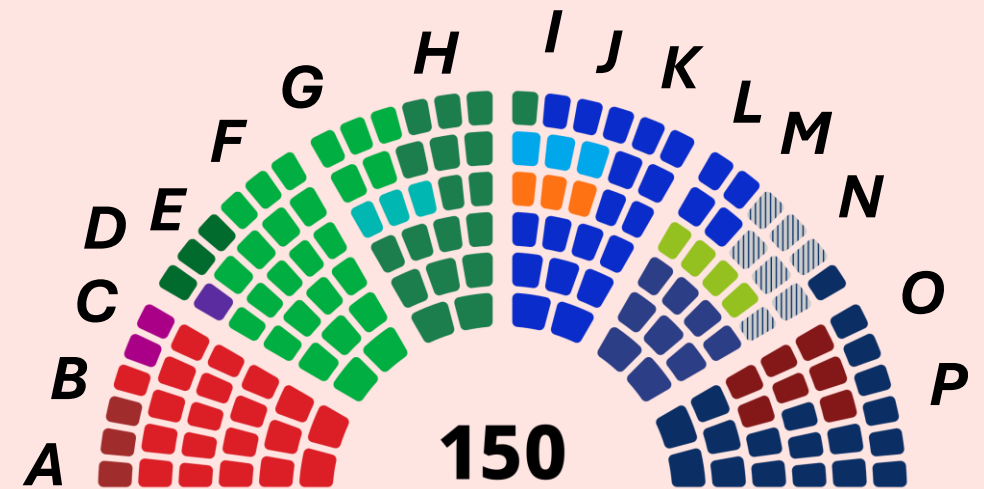


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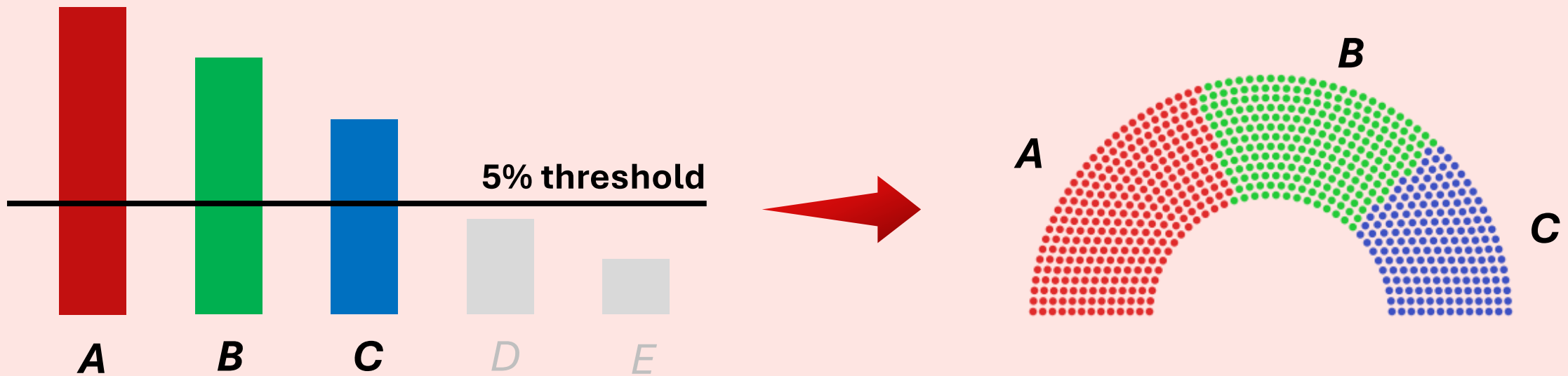
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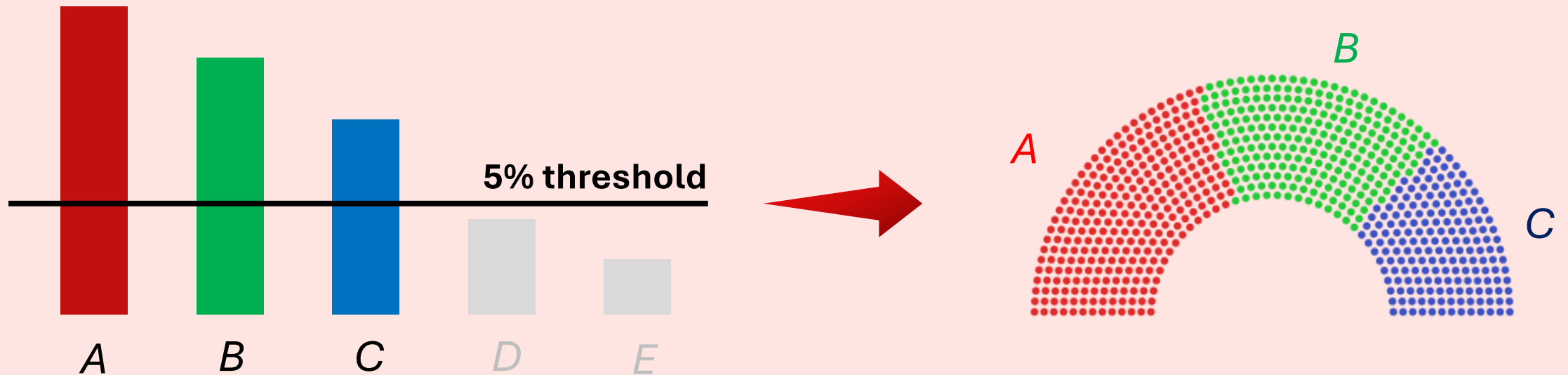
Problem: possible political fragmentation (many parties get a seat).

Proportional representation with *threshold*

Many countries impose an **electoral threshold** to reduce political fragmentation.



Proportional representation with *threshold*



- ➔ **Some votes are “lost”:** *D* and *E* supporters have no influence on the seat distribution.
- ➔ This incentivizes forms of **tactical voting**.

The “lost” votes



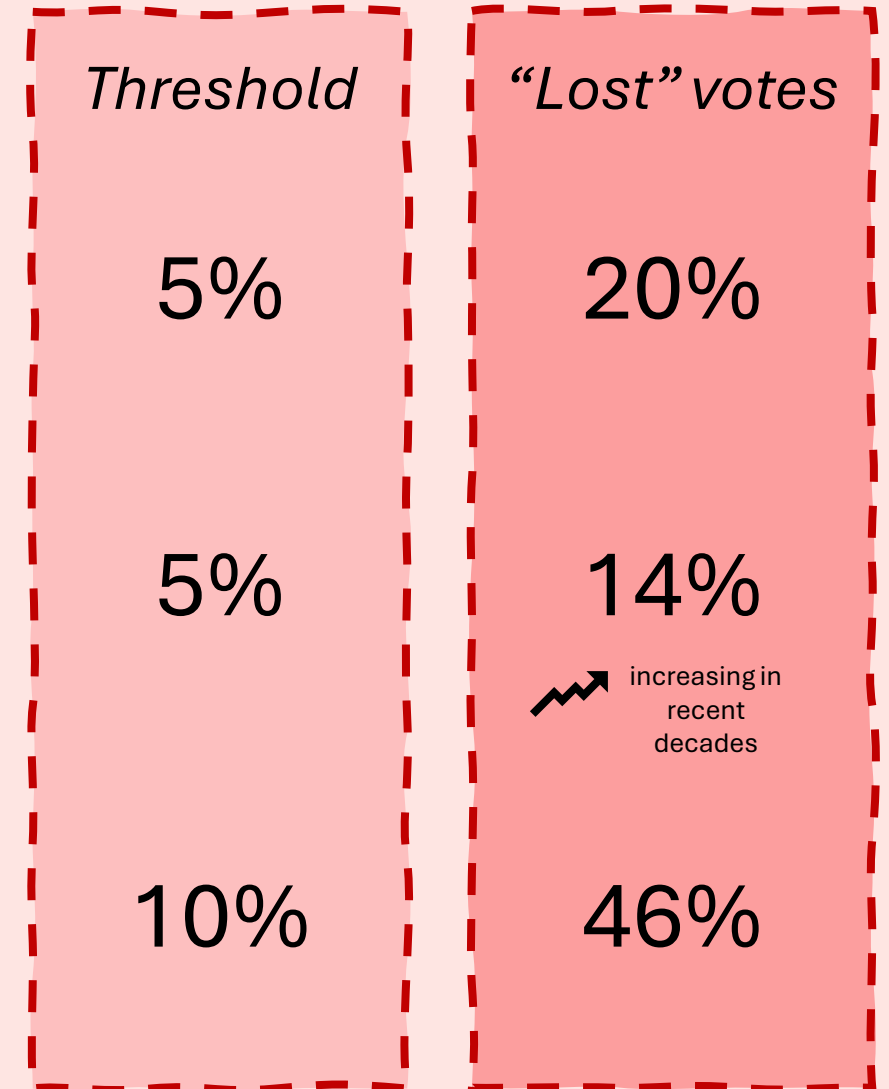
2019 election of the French representatives to the *EU Parliament*.



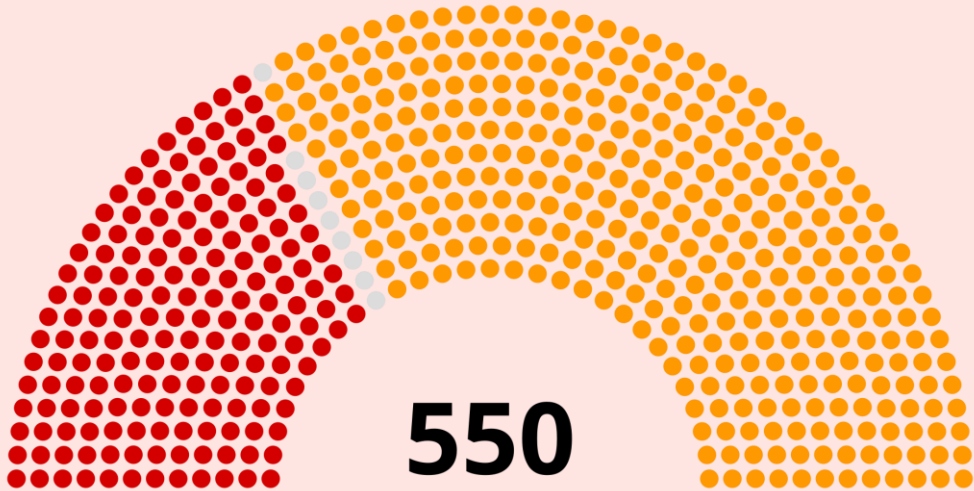
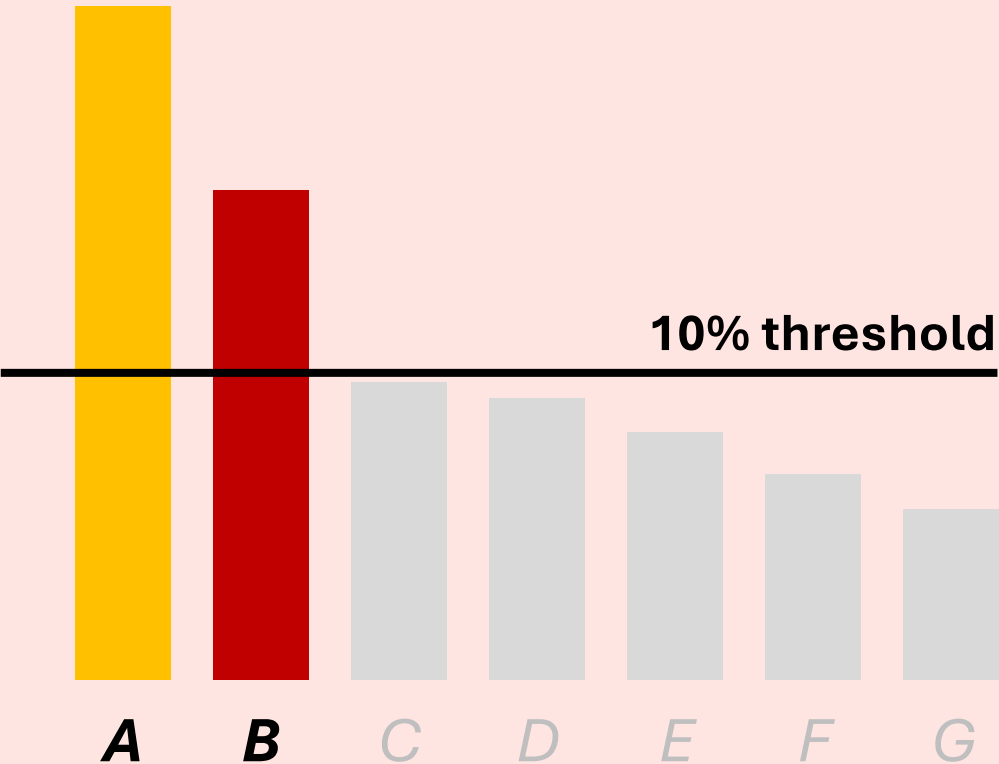
2025 election of the *Bundestag* members.



2002 election of the *Turkish Parliament* members.



The “lost” votes: 2002 Turkish Election



Idea: a *replacement* vote

We could let voters **indicate a second choice** to be used in case their first choice does not reach the threshold.

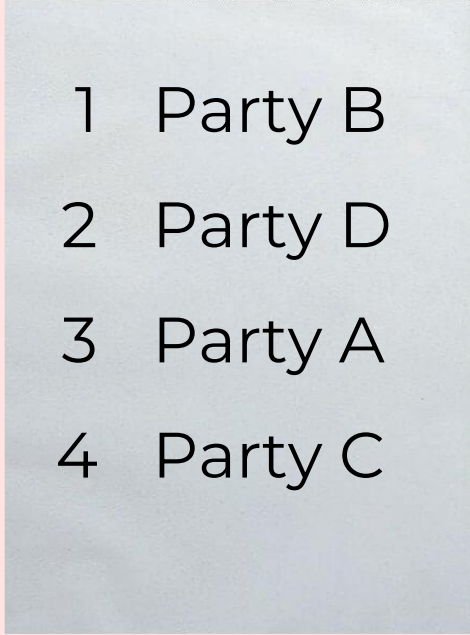


Idea: a *replacement* vote

We could ask voters to rank **two parties**

- 
- 1 Party B
 - 2 Party D

We could even ask for a **truncated ranking**

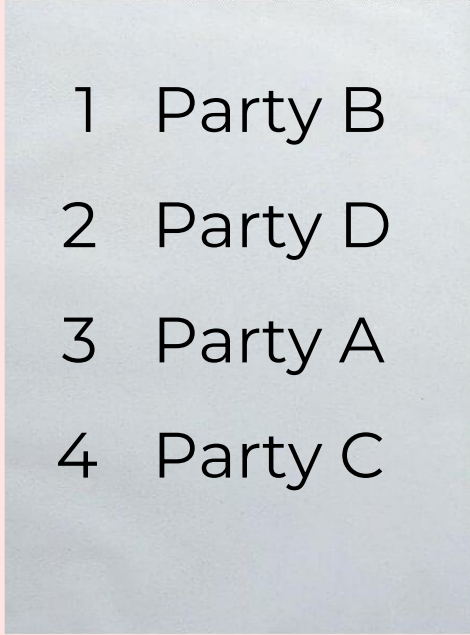
- 
- 1 Party B
 - 2 Party D
 - 3 Party A
 - 4 Party C

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- 1 Party B
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 - 3 Party A
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Question: how to decide which parties are “above the threshold”?

A not-so-easy problem

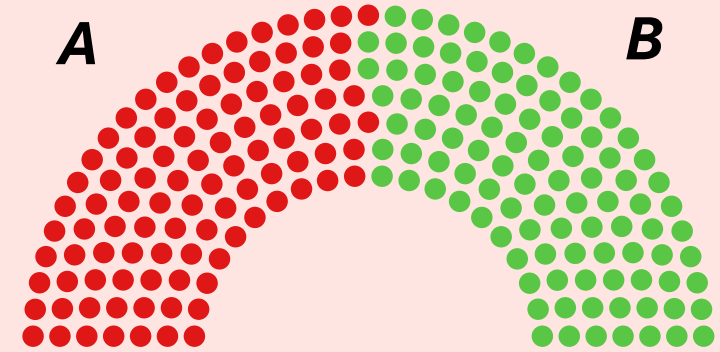
47% **A**

47% **B**

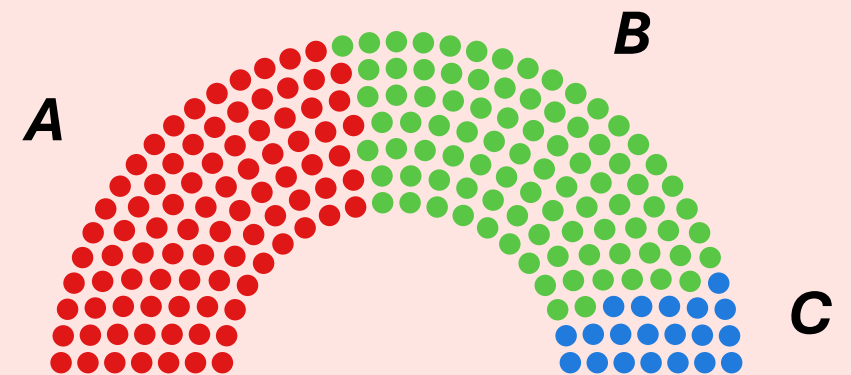
4% **C** \succ **D**

2% **D** \succ **C**

5% threshold



or



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Definitions and model

The **formal** model

We have:

- Set of **voters** $V = \{1, 2, \dots, n\}$ and **parties** $C = \{p_1, \dots, p_m\}$.
- A preference **profile** $P = (\succ_1, \dots, \succ_n)$ of truncated rankings of voters over parties.
- A given **threshold** τ (absolute number of voters).

We want:

- A set of selected parties $S \subseteq C$, called the **outcome**.
- Voters are **represented** by their most-preferred party in S (if any).
- An outcome S is **feasible** if every party represents $\geq \tau$ voters.
- We assume that parties in S get **a number of seats proportional** to the **share** of voters they represent.

Feasible outcomes

- Profile P :

6 × $A \succ B \succ D \succ C$

4 × $B \succ C \succ E \succ A \succ D$

3 × $C \succ B$

3 × $D \succ E \succ B \succ A \succ C$

2 × $C \succ A \succ E$

- Threshold $\tau = 5$.



Outcome $\{A\}$ is feasible.

Feasible outcomes

- Profile P :

6 × $A \succ B \succ D \succ C$

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- Threshold $\tau = 5$.



Outcome $\{A\}$ is feasible.



Outcome $\{E\}$ is feasible.

Feasible outcomes

- Profile P :

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3 × $D \succ E \succ B \succ A \succ C$

2 × $C \succ A \succ E$

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Outcome $\{A\}$ is feasible.



Outcome $\{E\}$ is feasible.



Outcome $\{A, C\}$ is feasible.

Feasible outcomes

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Outcome $\{A, B, C\}$ is feasible.

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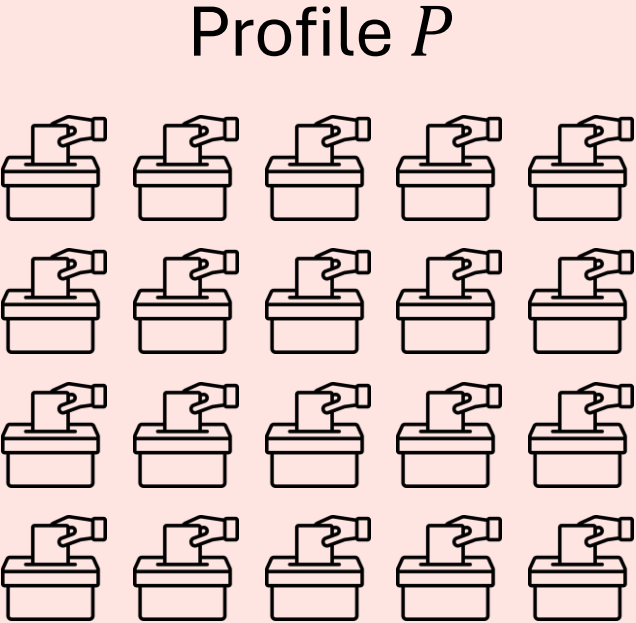
3 × $D \succ E \succ B \succ A \succ C$

2 × $C \succ A \succ E$

- Threshold $\tau = 5$.

- ✓ Outcome $\{A\}$ is feasible.
- ✓ Outcome $\{E\}$ is feasible.
- ✓ Outcome $\{A, C\}$ is feasible.
- ✓ Outcome $\{A, B, C\}$ is feasible.
- ✗ Outcome $\{B, D\}$ is not feasible.

Summary of the problem



Party
Selection
Rule f



Feasible
Outcome

$$S \subseteq C$$

+ Threshold τ

Special case: single-winner voting



If $\tau > n/2$ and with full rankings, this corresponds to the **single-winner voting model** (if we additionally want a non-empty outcome).

(This is because when $\tau > n/2$, only one candidate can be part of the outcome since each candidate in the outcome needs to represent more than τ voters).

Group activity selection problem

Voters are student that gives **preferences over possible courses/activities.**

Students ultimately **go their favorite course** among the one that are taking place.

A course/activity can take place if and only if **more than τ students selected it.**

[2022] Simplified group activity selection with group size constraints.

Andreas Darmann, Janosch Döcker, Britta Dorn, and Sebastian Schneckeburger.

International Journal of Game Theory 51, 1 (2022), 169–212.

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Party Selection Rules

Rule 1: Direct Winners Only (DO)

Rule: Direct Winners Only (DO)

The selected parties are all those which receive more first-place votes than required by the threshold.

35% **A** > C

3% **D** > E > F > C

30% **B**

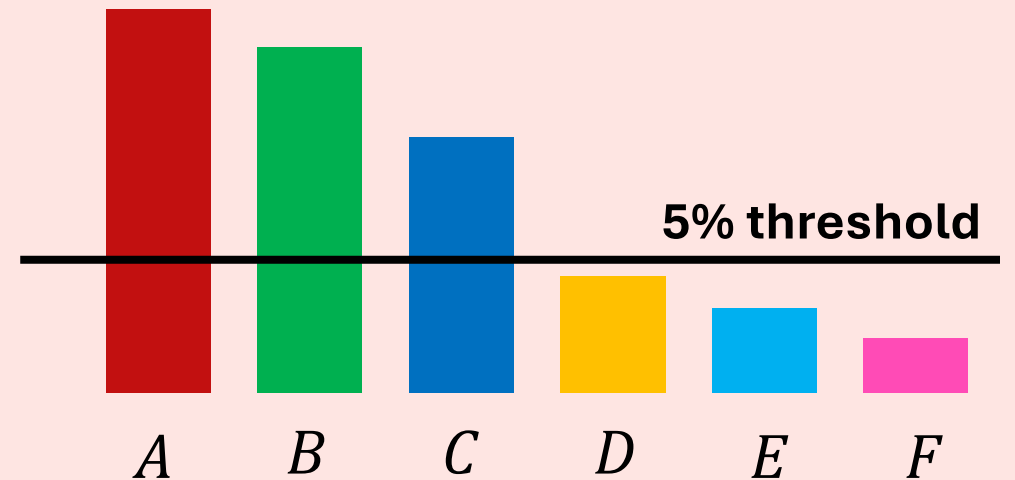
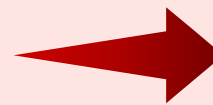
3% **E** > F > A

20% **C** > E > A

2% **F** > E > D

6% **C** > A > B

1% **D** > F > B



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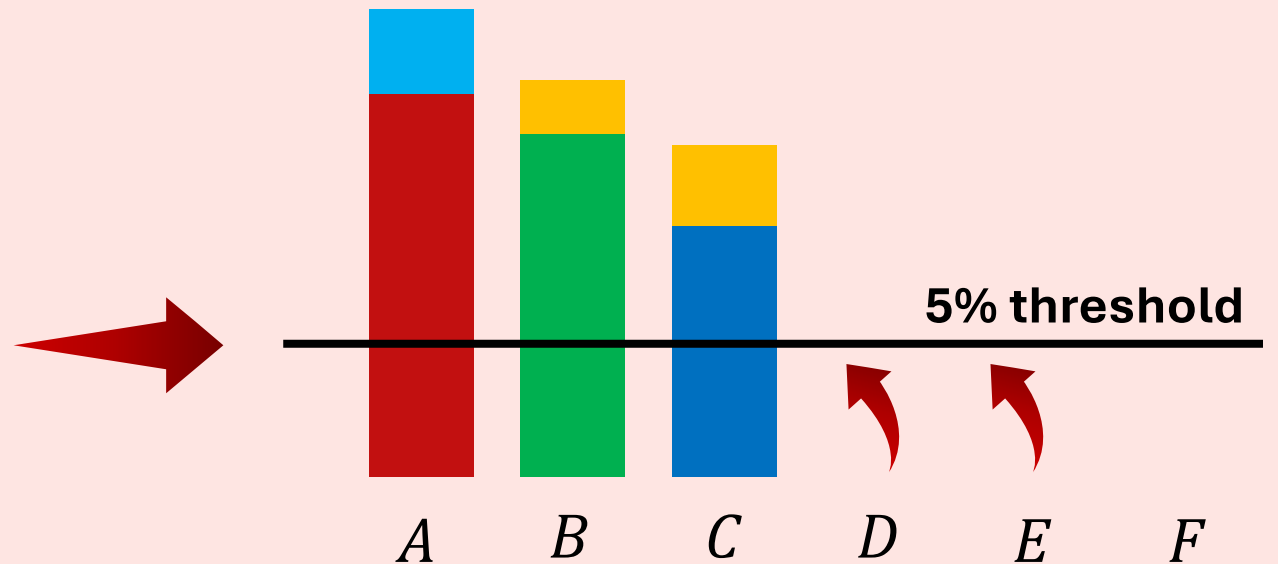
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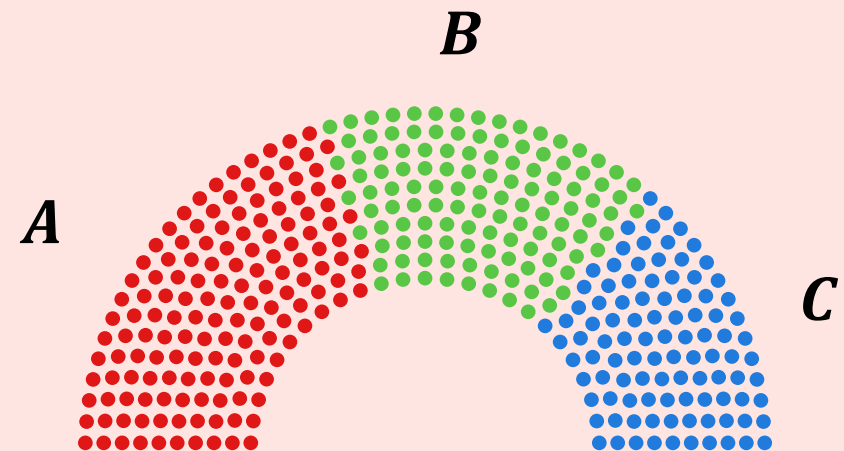
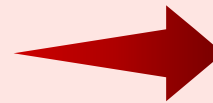
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Rule 2: Single Transferable Vote (STV)

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Parties that receive the fewest votes are successively eliminated until all parties receive more votes than required by the threshold.

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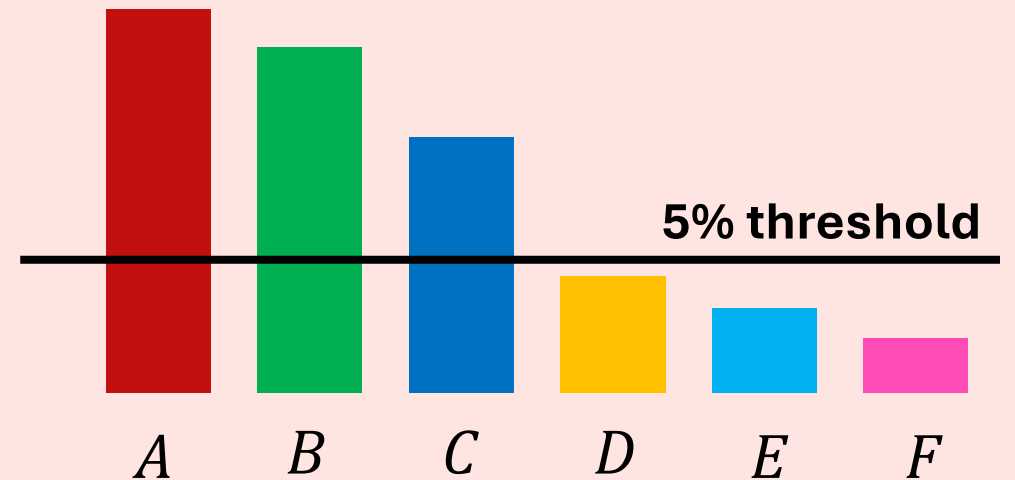
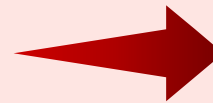
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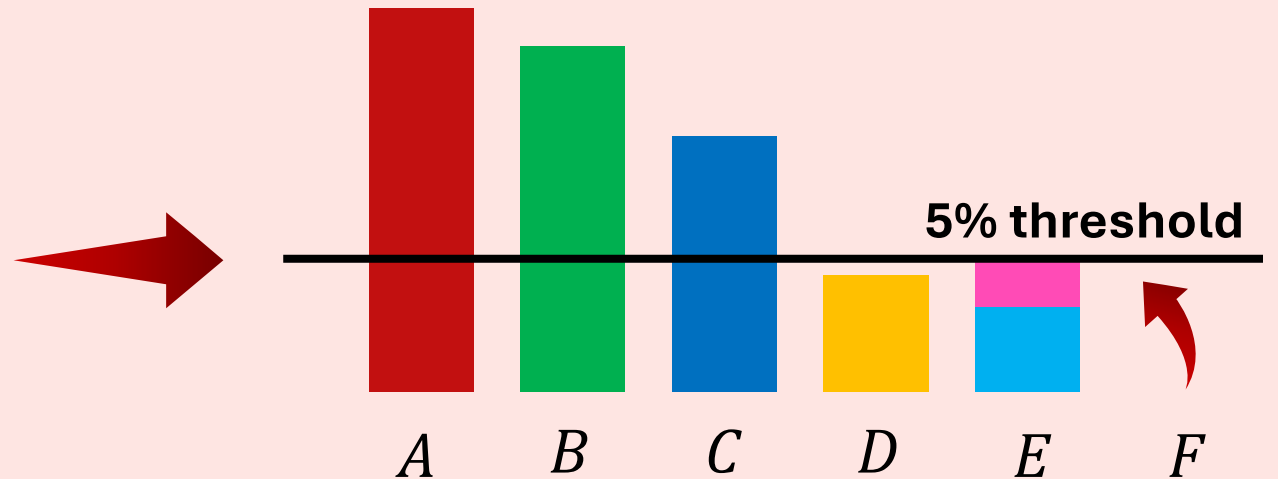
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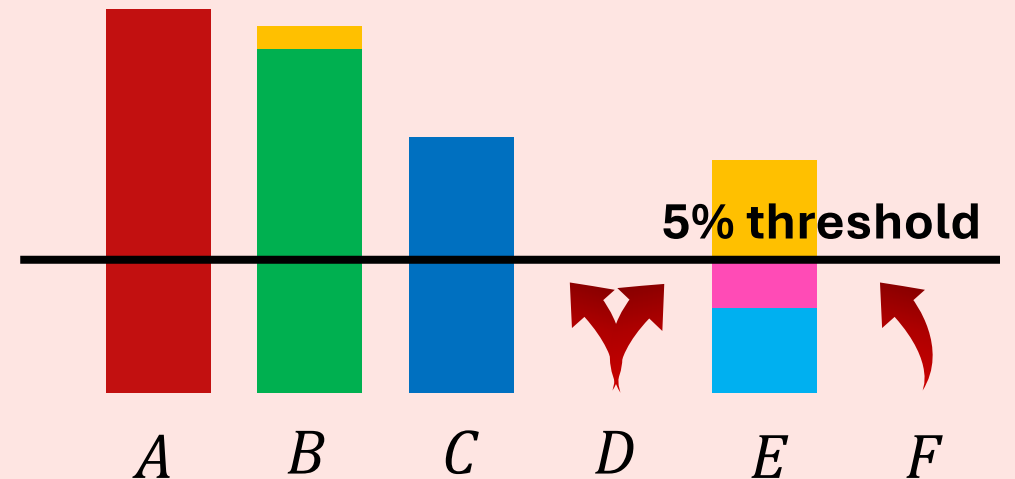
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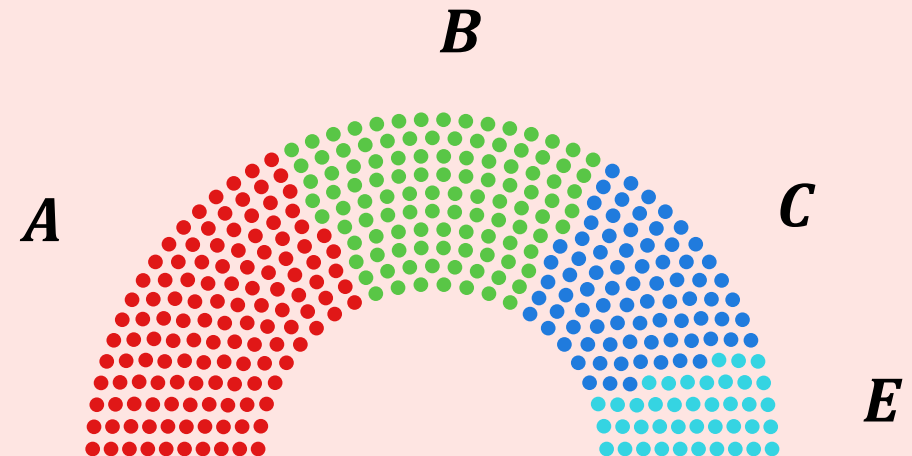
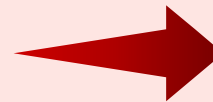
3% **E** > *F* > A

20% **C** > *E* > A

2% *F* > **E** > D

6% **C** > A > B

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Rule 3: Greedy Plurality (GP)

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Parties are considered in decreasing order of plurality score, and added to the outcome if it remains feasible.

35% **A** > C

3% *D* > *E* > *F* > C

30% *B*

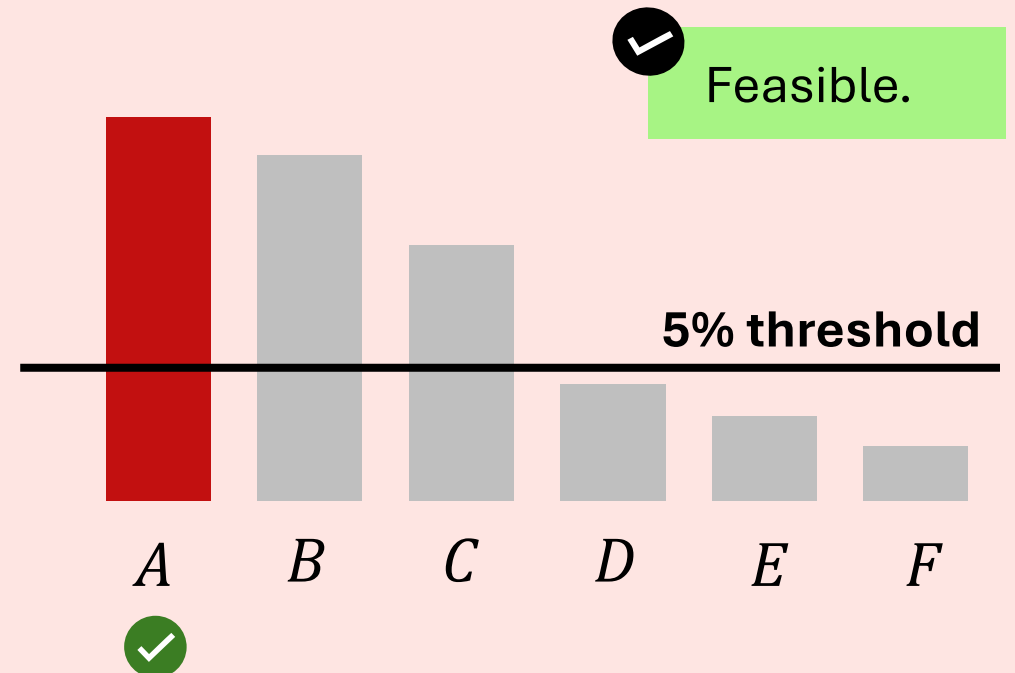
3% *E* > *F* > **A**

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2% *F* > *E* > *D*

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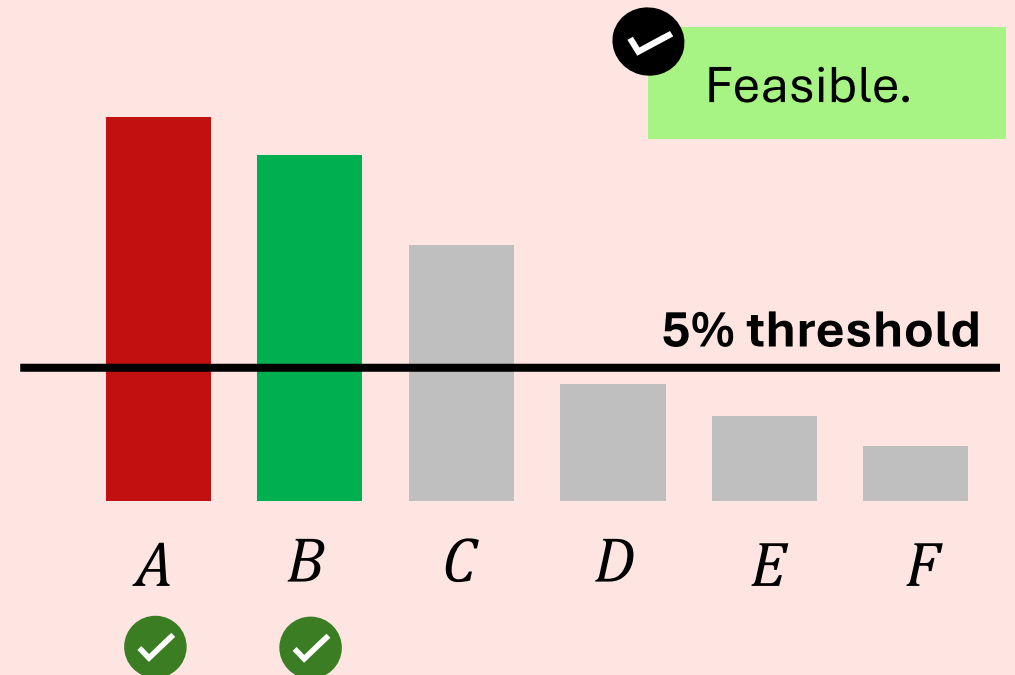
3% *E* > *F* > **A**

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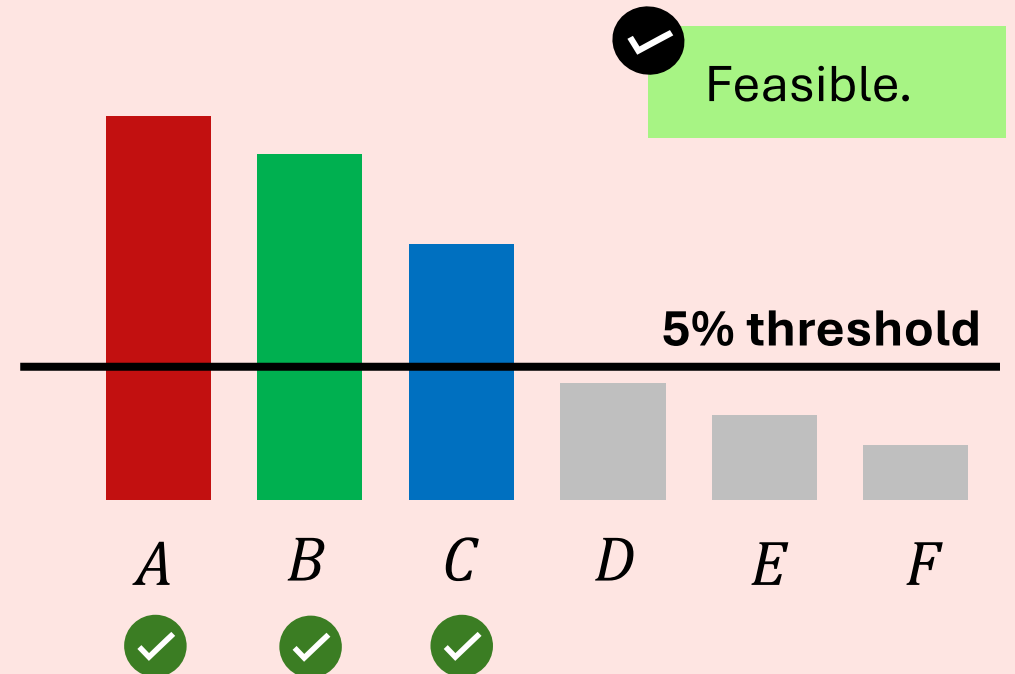
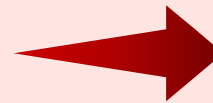
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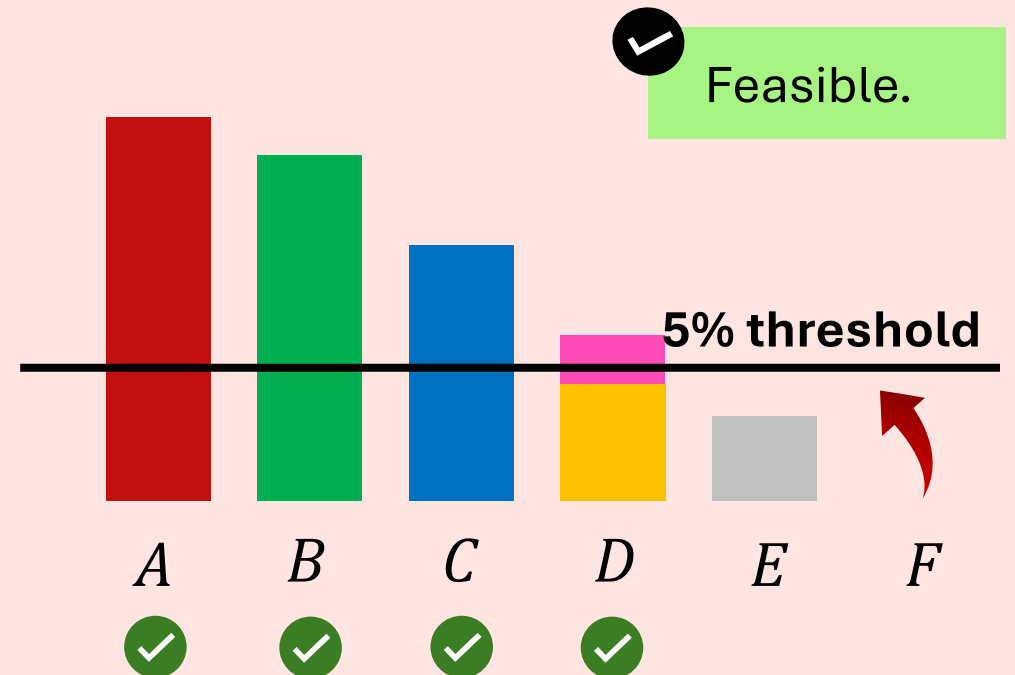
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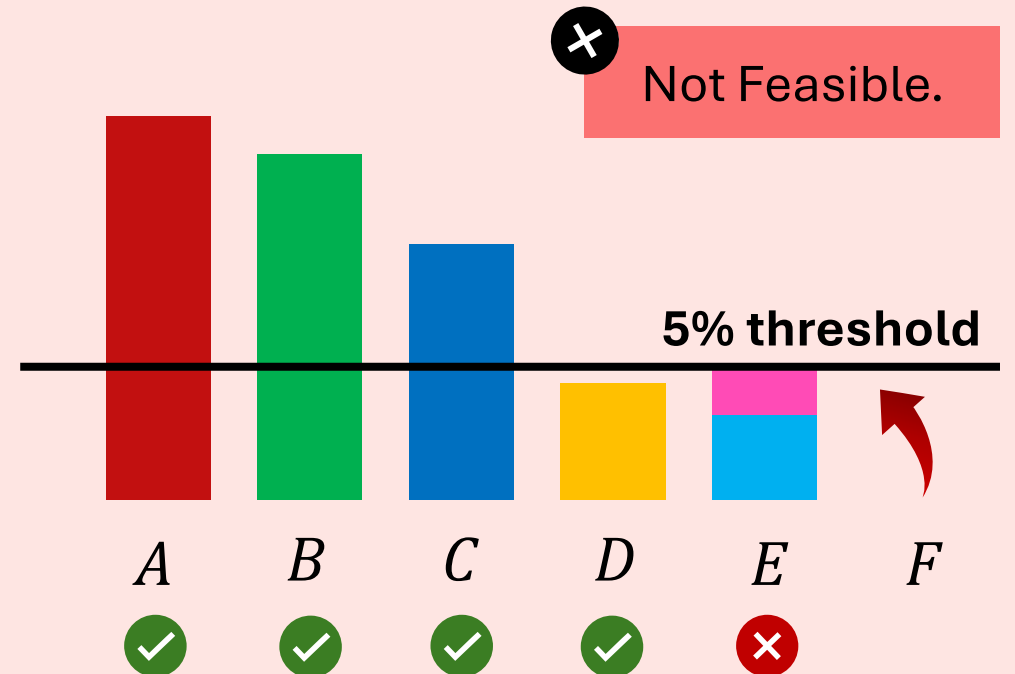
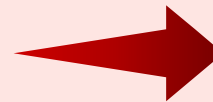
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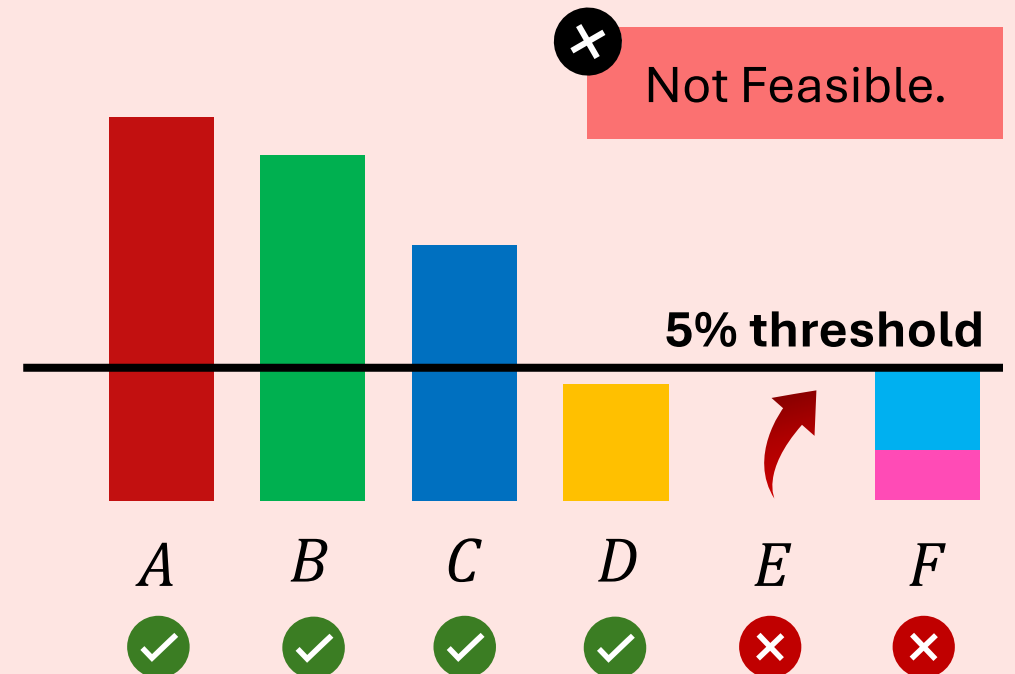


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30% B	3% E > F > A
20% C > E > A	2% F > E > D
6% C > A > B	1% D > F > B



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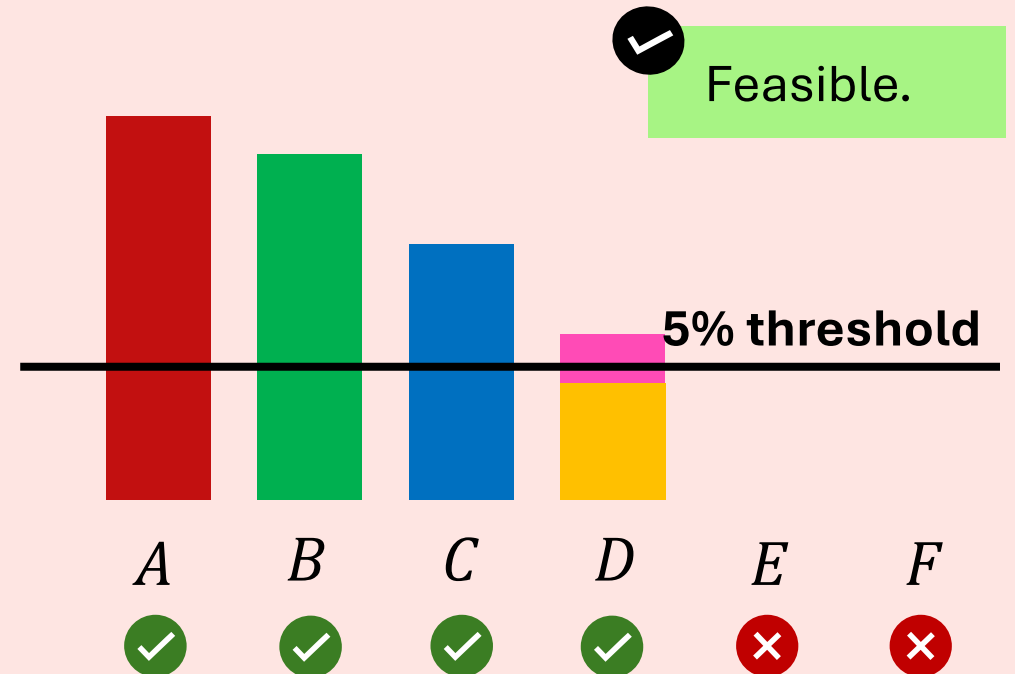
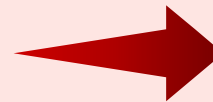
3% E > F > **A**

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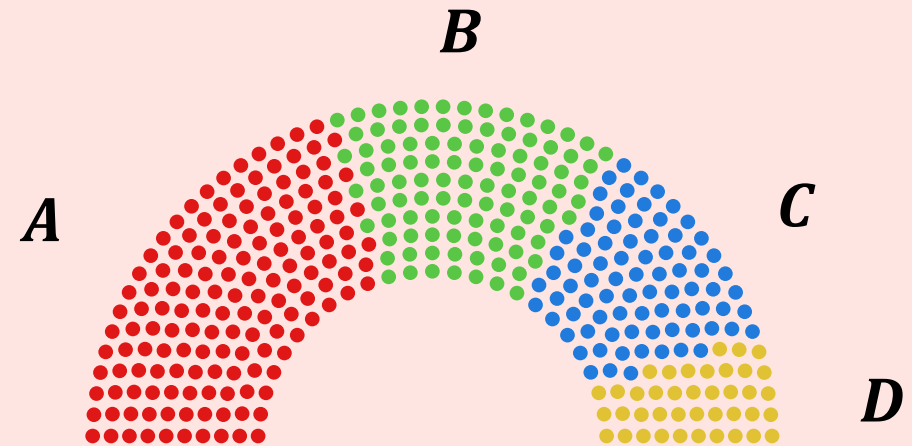
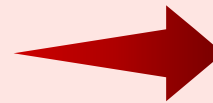
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Optimization-based rules

Rule: Maximum Plurality (MaxP)

Return the feasible outcome that maximizes the number of voters that are **represented by their first choice**.

Rule: Maximum Representation (MaxR)

Return the feasible outcome that maximizes the number of voters that are **represented**.

Computational Complexity

Theorem : The outcome of DO, STV and GP can be computed in polynomial time.

Theorem : The problem of computing the outcome of MaxR and MaxP is NP-hard.

(proof by reduction to the independent set problem)

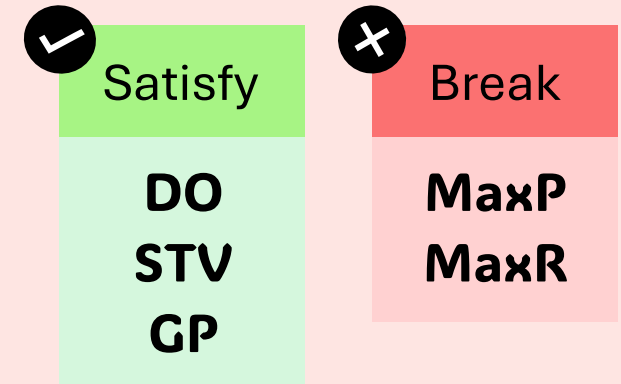
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Axiomatic Analysis

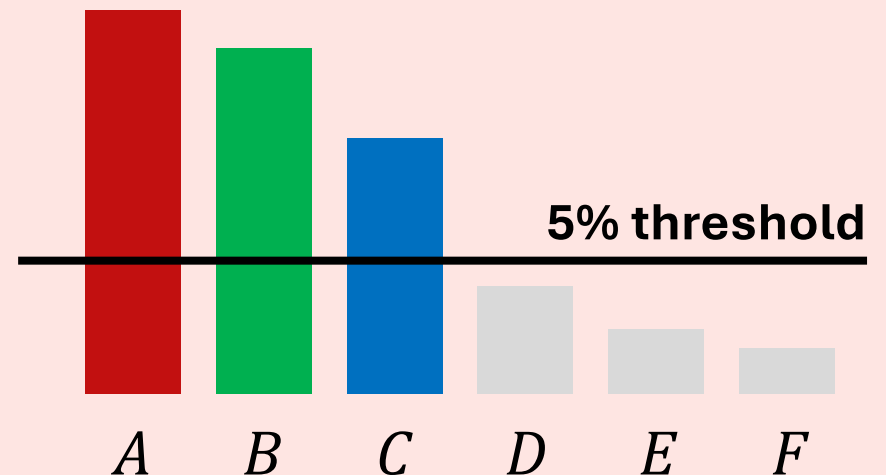
Inclusion of **Direct Winners**

Axiom: Inclusion of Direct Winners

If τ voters or more rank a party x on top of their rankings, this party should be selected.



35% $A \succ C$	4% $D \succ B \succ A$
30% B	3% $E \succ F \succ A$
26% $C \succ E \succ A$	2% $F \succ E \succ D$

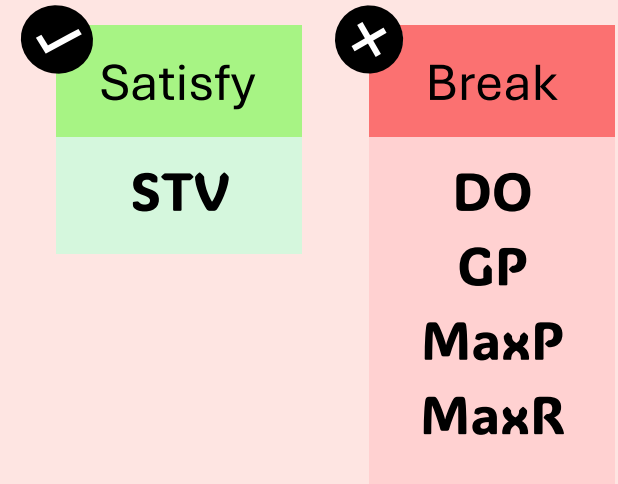


Representation of **Solid Coalitions**

Axiom: Representation of solid coalitions

If τ voters or more rank **a set** of parties T on top of their rankings, at least one of these parties should be selected.

Inspired by *Proportionality for Solid Coalitions* [Dummet 94]



35%	$A \succ C$	4%	$D \succ B \succ A$
30%	B	3%	$E \succ F \succ A$
26%	$C \succ E \succ A$	2%	$F \succ E \succ D$

= 5%

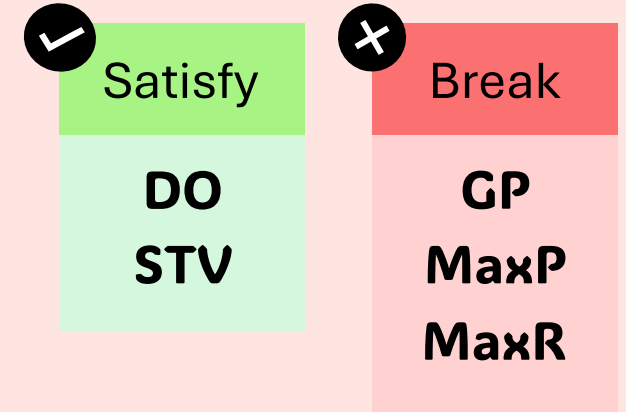


Either **E or F** should be selected

Varying the **Threshold**

Axiom: Threshold Monotonicity

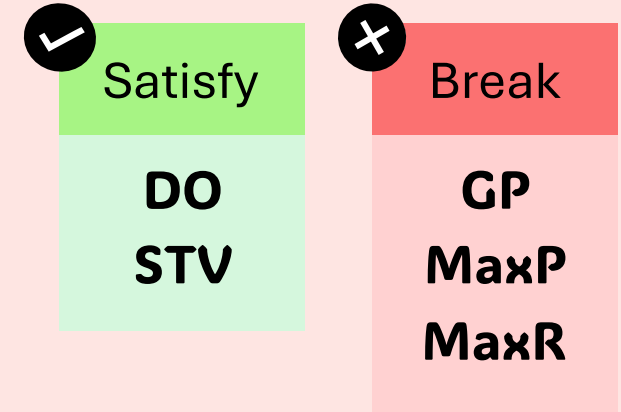
If a party is selected for threshold τ , then it is also selected for threshold $\tau' < \tau$.



Varying the **Threshold**

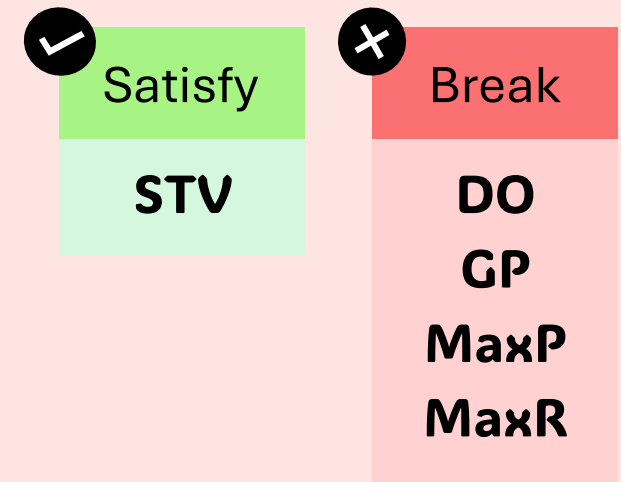
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Axiom: Independence of Definitely Losing Parties

Once some parties are losing at some threshold τ , then for all larger thresholds $\tau' > \tau$, the rule should behave **as if none of the losing parties had been available.**



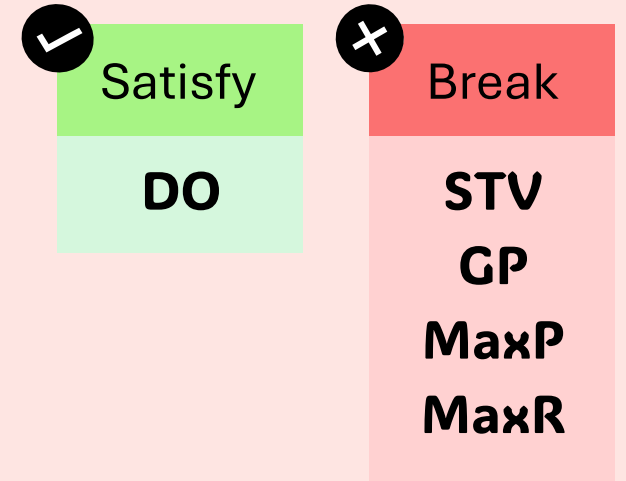
Varying the **Threshold**

Characterization Theorem: **STV** is the only party selection rule that satisfies *inclusion of direct winners* and *independence of definitely losing parties*.

Reinforcement

Axiom: Reinforcement for Winning Parties

If a party is selected for profile P_1 with threshold τ_1 and for profile P_2 with threshold τ_2 , then it should be selected for profile $P_1 + P_2$ with threshold $\tau_1 + \tau_2$.



Characterization Theorem: **DO** is the only party selection rule that satisfies *inclusion of direct winners* and *reinforcement for winning parties*.

Incentive Issues

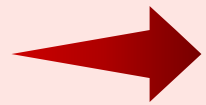
Axiom: Representative-strategyproofness

Voters cannot cause a party to be selected that they prefer to all currently selected parties by misreporting their preferences.

Axiom: Share-strategyproofness

Voters cannot cause a party to be selected that they prefer to all currently selected parties by misreporting their preferences **OR** increase the share of voters represented by their most-preferred selected party.

Incentive Issues



No hope for strategyproofness in general (Gibbard-Satterthwaite applies).

We thus consider scenarios in which **we require additional conditions** on the parties or on the possible manipulations.

Incentive Issues

Say that a party is **(all with respect to a voter i)**...

- ...**Safe** if it is *always* selected no matter how i votes.
- ...**Risky** if it *might or might not* be selected depending on how i votes.
- ...**Out** if it is *always not* selected no matter how i votes.

$\ll \tau$ votes

Out

$\sim \tau$ votes

Risky

$\gg \tau$ votes

Safe

Incentive Issues



Proposition: GP, MaxP and MaxR satisfy representative-strategyproofness when there is *at most one risky party* from the perspective of each voter. (DO and STV do not.)

Incentive Issues

$\ll \tau$ votes

Out

$\sim \tau$ votes

Risky

At least 1 in
every top-2

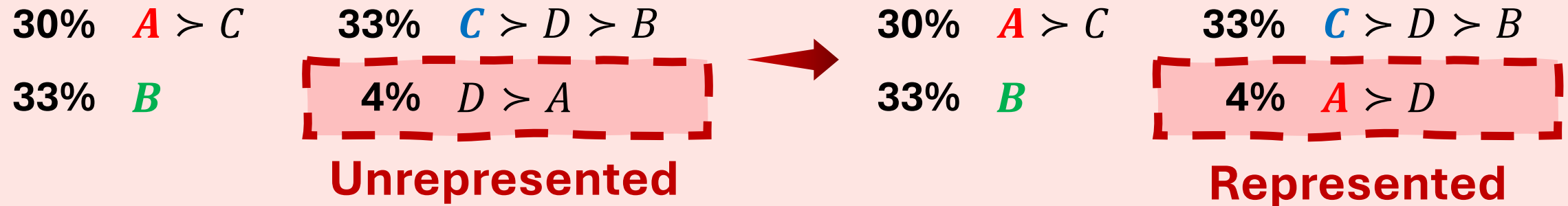
$\gg \tau$ votes

Safe

Proposition: **DO** satisfies share-strategyproofness when every voter *has a safe party as one of their two most-preferred parties*. (GP, STV, MaxP and MaxR do not.)

Incentive Issues

In the (current) uninominal system, voters are incentivized **to vote for their favorite party among the ones that will be selected:**



Proposition: **DO** and **GP** satisfy share-strategyproofness under the restriction that voters can only misreport by *promoting their most-preferred selected party* into first place. (STV, MaxP and MaxR do not)

Summary of the Axiomatic Analysis

	DO	STV	GP	MaxP	MaxR
Set-Maximality			✓	✓	✓
Inclusion of Direct Winners	✓	✓	✓		
Representation of Solid Coalitions		✓			
Threshold Monotonicity	✓	✓			
Ind. of Definitely Losing Parties		✓*			
Ind. of Clones		✓			✓
Reinforcement for Winning Parties	✓*				
Monotonicity	✓				
Rep-SP (one risky party)			✓	✓	✓
Share-SP (safe party first or second)	✓				
Share-SP (rep. ranked first)	✓		✓		

- 4 -

Empirical Analysis

Context of the Experiment

To collect appropriate preference data, **we ran a voting experiment** during the 2024 election of the French representative to the EU parliament.

Candidate parties: 38

Threshold: 5%

Parties above the threshold: 7

Lost votes: 12.1%



Experimental Setup

- 1 Explanation of the issues caused by the threshold.
- 2 Presentation of the candidate lists.
- 3 **Vote with alternative voting methods.**
- 4 Questionnaire.

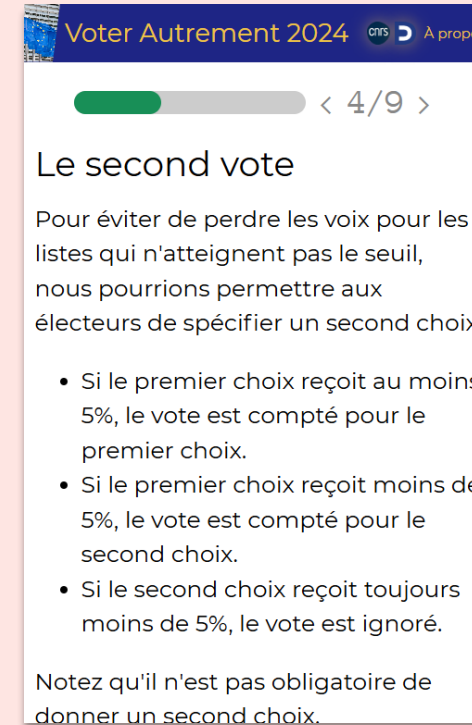


Fig. Screenshot of [the website of the experiment](#) conducted during the 2024 election of the French representative to the EU Parliament.

Two Samples of Participants

1 Self-selected sample

- 3046 participants in a week.
- Recruited through social media, unpaid.
- Overrepresentation of left-wing, young and educated people.

2 Representative sample

- 1000 participants.
- Recruited via a polling institute and paid a fixed amount to participate.
- Representative of the French population.

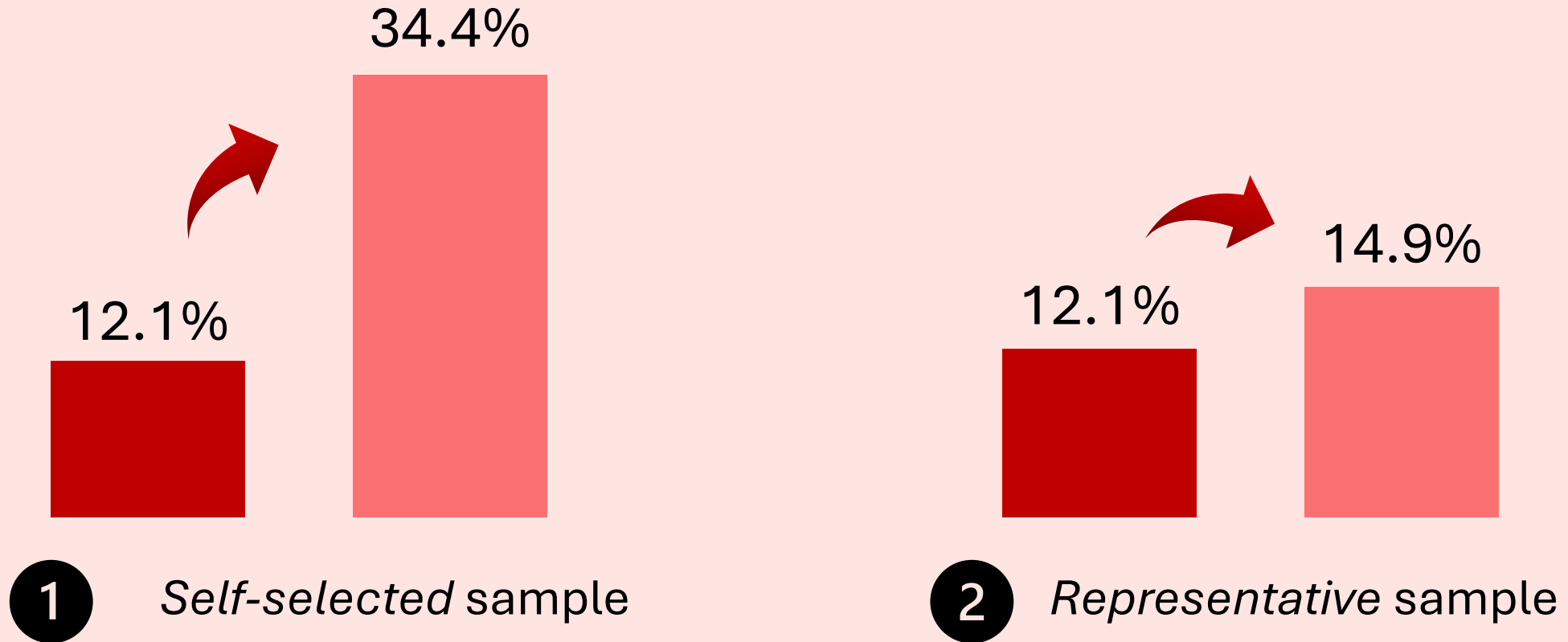
Two Samples of Participants



We assign weights to the voters to reduce the biases: weights are selected based on the vote of the participant at the actual election, to match the share of votes received by each party.

Example: if 5% of participants said they voted RN in the experiment, but the RN got 20% of the votes, each of these participants **is assigned a weight of 4.**

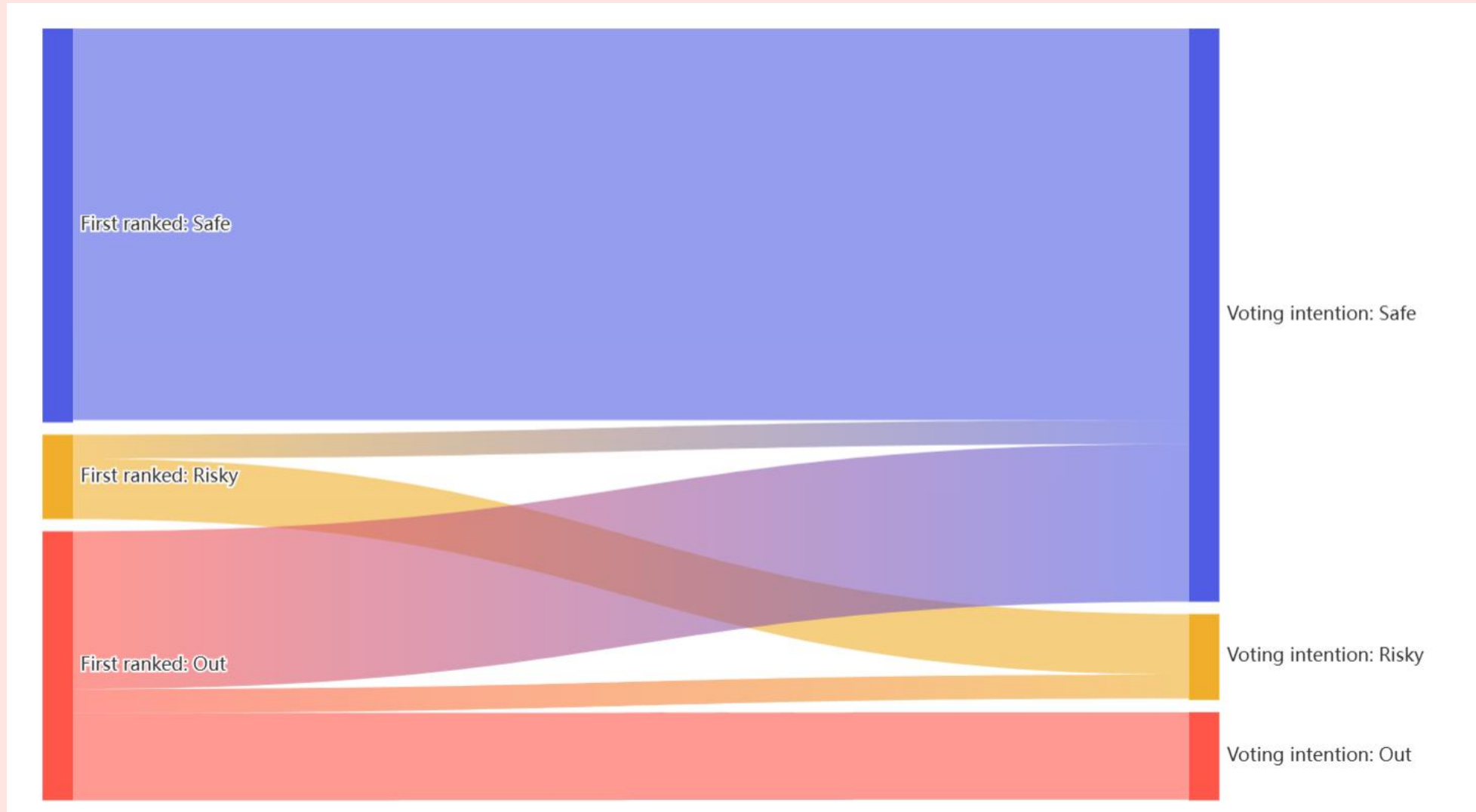
Observation 1: More votes for “small parties”



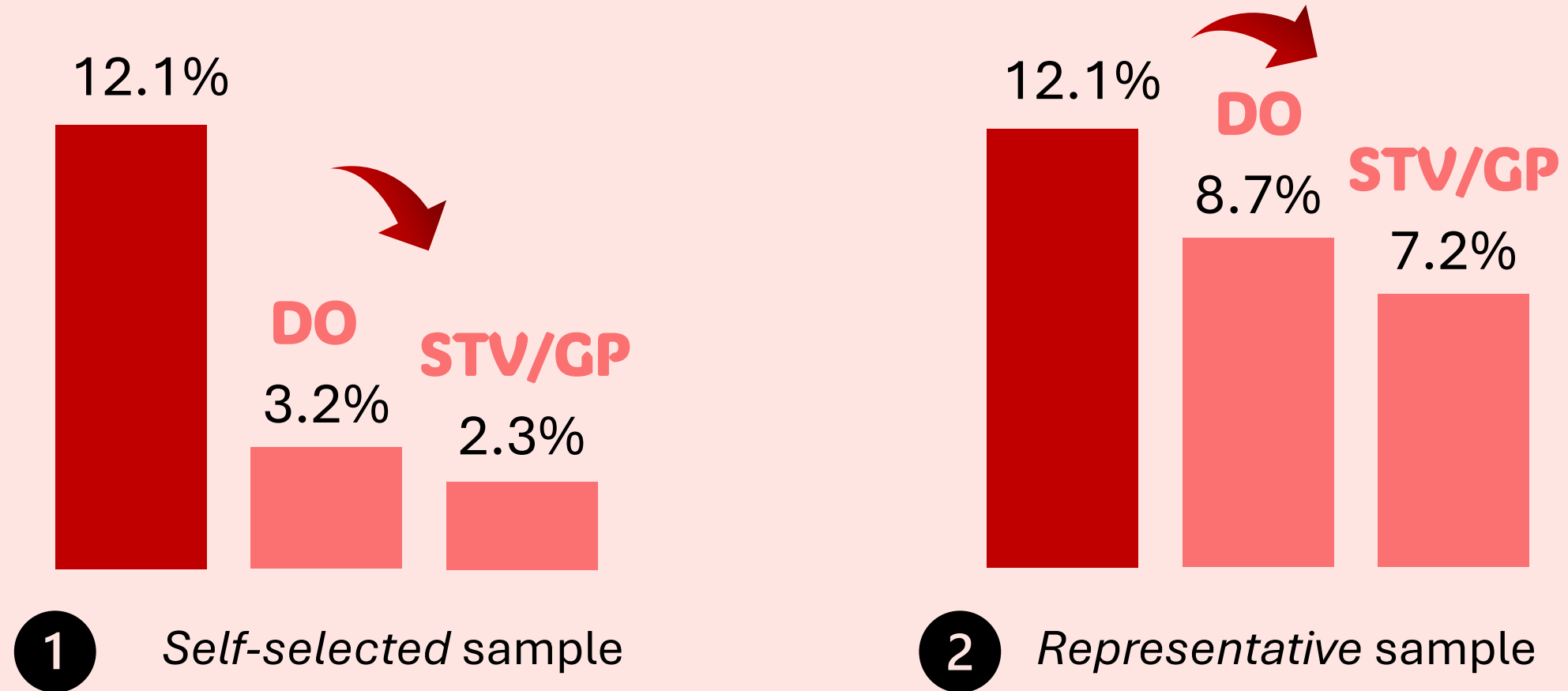
■ Share of voters that vote for a “small” party in the actual election.

■ Share of voters that put a “small” party first in their ranking.

Observation 1: More votes for “small parties”



Observation 2: Less “lost votes”



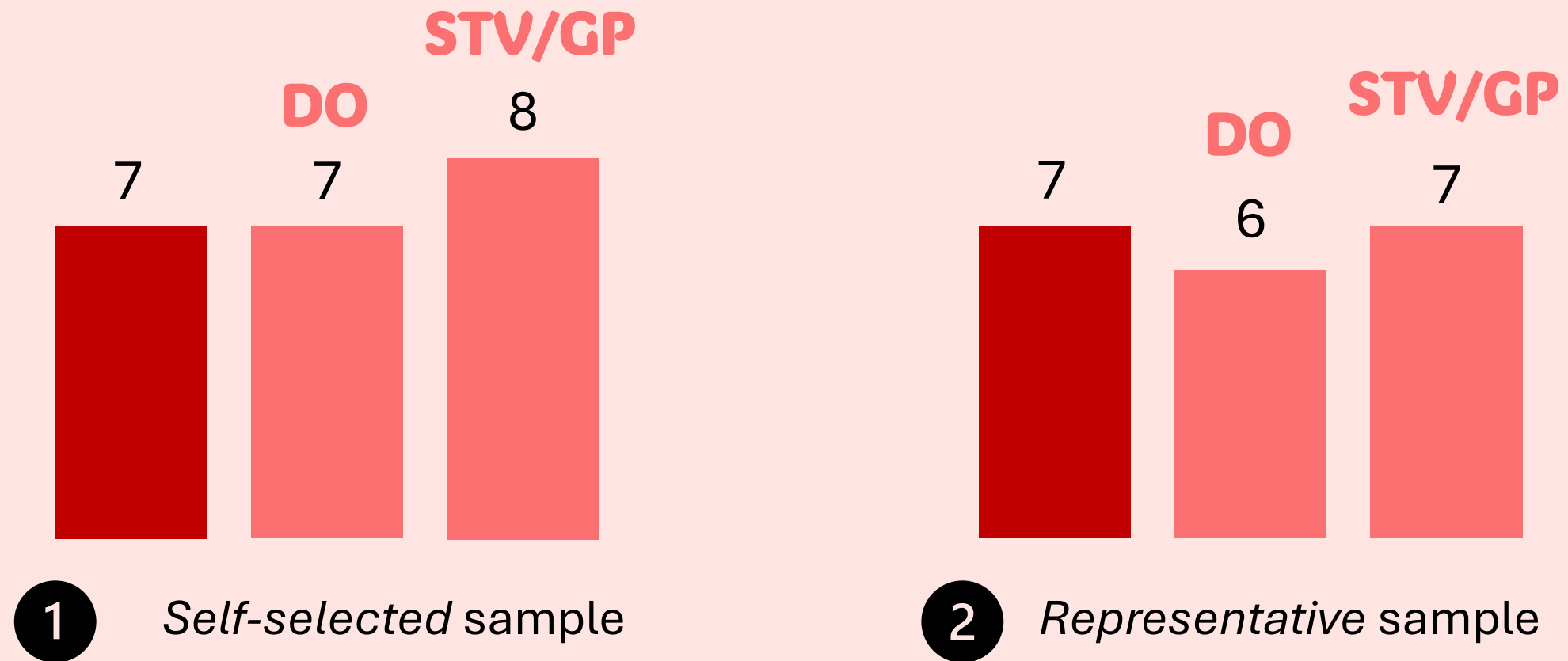
1 *Self-selected* sample

2 *Representative* sample

■ Share of unrepresented voters in the actual election.

■ Share of unrepresented voters with ranking-based rules.

Observation 3: No political fragmentation



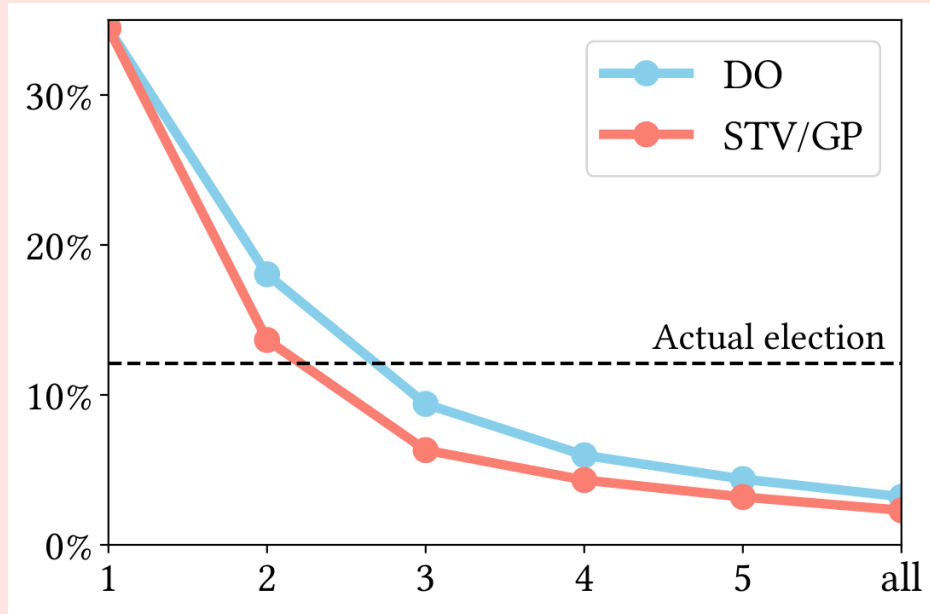
1 *Self-selected* sample

2 *Representative* sample

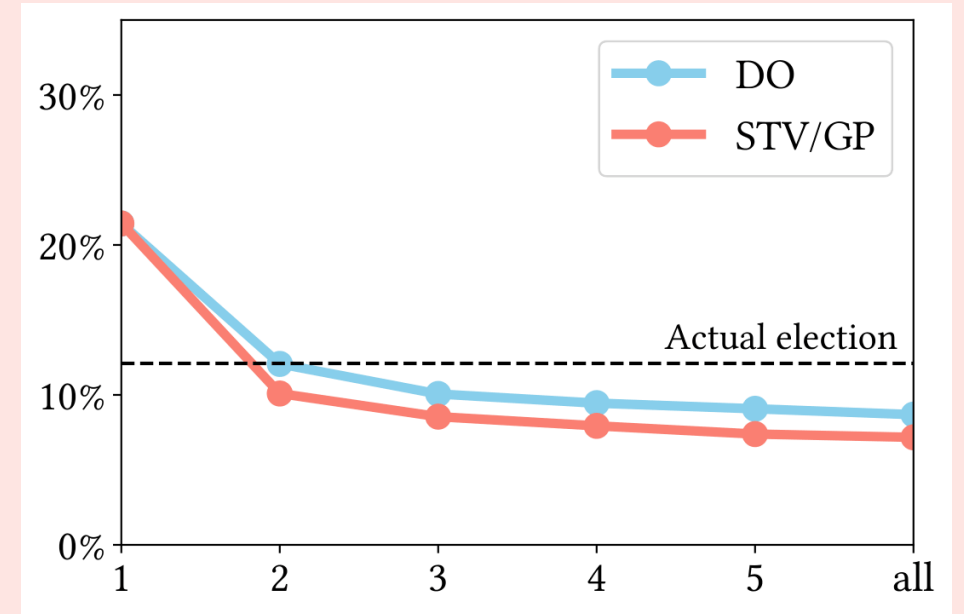
■ Number of parties receiving a seat in the actual election.

■ Number of parties receiving a seat with ranking-based rules.

Observation 4: We can ask for short rankings



① *Self-selected* sample



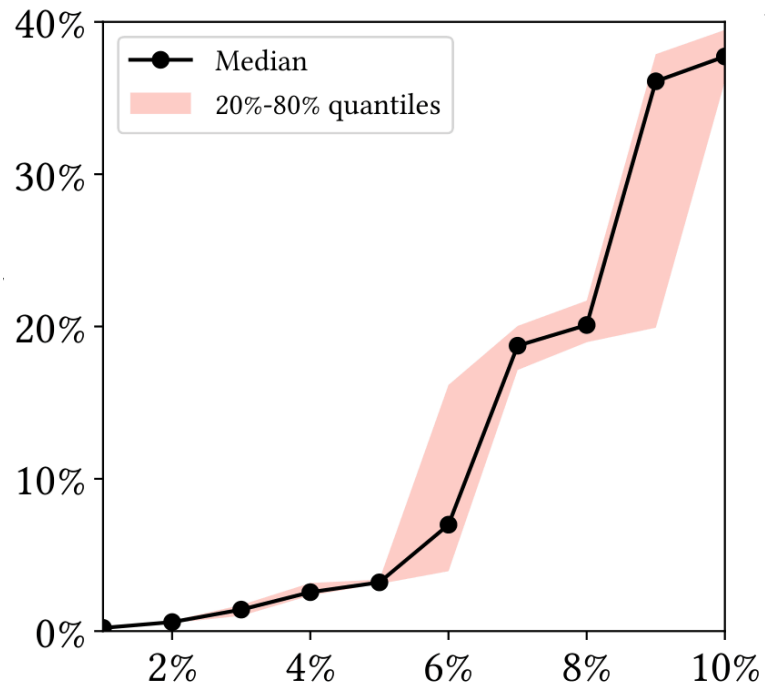
② *Representative* sample



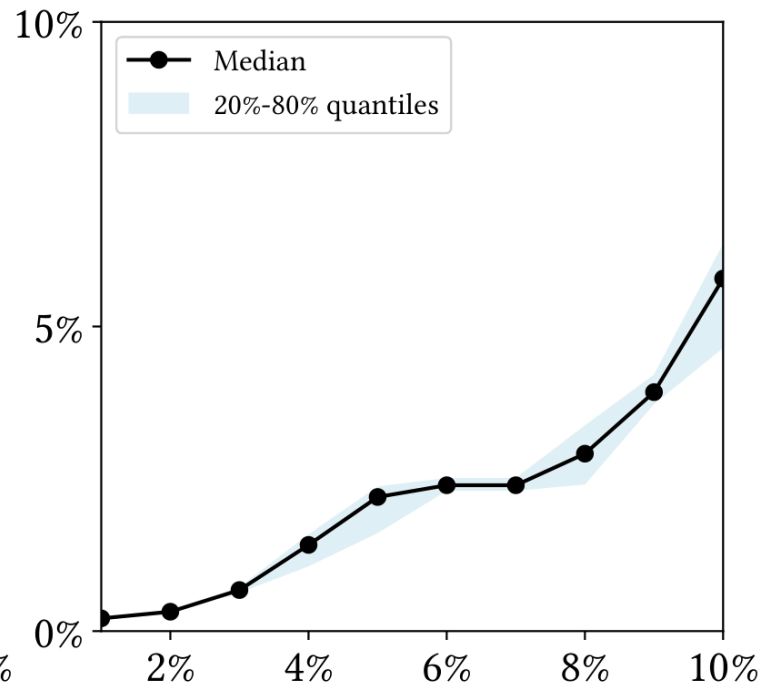
Share of unrepresented voters if all rankings are **truncated** to rank k (x-axis).

Robustness tests

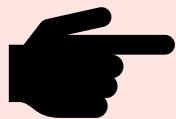
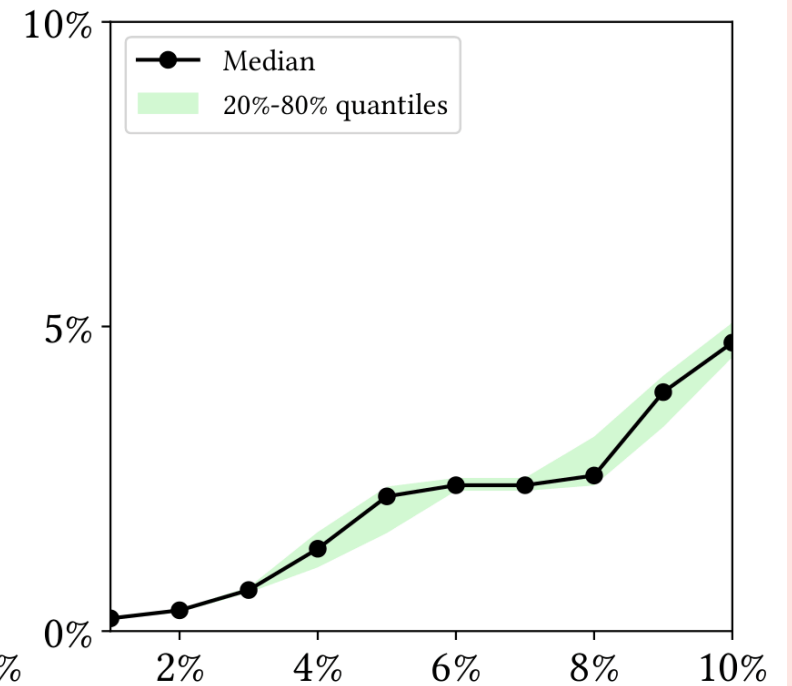
DO



STV



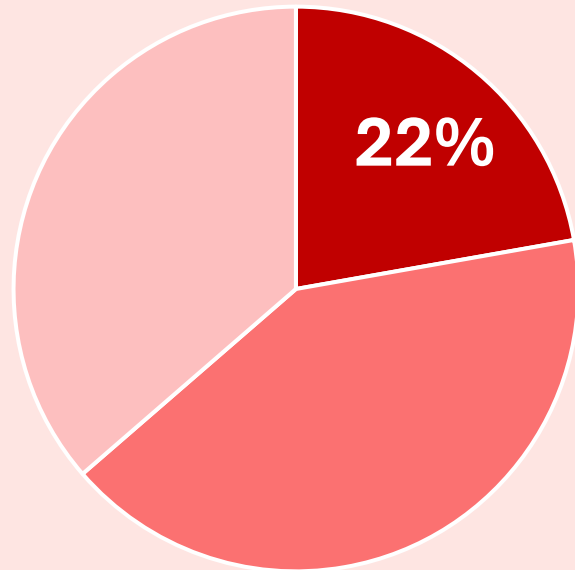
GP



Share of unrepresented voters with **different threshold values** and with **random noise** added to the preferences (*self-selected sample*).

Opinions of the participants

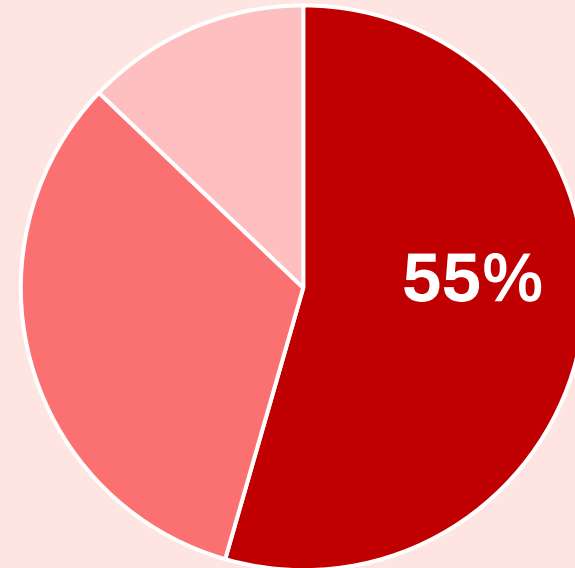
“Which system do you think is better suited for the election of your representatives to the EU parliament?”



1 *Self-selected sample*

■ Current system

■ Two choices



2 *Representative sample*

■ Truncated ranking

- / -

Conclusion

Conclusion

We axiomatically and empirically studied rules for electing parliaments with electoral thresholds.

Main takeaway: We can significantly increase representativeness by allowing voters to **rank parties**.



STV and GP leave **fewer voters unrepresented** than DO.



DO and GP have **stronger strategyproofness guarantees** than STV.



STV satisfies **independence of clones and represents solid coalitions**.

Thanks for your attention! **Questions?**

Reallocating Wasted Votes in Proportional Parliamentary Elections with Thresholds

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Paris School of Economics

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