Generalizing Instant Runoff Voting to Allow Indifferences

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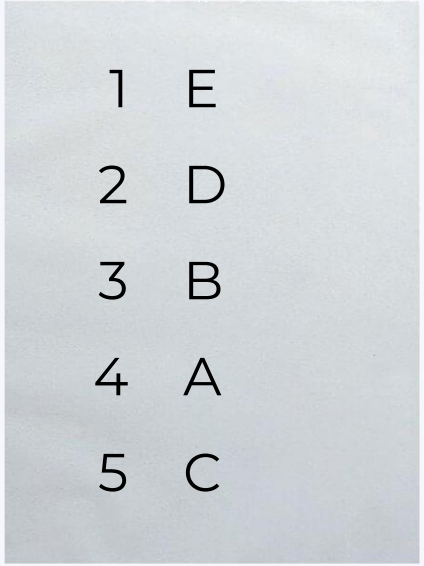
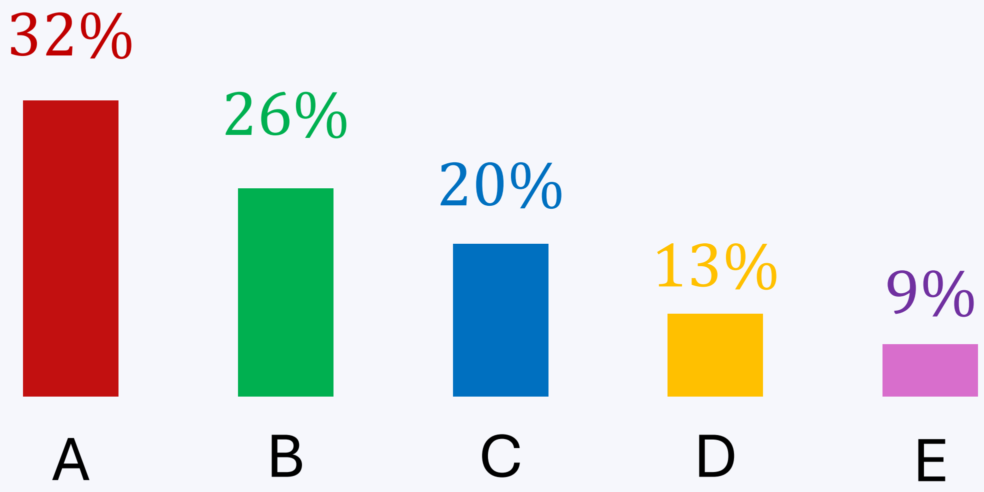
Based on a joint work with *Dominik Peters*

Reminder: Instant Runoff Voting (IRV)

Voters provide
a **ranking** of the candidates

The score of each candidate is
their number of 1st place





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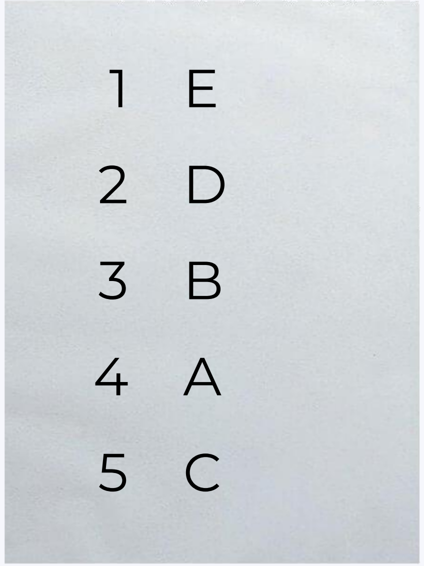

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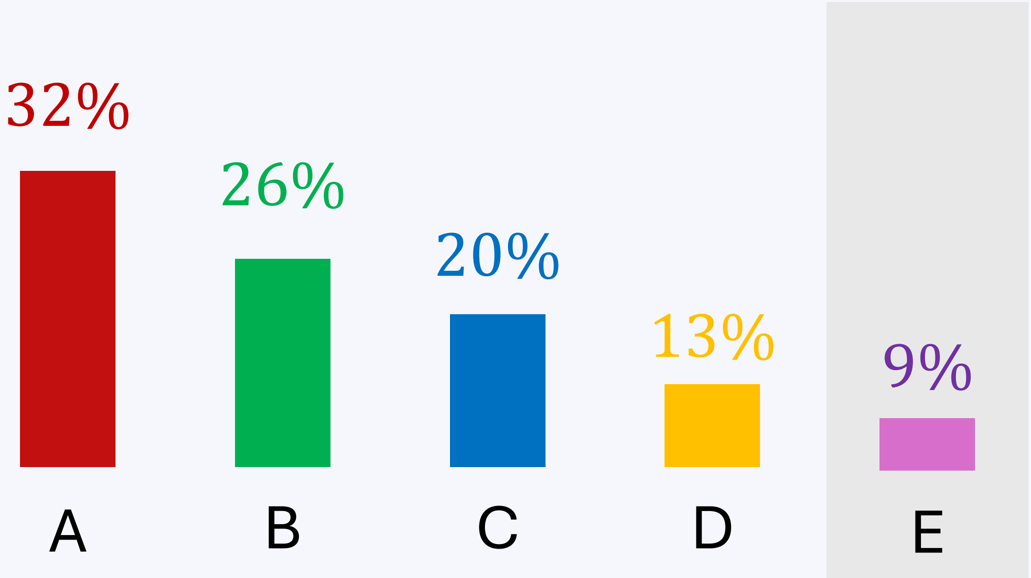

The candidate with the lowest score is **eliminated**, and their votes are transferred

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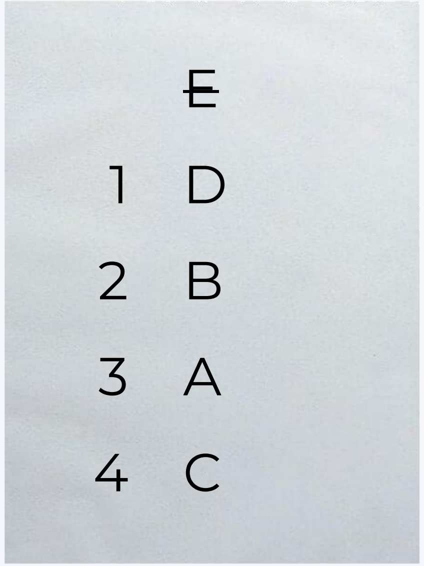

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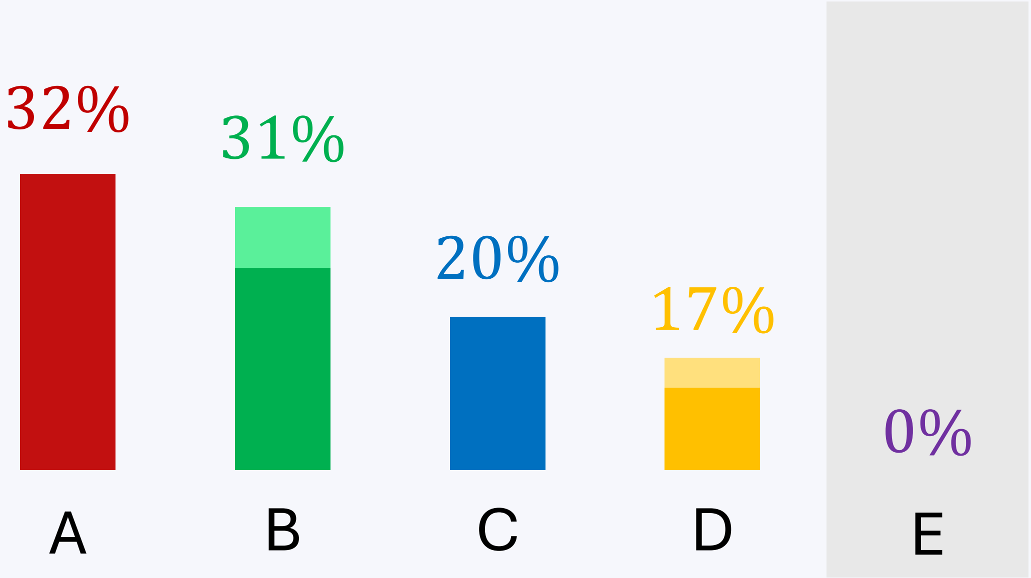

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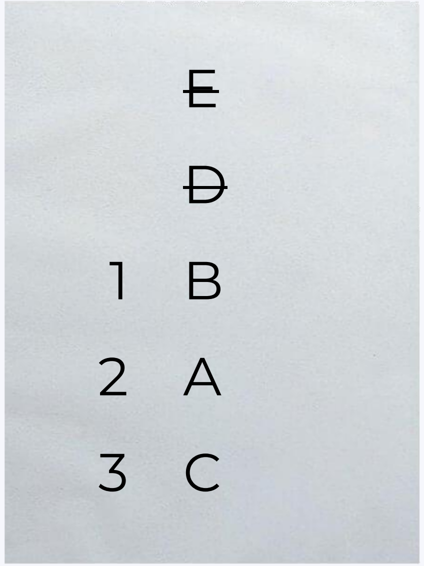

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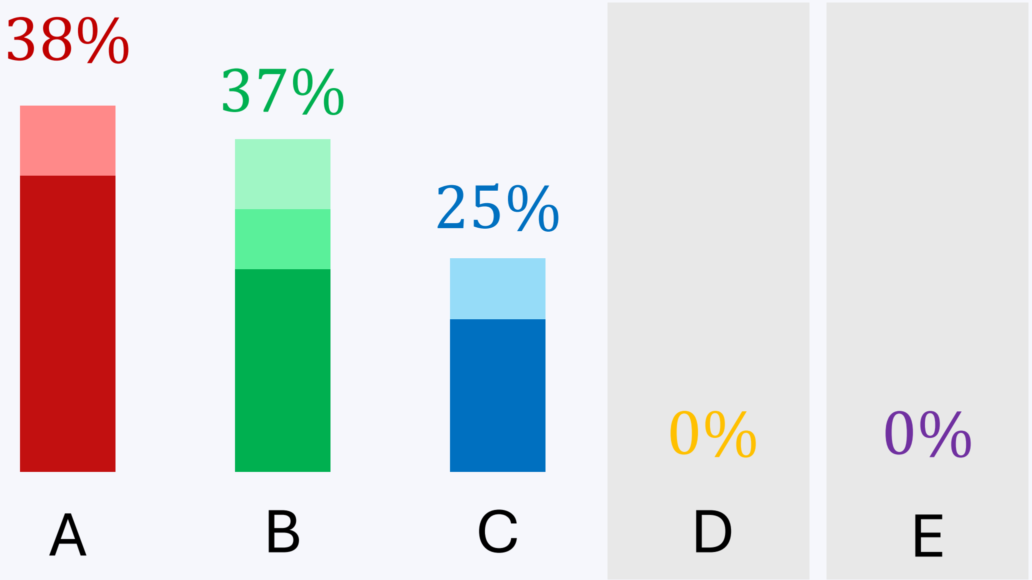

And so on, **until one candidate remains**

Reminder: Instant Runoff Voting (IRV)

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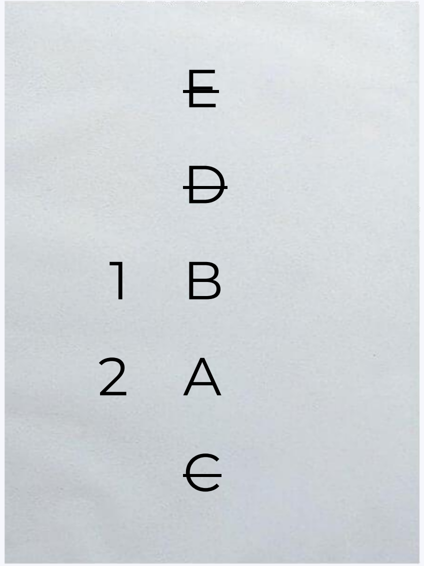

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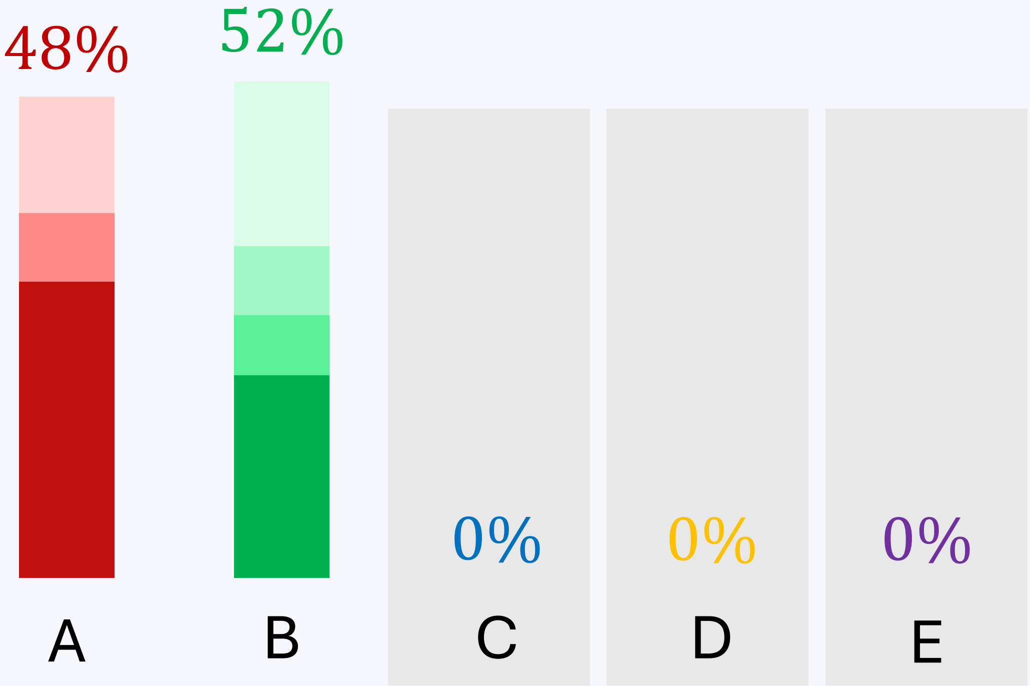

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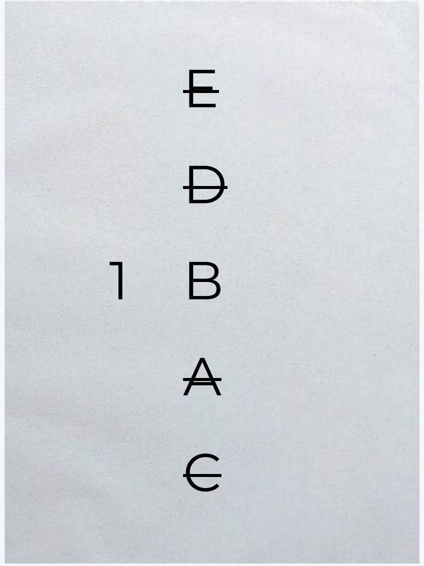

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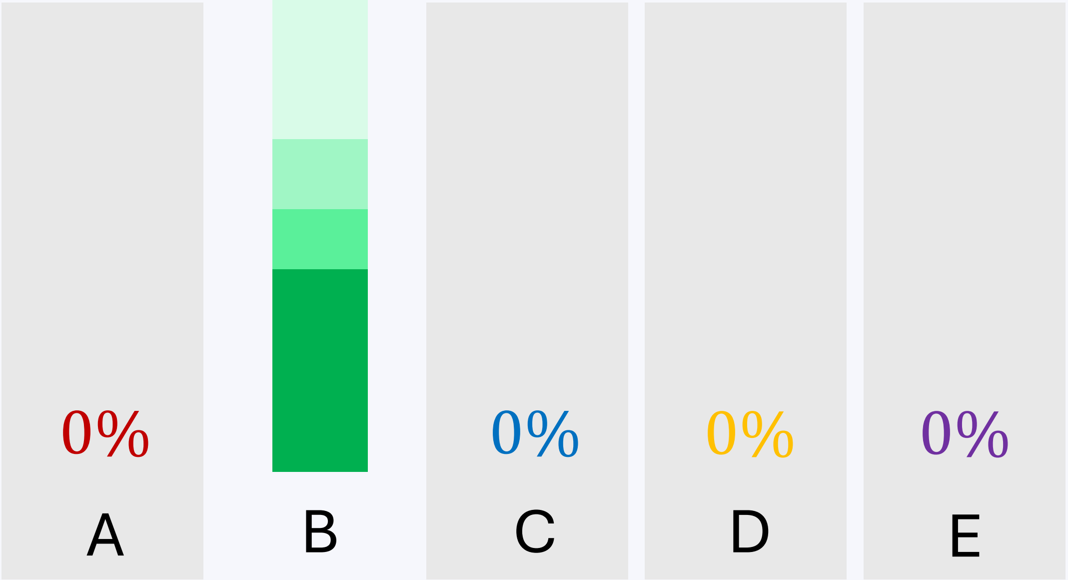
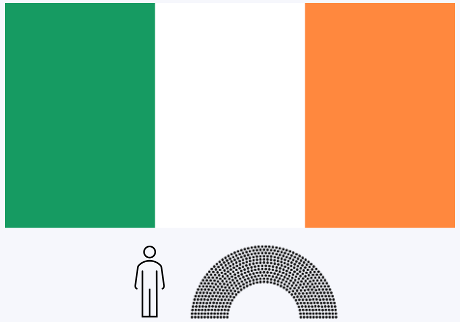


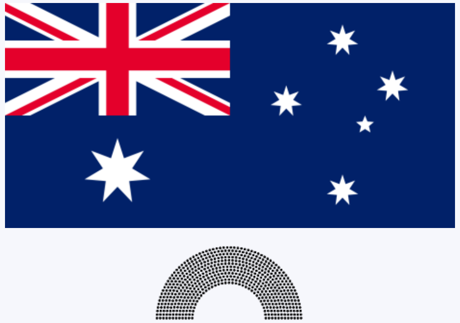
And so on, **until one candidate remains**

Reminder: Instant Runoff Voting (IRV)

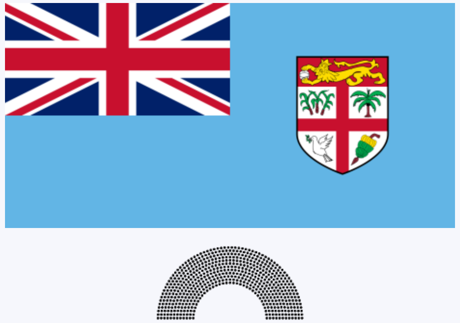
Ireland (since 1937)



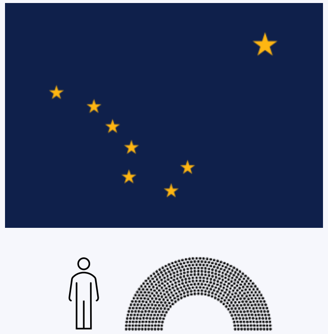
Australia (since 1918)



Fiji (since 1999)

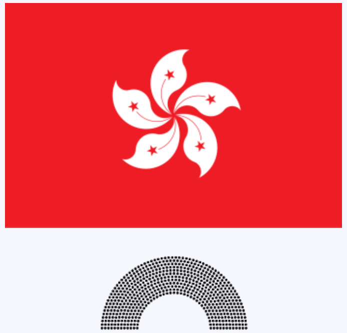


Alaska, USA (since 2022)



Maine, USA (since 2018)

Hong Kong (since 1998)



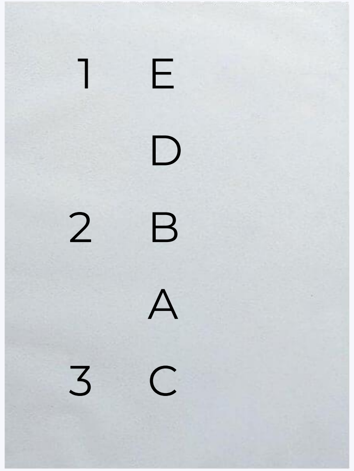
Also used for **primaries** in some countries

Properties of IRV

- ✓ **Independence of clones:** adding/removing a clone of a candidate does not change the results of the election.
- ✓ **Majority criterion:** if a majority of voters rank one candidate first, this candidate should win.
- ✗ **Monotonicity:** if a candidate is the winner and we improve its rank in some rankings, it should remain the winner.

Allowing for Indifferences

What if a voter is **indifferent** between several candidates?



Voters can cast **weak orders**

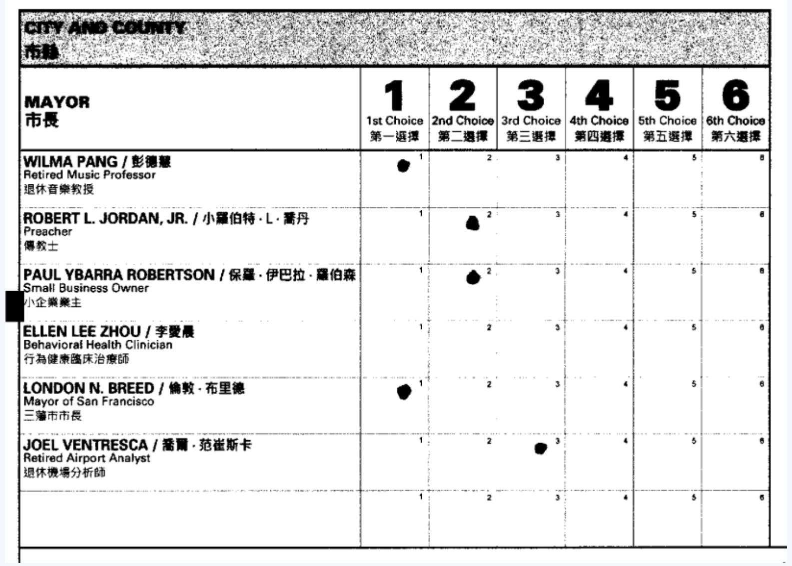


Fig. A ballot cast in San Francisco mayor election, containing indifferences

Question:
How to generalize Instant Runoff Voting to weak orders?

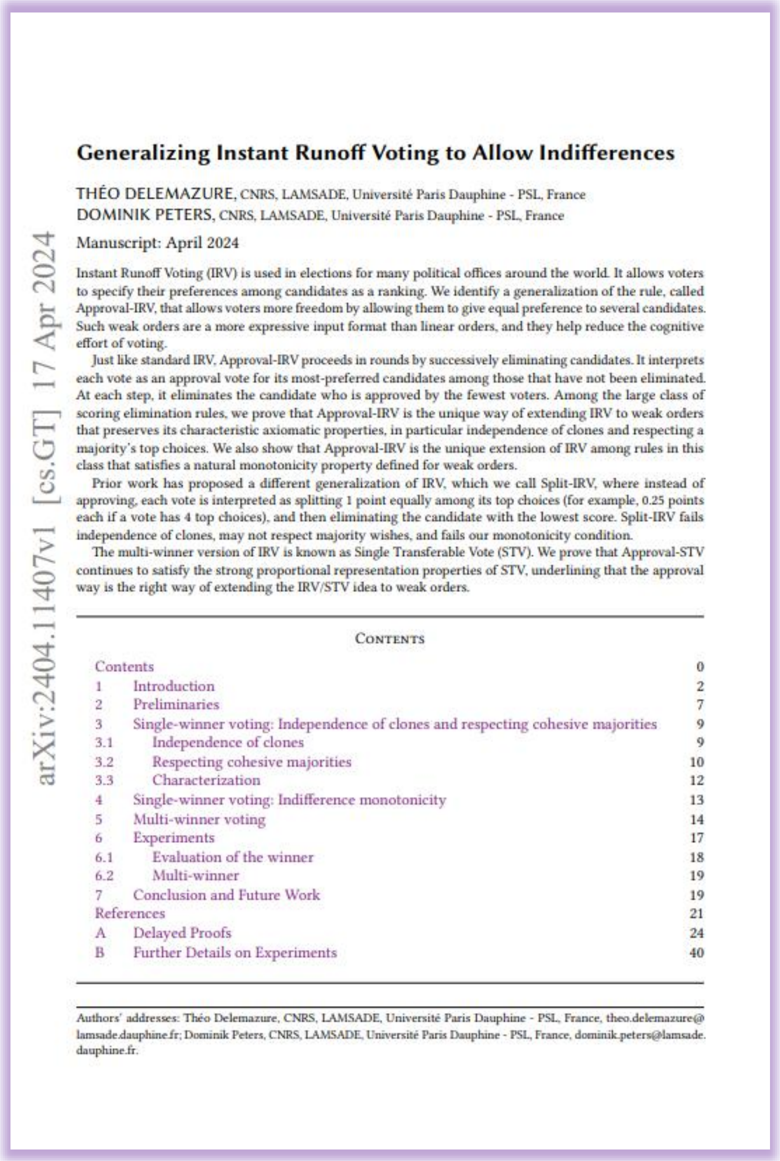
Outline of the talk

Part 1.
The different solutions

Part 2.
Axiomatic analysis

Part 3.
Experimental analysis

This talk is based on a paper published at **EC' 2024**, which is a joint work with *Dominik Peters, Paris-Dauphine University*.



arXiv:2404.11407v1 [cs.GT] 17 Apr 2024

Generalizing Instant Runoff Voting to Allow Indifferences

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Manuscript: April 2024

Instant Runoff Voting (IRV) is used in elections for many political offices around the world. It allows voters to specify their preferences among candidates as a ranking. We identify a generalization of the rule, called Approval-IRV, that allows voters more freedom by allowing them to give equal preference to several candidates. Such weak orders are a more expressive input format than linear orders, and they help reduce the cognitive effort of voting.

Just like standard IRV, Approval-IRV proceeds in rounds by successively eliminating candidates. It interprets each vote as an approval vote for its most-preferred candidates among those that have not been eliminated. At each step, it eliminates the candidate who is approved by the fewest voters. Among the large class of scoring elimination rules, we prove that Approval-IRV is the unique way of extending IRV to weak orders that preserves its characteristic axiomatic properties, in particular independence of clones and respecting a majority's top choices. We also show that Approval-IRV is the unique extension of IRV among rules in this class that satisfies a natural monotonicity property defined for weak orders.

Prior work has proposed a different generalization of IRV, which we call Split-IRV, where instead of approving, each vote is interpreted as splitting 1 point equally among its top choices (for example, 0.25 points each if a vote has 4 top choices), and then eliminating the candidate with the lowest score. Split-IRV fails independence of clones, may not respect majority wishes, and fails our monotonicity condition.

The multi-winner version of IRV is known as Single Transferable Vote (STV). We prove that Approval-STV continues to satisfy the strong proportional representation properties of STV, underlining that the approval way is the right way of extending the IRV/STV idea to weak orders.

Contents

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Part 1
Generalizations of IRV with indifferences

Model and notations

We have a set of **voters** $V = \{1, \dots, n\}$ and of **candidates** $C = \{c_1, \dots, c_m\}$.

A **weak order** $\succcurlyeq$ is a complete pre-order (reflexive, transitive and complete binary relation) over the set of candidates C.

We denote $\succcurlyeq = C_1 \succ C_2 \succ \cdots \succ C_k$ with $C_j \subseteq C$. We call the C_j the **indifference classes** of $\succcurlyeq$ and $\tau_{\succcurlyeq} = (|C_1|, \dots, |C_k|)$ its **order type**.

A **preference profile** is a collection of weak orders $P = (\succcurlyeq_1, \dots, \succcurlyeq_n)$.

A **voting rule** is a function that associates each profile to one or (in case of ties) several winning candidates.

Example

Preferences	Order type
$\{A, B\} \succ \{C\} \succ \{D\}$	$(2, 1, 1)$
$\{A, B, D\} \succ \{C\}$	$(3, 1)$
$\{B\} \succ \{A, C\} \succ \{D\}$	$(1, 2, 1)$
$\{C\} \succ \{A\} \succ \{B, D\}$	$(1, 1, 2)$
$\{D\} \succ \{A\} \succ \{B\} \succ \{C\}$	$(1, 1, 1, 1)$

How to design generalization?

We want to keep the idea of **repeatedly eliminating a candidate with the lowest score**, with score being computed at each step by looking at the **top indifference class** of every order.

Question: how should the score be computed?

$$\{\boldsymbol{A}, \boldsymbol{B}\} \succ \{C\} \succ \{D\}$$

$$\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{D}\} \succ \{C\}$$

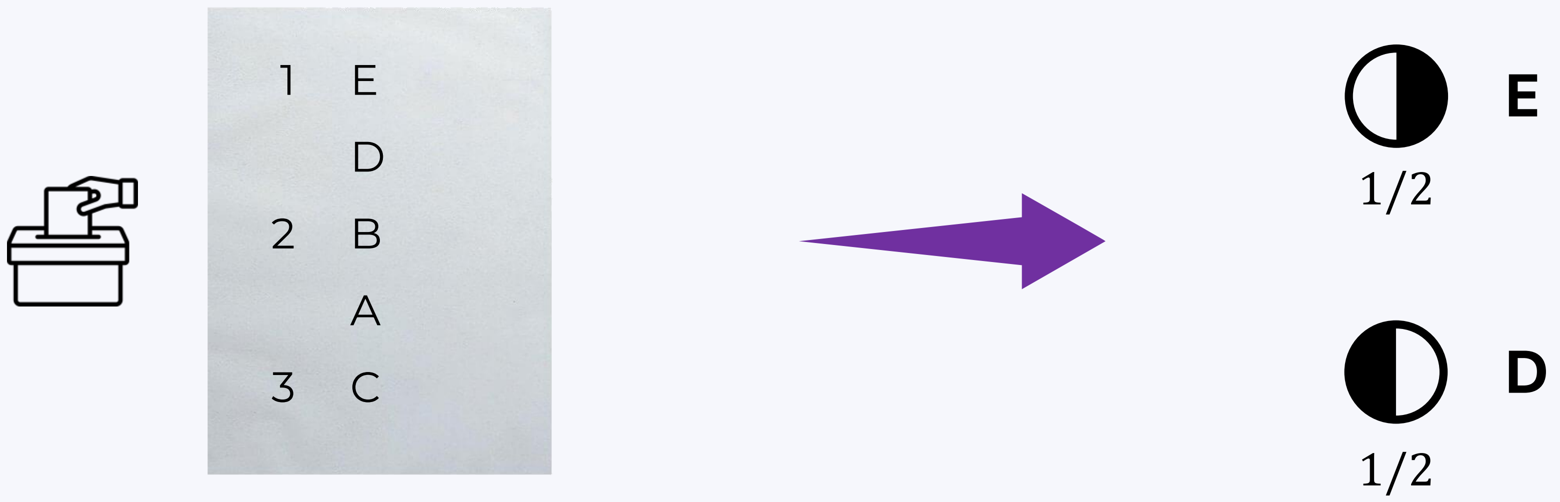
$$\{\boldsymbol{B}\} \succ \{A, C\} \succ \{D\}$$

$$\{\boldsymbol{C}\} \succ \{A\} \succ \{B, D\}$$

$$\{\boldsymbol{D}\} \succ \{A\} \succ \{B\} \succ \{C\}$$

Rule 1: Split-IRV

> Split-IRV *(Meek and Hill, 1994)*
>
> Each voter gives $1/k$ point to the k candidates that are tied as first among the remaining candidates in their ranking.

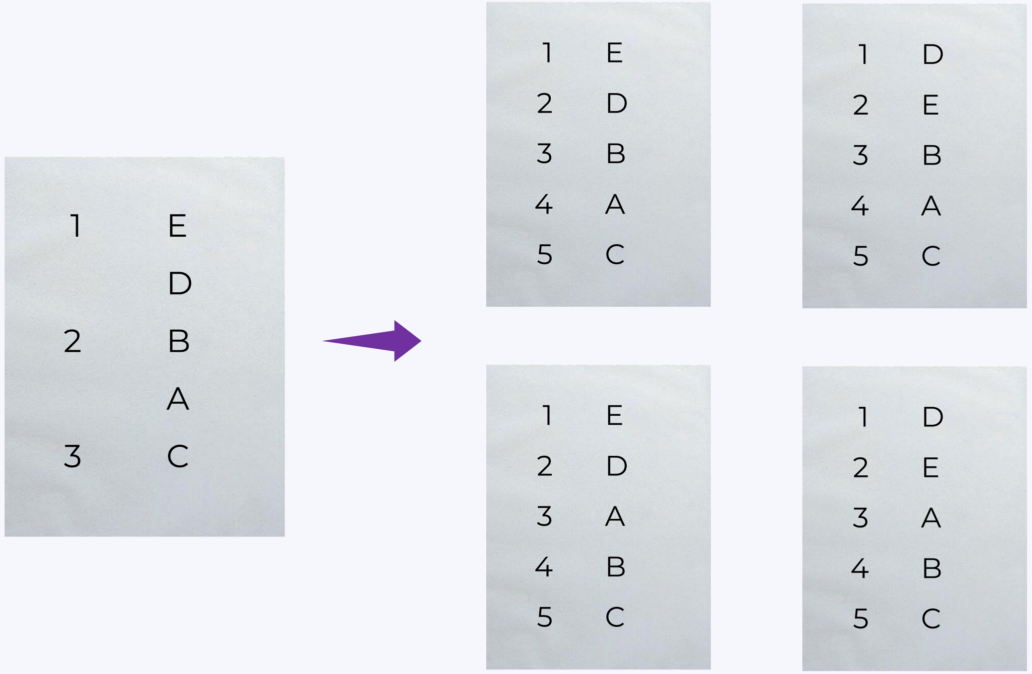


Rule 1: Split-IRV

Split-IRV *(Meek and Hill, 1994)*

Each voter gives $1/k$ point to the k candidates that are tied as first among the remaining candidates in their ranking.

Equivalent definition: if a weak order admit ℓ possible completions into a full ranking, replace every weak order by all its possible completions each with weight $1/\ell$ and compute the winner.



Rule 2: Approval-IRV

> Approval-IRV *(Janson, 2016)*
>
> Each voter gives 1 point to the k candidates that are tied as first among the remaining candidates in their ranking.

Example

		Approval-IRV	Split-IRV
$\{A, B\} \succ \{C\} \succ \{D\}$			
$\{A, B, D\} \succ \{C\}$	4^{th}		
$\{B\} \succ \{A, C\} \succ \{D\}$	3^{rd}		
$\{C\} \succ \{A\} \succ \{B, D\}$	2^{nd}		
$\{D\} \succ \{A\} \succ \{B\} \succ \{C\}$	1^{st}		

A: B: C: D:

Example

$\{A, B\} \succ \{C\} \succ \{D\}$

$\{A, B, D\} \succ \{C\}$

$\{B\} \succ \{A, C\} \succ \{D\}$

$\{C\} \succ \{A\} \succ \{B, D\}$

$\{D\} \succ \{A\} \succ \{B\} \succ \{C\}$

	Approval-IRV	Split-IRV
4th		
3rd		
2nd		
1st		

A: 2 B: 3 C: 1 D: 2

Example

$\{A, B\} \succ \{D\}$

$\{A, B, D\}$

$\{B\} \succ \{A\} \succ \{D\}$

$\{A\} \succ \{B, D\}$

$\{D\} \succ \{A\} \succ \{B\}$

	Approval-IRV	Split-IRV
4th	C	
3rd		
2nd		
1st		

A: 3 B: 3 C: 0 D: 2

Example

$\{A, B\}$

$\{A, B\}$

$\{B\} \succ \{A\}$

$\{A\} \succ \{B\}$

$\{A\} \succ \{B\}$

	Approval-IRV	Split-IRV
4th	C	
3rd	D	
2nd		
1st		

A: 4 B: 3 C: 0 D: 0

Example

$\{A, B\}$

$\{A, B\}$

$\{B\} \succ \{A\}$

$\{A\} \succ \{B\}$

$\{A\} \succ \{B\}$

	Approval-IRV	Split-IRV
4th	C	
3rd	D	
2nd	B	
1st	A	

$A: 4$ $B: 3$ $C: 0$ $D: 0$

Example

$\{A, B\} \succ \{C\} \succ \{D\}$

$\{A, B, D\} \succ \{C\}$

$\{B\} \succ \{A, C\} \succ \{D\}$

$\{C\} \succ \{A\} \succ \{B, D\}$

$\{D\} \succ \{A\} \succ \{B\} \succ \{C\}$

	Approval-IRV	Split-IRV
4^{th}	C	
3^{rd}	D	
2^{nd}	B	
1^{st}	A	

A: 1/2 + 1/3 B: 1/2 + 1/3 + 1 C: 1 D: 1/3 + 1

Example

$\{B\} \succ \{C\} \succ \{D\}$

$\{B, D\} \succ \{C\}$

$\{B\} \succ \{C\} \succ \{D\}$

$\{C\} \succ \{B, D\}$

$\{D\} \succ \{B\} \succ \{C\}$

	Approval-IRV	Split-IRV
4th	C	A
3rd	D	
2nd	B	
1st	A	

A: 0 B: 1 + 1/2 + 1 C: 1 D: 1/2 + 1

Example

$\{B\} \succ \{D\}$

$\{B, D\}$

$\{B\} \succ \{D\}$

$\{B, D\}$

$\{D\} \succ \{B\}$

	Approval-IRV	Split-IRV
4th	C	A
3rd	D	C
2nd	B	
1st	A	

$A: 0$ $B: 3$ $C: 0$ $D: 2$

Example

$\{B\} \succ \{D\}$

$\{B, D\}$

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$\{B, D\}$

$\{D\} \succ \{B\}$

	Approval-IRV	Split-IRV
4^{th}	C	A
3^{rd}	D	C
2^{nd}	B	D
1^{st}	A	B

$A: 0$ $\quad B: 3$ $\quad C: 0$ $\quad D: 2$

The family of **runoff scoring rules**

A **scoring system** is a function s that associates each order type τ to a scoring vector of the same length $s(\tau) = \left(s(\tau)_1, \ldots, s(\tau)_{|\tau|}\right)$. Given a weak order $\succcurlyeq\, = C_1 \succ C_2 \succ \cdots \succ C_k$ of order type τ, **candidates in indifference class C_j receive $s(\tau)_j$ points.**

A **runoff scoring rule** based on the scoring system s works by step and at each step it eliminates the candidate with the lowest score.

Approval-IRV is based on the scoring system such that $s(\tau) = (1,0, \ldots, 0)$ for all order types τ.

Split-IRV is based on the scoring system such that $s(\tau) = (1/\tau_1, 0, \ldots, 0)$ for all order types τ.

Part 2

An axiomatic comparison

Axiom 1: Independence of Clones

Independence of clones *(Tideman, 1987)*

Adding a *"clone"* of a candidate should not change significantly the result of the election.

$\{A, B\} \succ \{C\} \succ \{C'\}$

$\{A\} \succ \{C, C'\} \succ \{B\}$

$\{B, C, C'\} \succ \{A\}$

$\{C\} \succ \{C'\} \succ \{B\} \succ \{A\}$

The rule returns the same candidates in these two profiles

$\{A, B\} \succ \{C\}$

$\{A\} \succ \{C\} \succ \{B\}$

$\{B, C\} \succ \{A\}$

$\{C\} \succ \{B\} \succ \{A\}$

Axiom 1: Independence of Clones

> Independence of clones *(Tideman, 1987)*
>
> Adding a *"clone"* of a candidate should not change significantly the result of the election.

- ✓ When we restrict the profile to **full rankings**, **IRV** satisfies this axiom.
- ✓ **Approval-IRV** satisfies this axiom.
- ✗ **Split-IRV** fails this axiom.

Axiom 2: Respect for Cohesive Majorities

> Majority Criterion *(Lepelley, 1992)*
>
> If a majority of voters rank one candidate first, this candidate should be the winner.

✓ When we restrict the profile to **full rankings**, **IRV** satisfies this axiom.

✗ This axiom is **too strong** for the weak order case (and not desirable).

Axiom 2: Respect for Cohesive Majorities

> Respect for cohesive majorities
>
> If a majority of voters rank one candidate first, **the winner should be ranked first by one of these voters.**

- ✓ When we restrict the profile to **full rankings**, **IRV** satisfies this axiom.
- ✓ **Approval-IRV** satisfies this axiom.
- ✗ **Split-IRV** fails this axiom.

Characterization (1) of Approval-IRV

First characterization of Approval-IRV

Approval-IRV is the **only** runoff scoring rule for weak orders that satisfies **both** independence of clones and respect for cohesive majorities.

Axiom 3: Indifference Monotonicity

> Monotonicity *(Fishburn, 1982)*
>
> If some candidate is the winner, and we increase the rank of this candidate in one ranking, it should still win.

✕ IRV fails monotonicity

1	Eddy
2	Dan
3	Bob
4	Ann
5	Cora

Bob wins

→

1	Eddy
2	**Bob**
3	**Dan**
4	Ann
5	Cora

Bob still wins

Axiom 3: Indifference Monotonicity

> **Weak monotonicity**
>
> If some candidate is the winner, and we increase the rank of this candidate in one ranking **in which this candidate is not tied**, it should still win.

✗	Split-IRV
✓	Approval-IRV

1	Eddy
	Dan
2	Bob
3	Ann
	Cora

Bob wins →

1	Eddy
	Dan
	Bob
2	Ann
	Cora

Bob still wins

Characterization (2) of Approval-IRV

Second characterization of Approval-IRV

Approval-IRV is the **only** runoff scoring rule for weak orders that generalizes IRV and satisfies weak monotonicity

Part 3
Experimental analysis

What experiments to run?

1. **Generate** preference profiles.

2. **Compute** the Approval-IRV and Split-IRV winners.

3. **Analyze** the results:
   - How similar are the two rules in practice?
   - How similar are these rules to known SCF?
   - Which rule return the “best” winner?

Generating datasets

Step 1: Generate full rankings

How?

Step 2: Introduce indifferences

How?

Generating datasets

Step 1: Generate full rankings

How?

➜ **Synthetic data:**
Probabilistic models

➜ **Real data**

Step 2: Introduce indifferences

How?

Synthetic data: The impartial culture (IC)

In the **Impartial Culture (IC) model**, every possible ranking as the same probability to be sampled i.i.d. for each voter:

$$P(\succ_i = \succ) = \frac{1}{m!}$$

For all ranking $\succ \in L(A)$ and all voter $i \in V$.

Remark: IC is very simplistic and unrealistic so it should not be the only model used, but it is a frequently used model, so it serves as a baseline.

Synthetic data: Mallows' model

In a **Mallows' model**, all rankings are noisy approximations of a ground truth ranking. More formally, there exists a **central ranking** $\succ^*$ such that it is more likely to sample rankings closer to $\succ^*$. The distance between rankings is computed with the **Kendall-tau distance**:

$$d_{KT}(\succ_1, \succ_2) = |\{x, y \in C : x \succ_1 y \text{ and } y \succ_2 x\}|$$

Example: The KT distance between $a \succ_1 b \succ_1 c \succ_1 d$ and $c \succ_2 b \succ_2 a \succ_2 d$ is 3 because the rankings are disagreeing on 3 pairs of candidates.

Synthetic data: **Mallows' model**

Then, we sample rankings based on the central ranking $\succ^*$ and a **dispersion parameter** $\phi \in [0,1]$:

$$P(\succ_i = \succ \mid \succ^*, \phi) = \frac{\phi^{d_{KT}(\succ,\succ^*)}}{K}$$

with K a normalization constant.

Question: what happens when $\phi = 0$? And when $\phi = 1$?

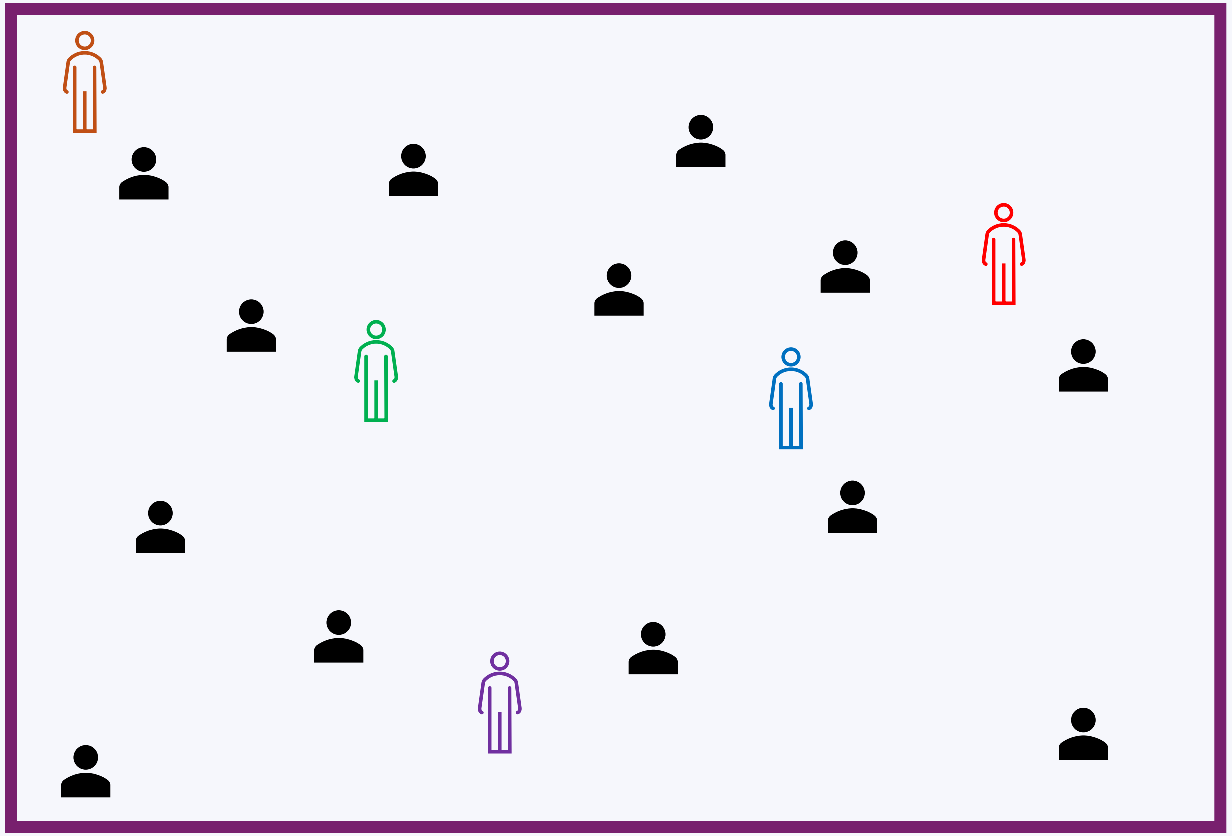
Mixture of Mallows

In a **mixture of k Mallows**, there are k central rankings $(\succ_1^*, \dots, \succ_k^*)$ and probabilities $(p_1, \dots, p_k)$ with $\sum p_j = 1$. For each voter, we select one Mallows according to the probabilities $(p_j)_j$ and we draw a random ranking according to the Mallows model with central ranking $\succ_j^*$ and dispersion ϕ.

This enables to have **more diversity** in the preferences.

Synthetic data: Euclidean Preferences

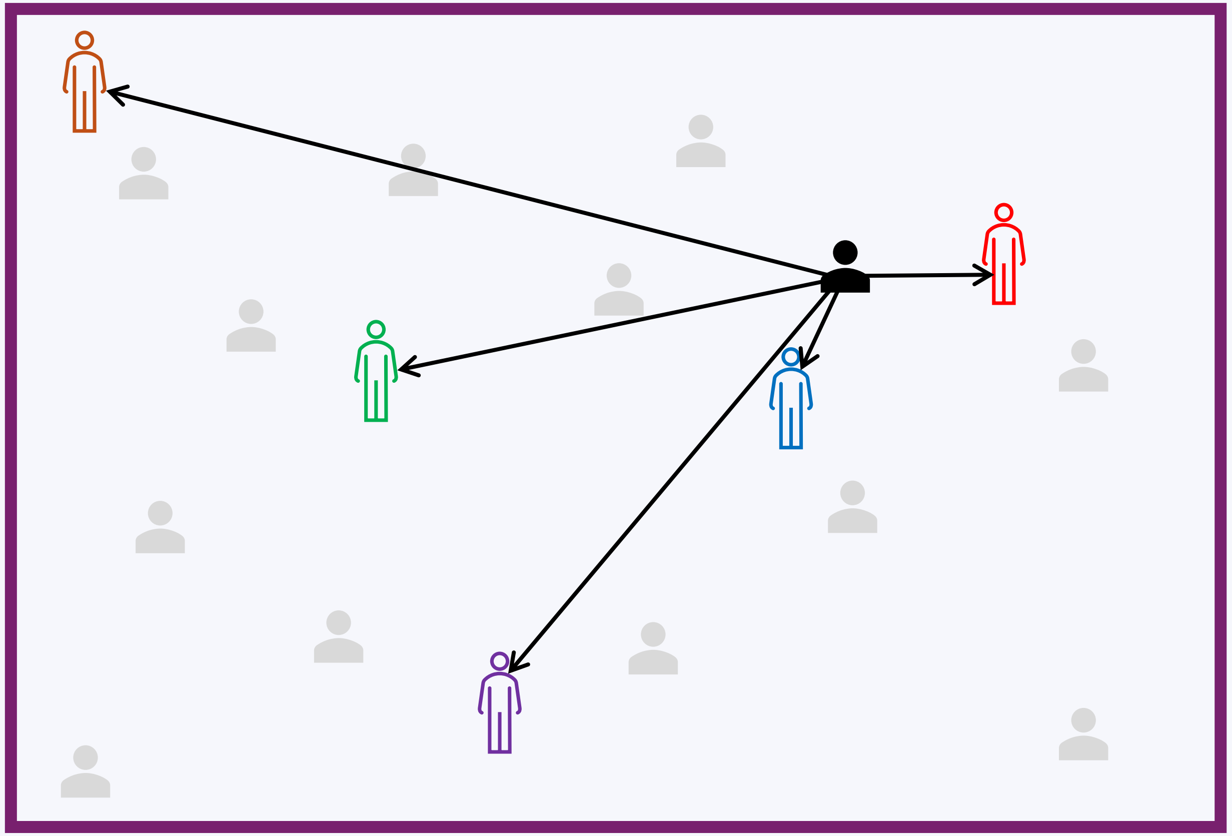
Voters   Candidates



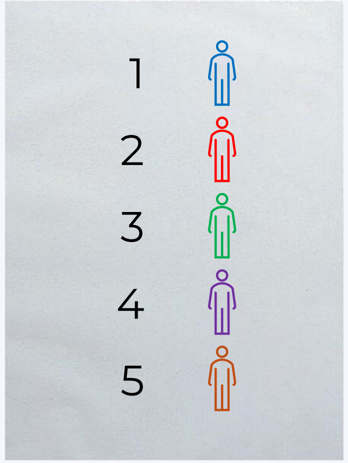
» Positions of voters and candidates are sampled randomly in the space.

Synthetic data: Euclidean Preferences

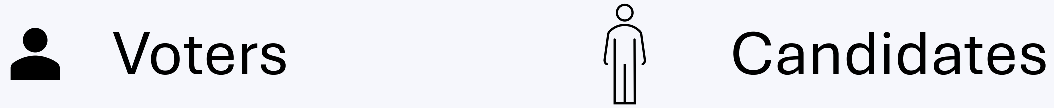
Voters Candidates

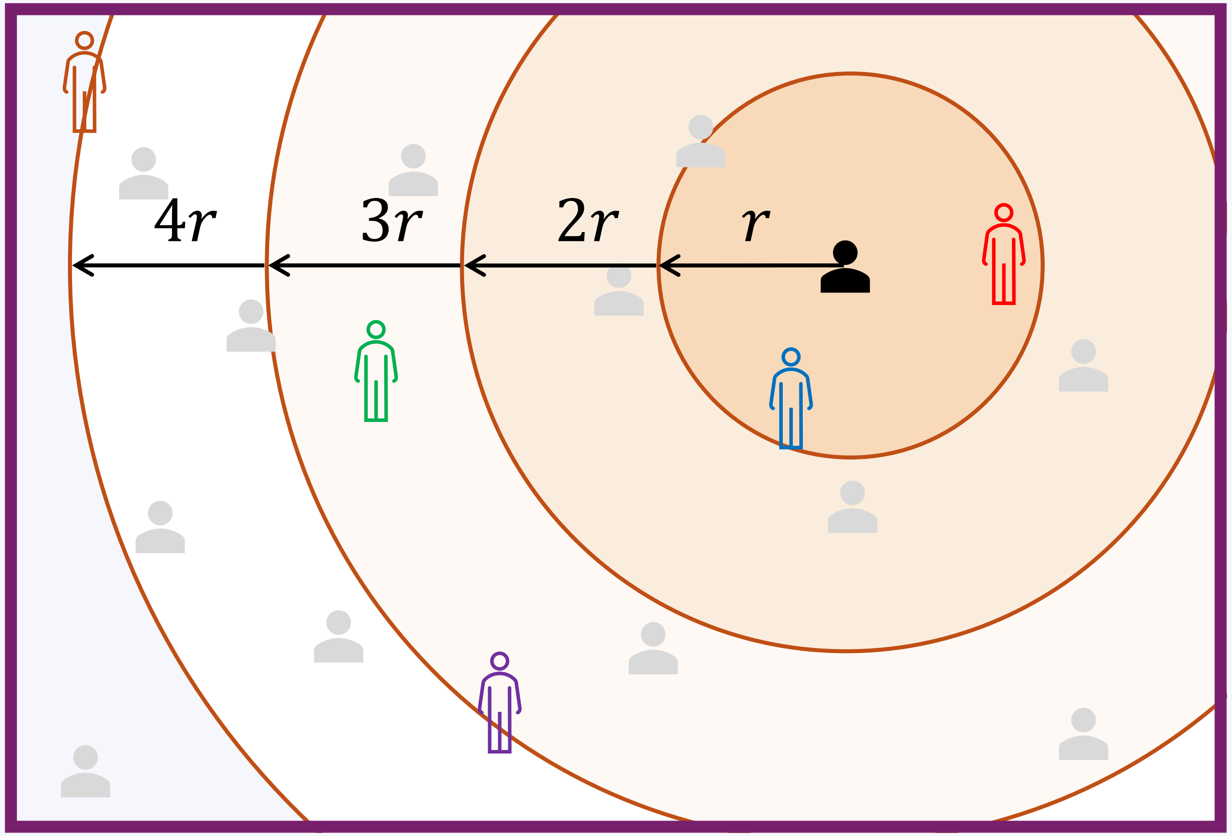


Voters prefer candidates that are closer to them:

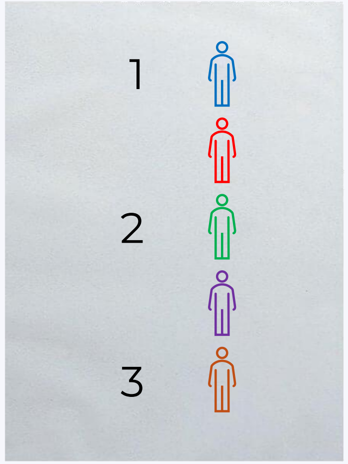


Synthetic data: Euclidean Preferences





- We can also obtain weak orders:



Synthetic data: The map of elections

- Preference profiles sampled from **various probabilistic models**.
- Similar profiles appear **close to each other** on the map of elections.

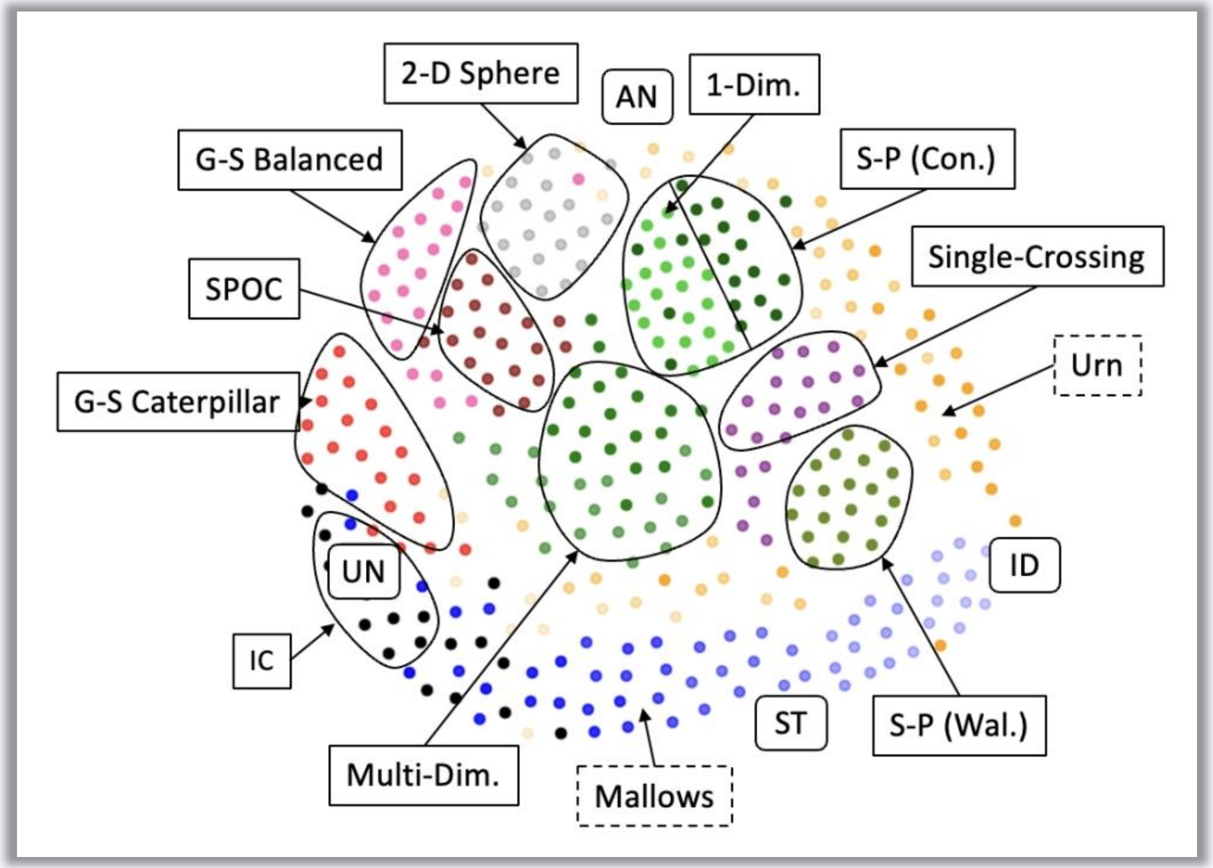
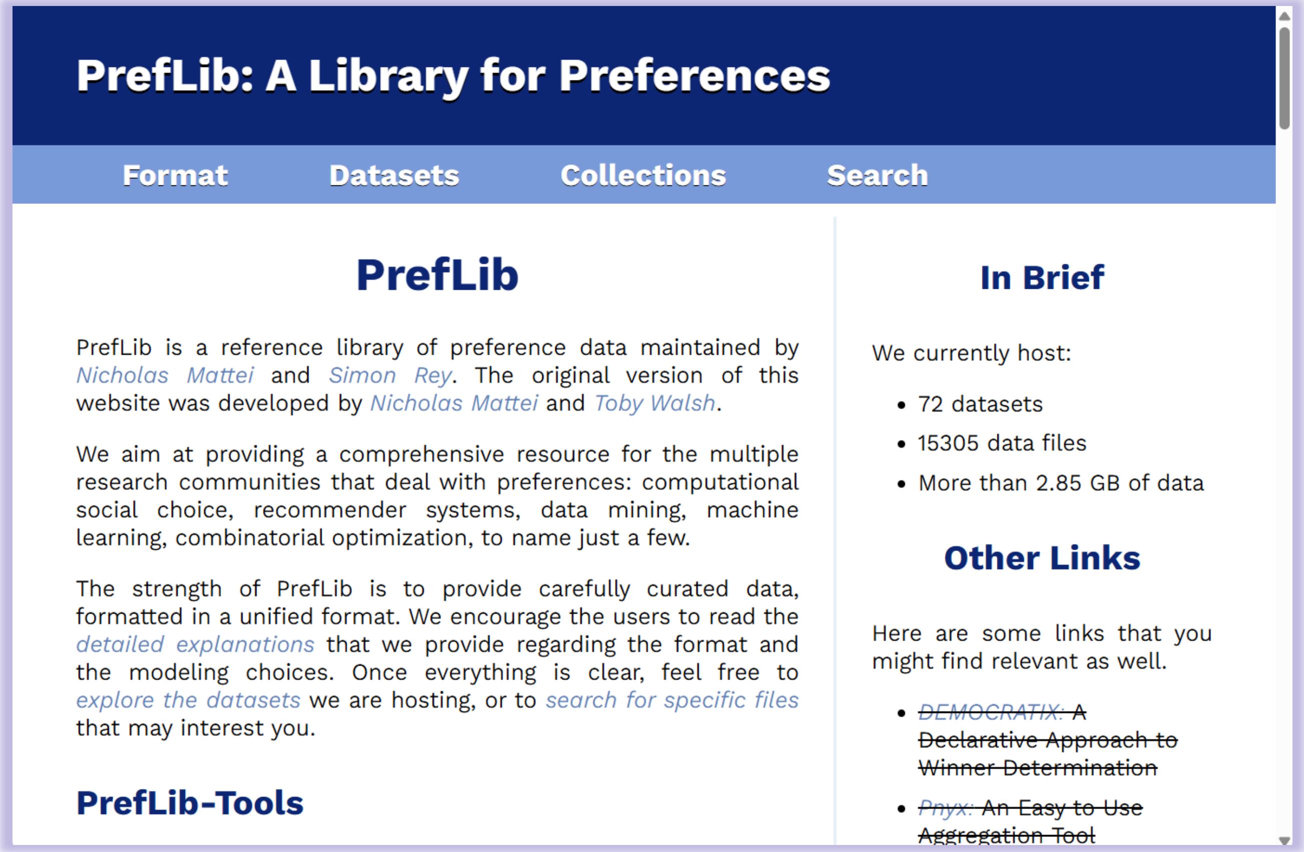


Figure. Map of elections with the isomorphic swap distance. Picture from *Boehmer et al. (2022b)*

Real data: **Preflib.org**

- Preference profiles from **real-world** elections.
- Also contains **tools** for computational social choice.

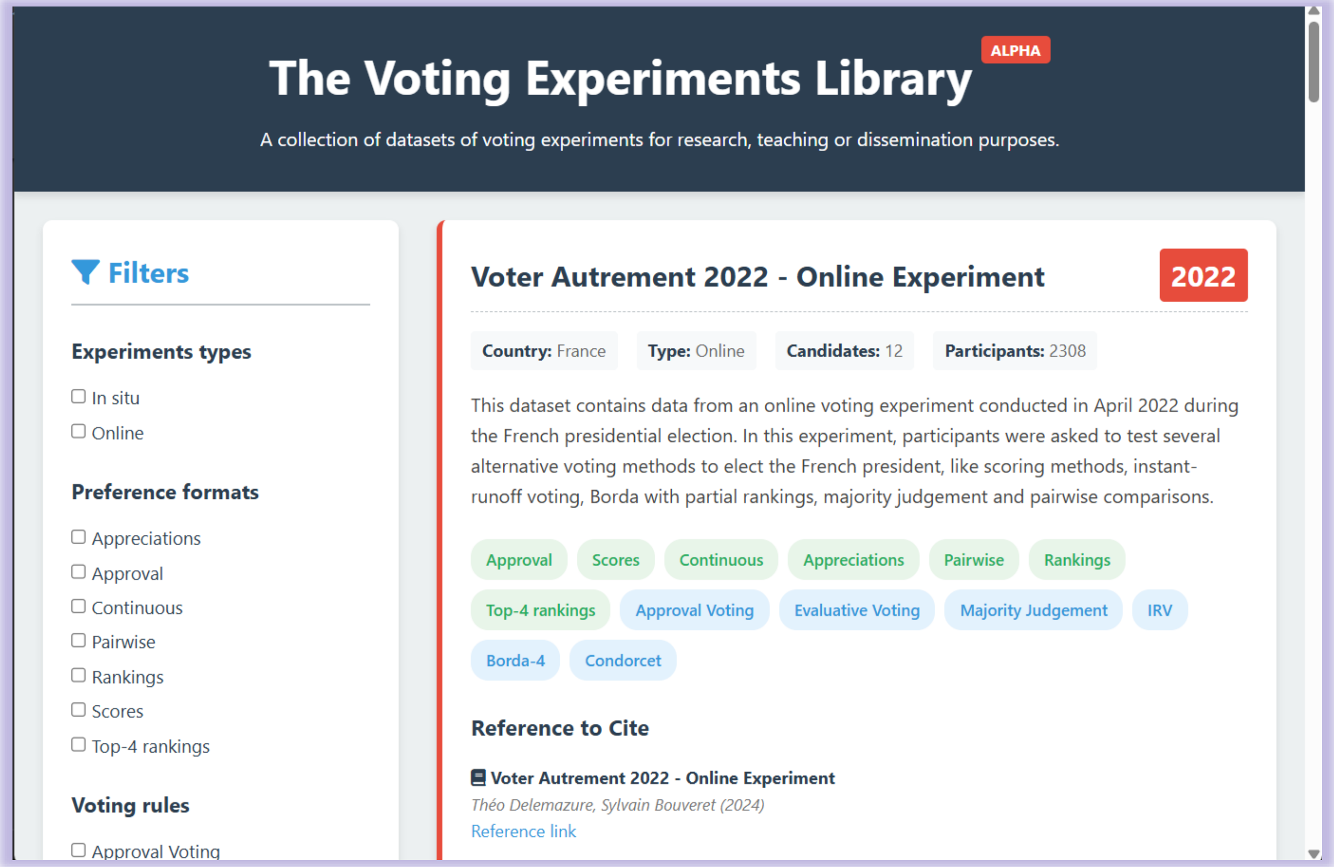
We take the **Irish** dataset from Preflib.



Screenshot from preflib.org

Real data: *Voter Autrement*

- Preference profiles from **voting experiments** in parallel to large-scale political elections.



Screenshot from [theo.delemazure.fr/datasets/]

Real data: Collect your own dataset

- If you want a very **specific preference format** or data type, you can run your own experiments.
- You can either **build a website** (with helps of LLM) or use simple tools such as Google form or pref.tools/vote/
- You can **share it to friends or on social media** (but it will not yield a representative sample).

From rankings to weak orders

Given a full ranking and a parameter $p \in [0,1]$, we consider each pair of successive candidates in the ranking and **add a tie between them with probability p.**

$$A \succ B \succ C \succ D \succ E$$

$$p \quad p \quad p \quad p$$

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Given a full ranking and a parameter $p \in [0,1]$, we consider each pair of successive candidates in the ranking and **add a tie between them with probability p.**

$$A \sim B \succ C \succ D \sim E$$

$$p \quad p \quad p \quad p$$

$$\{A, B\} \succ C \succ \{D, E\}$$

Experiment **settings**

For each dataset, average over **10 000 sampled profiles.**

Profiles of $n = 500$ voters and $m = 10$ candidates.

Indifference parameters p and r varying between 0 (full rankings) and 1 (complete indifference).

Results: Similarities between rules

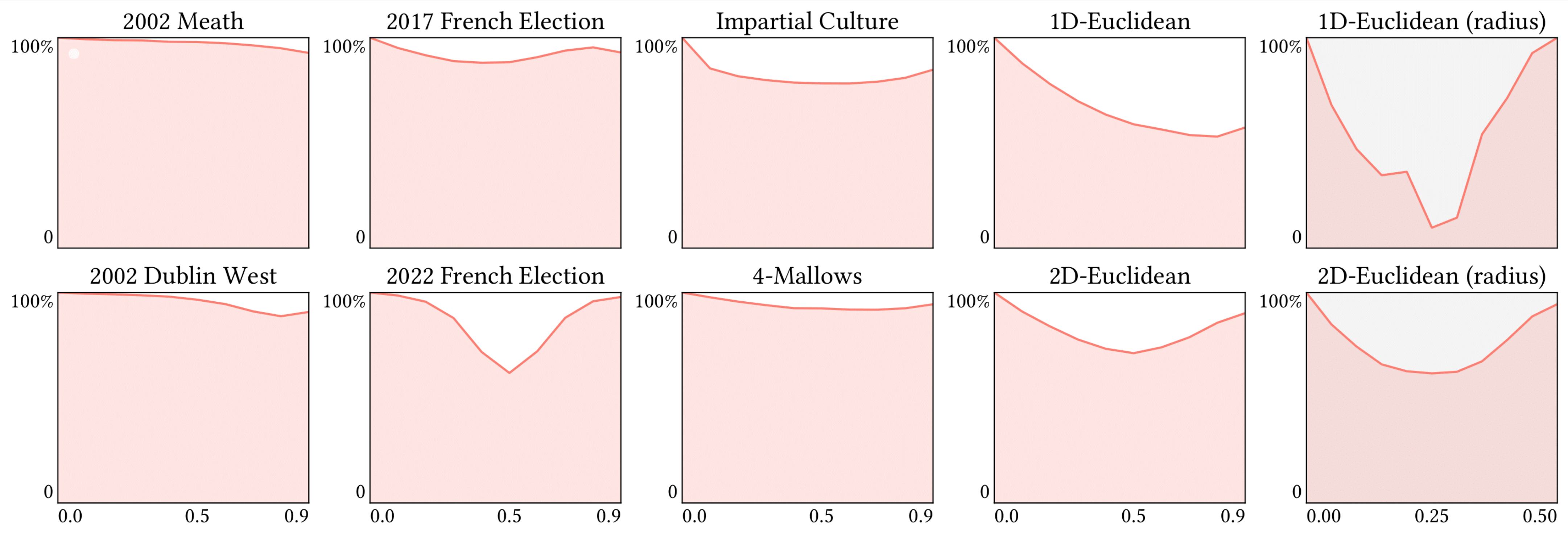


Figure. Frequency of agreement between Approval-IRV and Split-IRV on our datasets (depending on the value of the indifference parameter p or r).

Results: Similarities with classical SCF

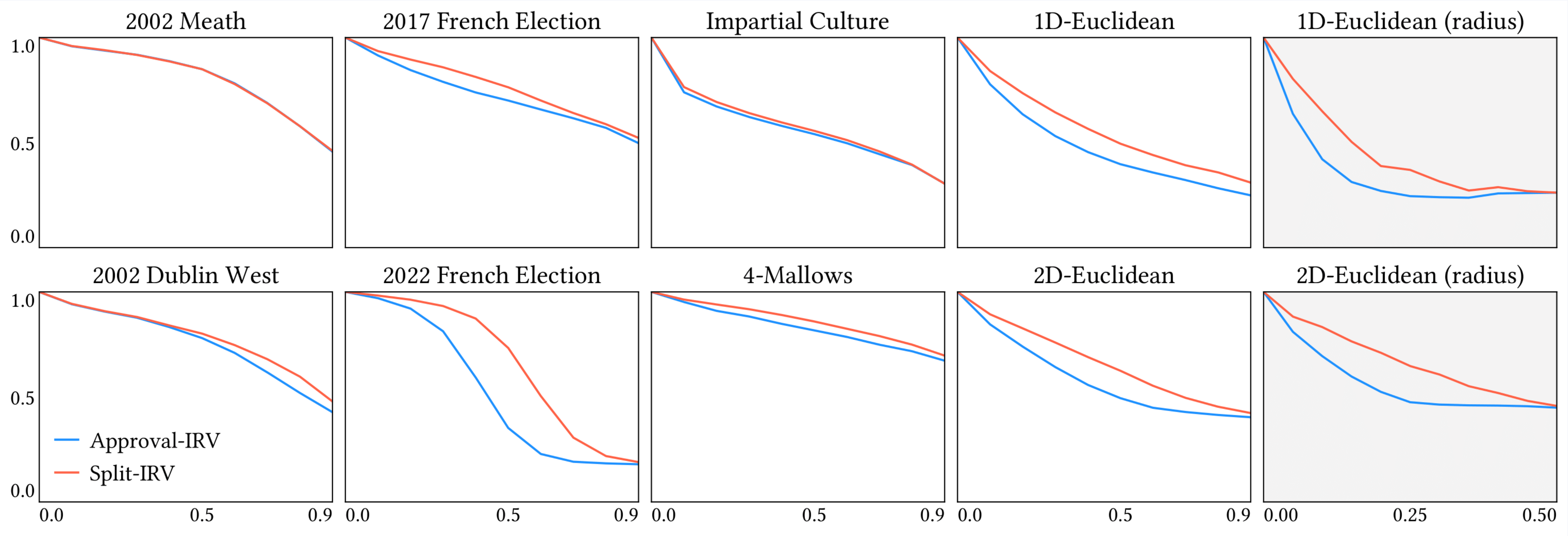


Figure. Frequency of agreement between the rules and **linear-order IRV** (based on full rankings) on our datasets. (depending on the value of the indifference parameter p or r).

Results: Similarities with classical SCF

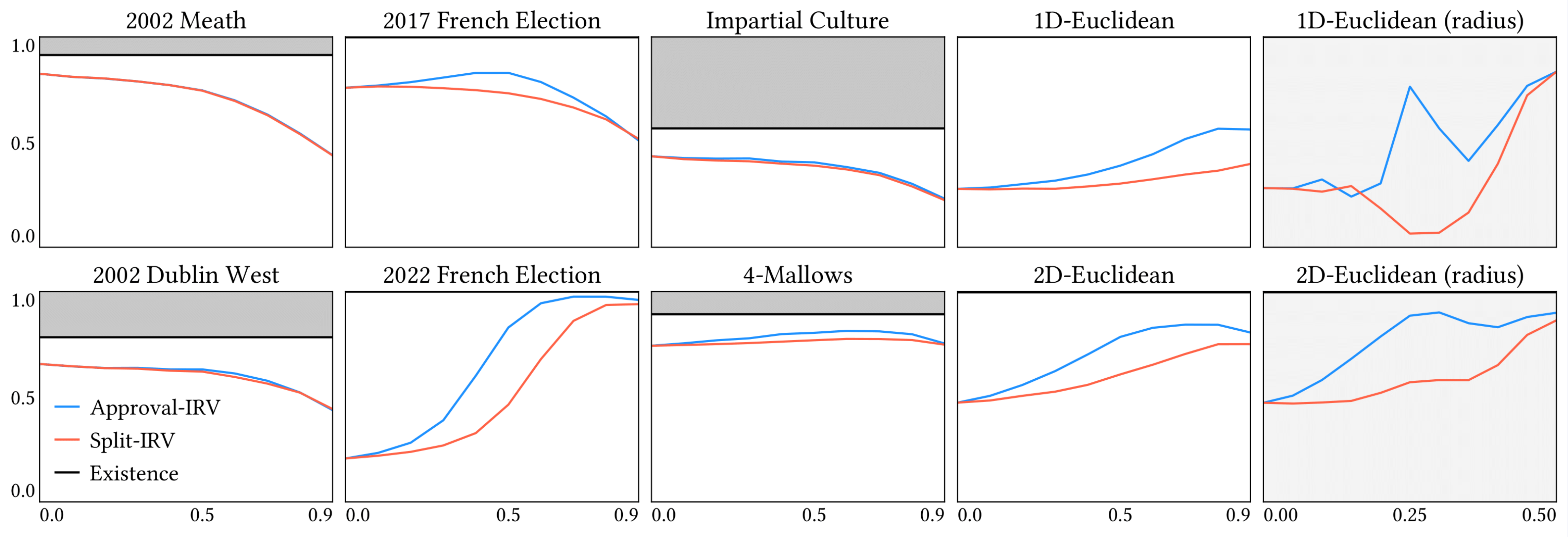


Figure. Frequency of finding the Condorcet winner, and frequency of such candidate existing on our datasets. (depending on the value of the indifference parameter p or r).

Results: Similarities with classical SCF

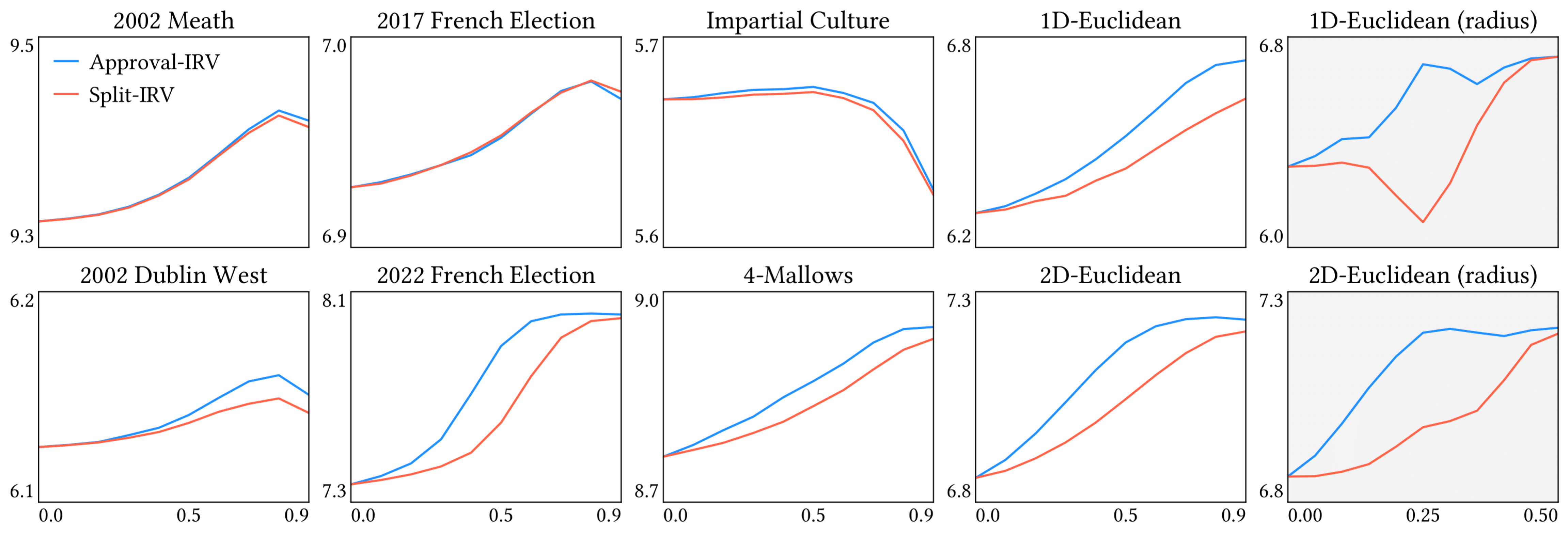
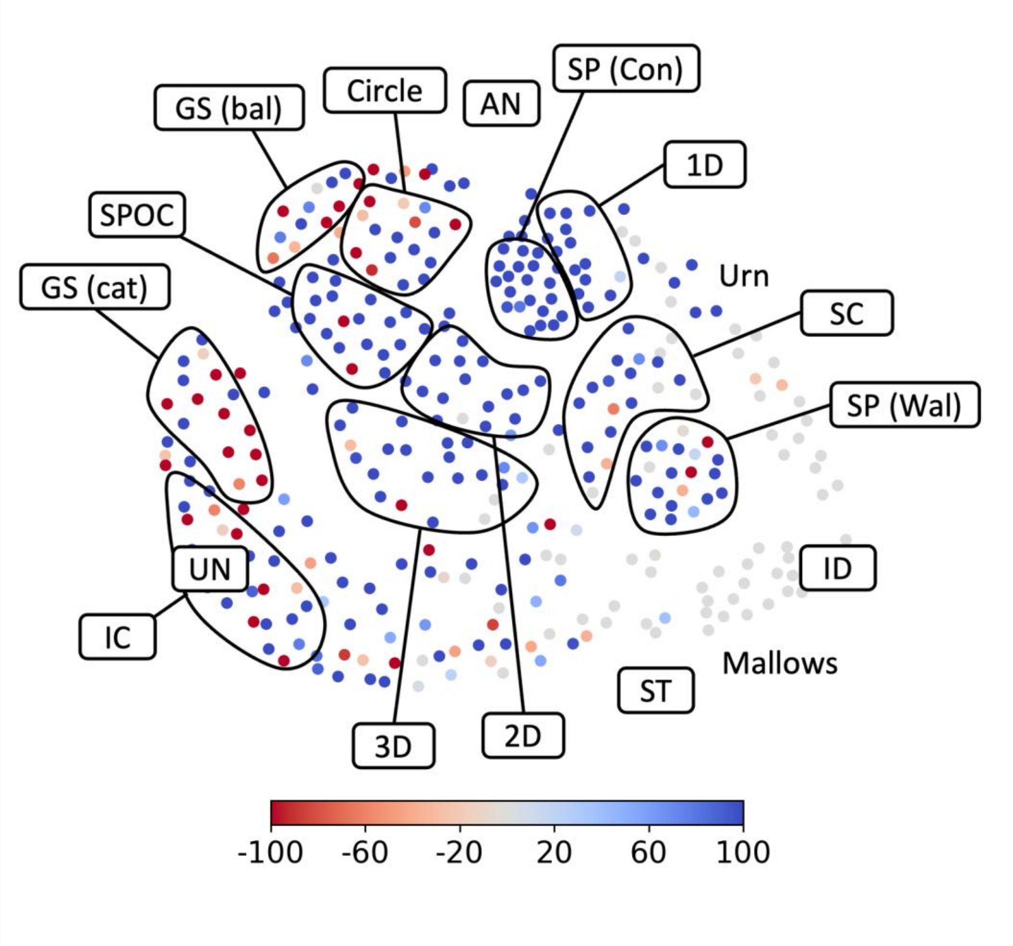


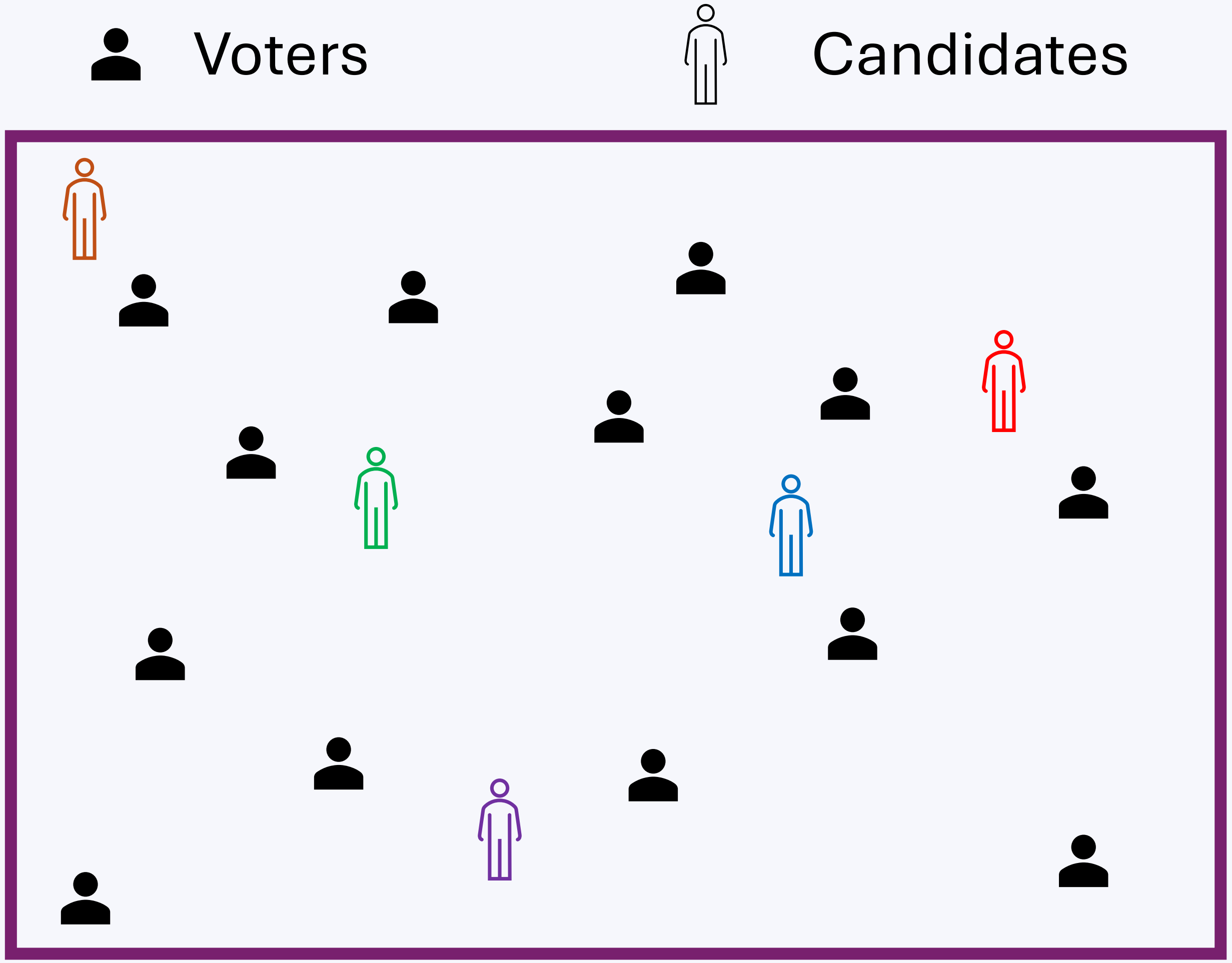
Figure. Average Borda score of the winner (normalized by dividing by the number of voters) on our datasets. (depending on the value of the indifference parameter p or r).

Results: Similarities with classical SCF

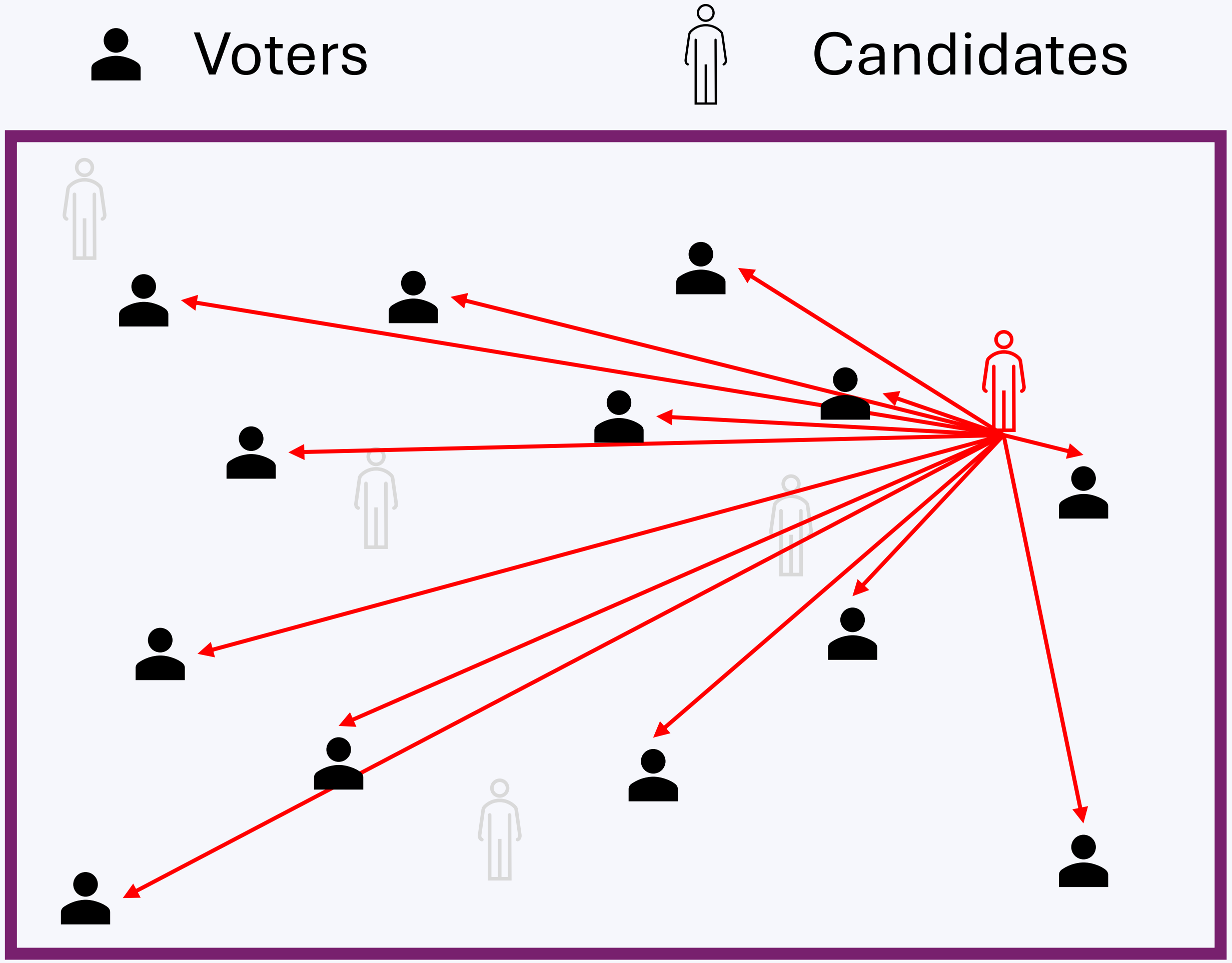
Figure. Map of elections, showing the difference in Borda score between the Approval-IRV and Split-IRV winners in the coin-flip model, with blue dots indicating that Approval-IRV selected on average a winner with higher Borda score.



Results: Utilitarian perspective



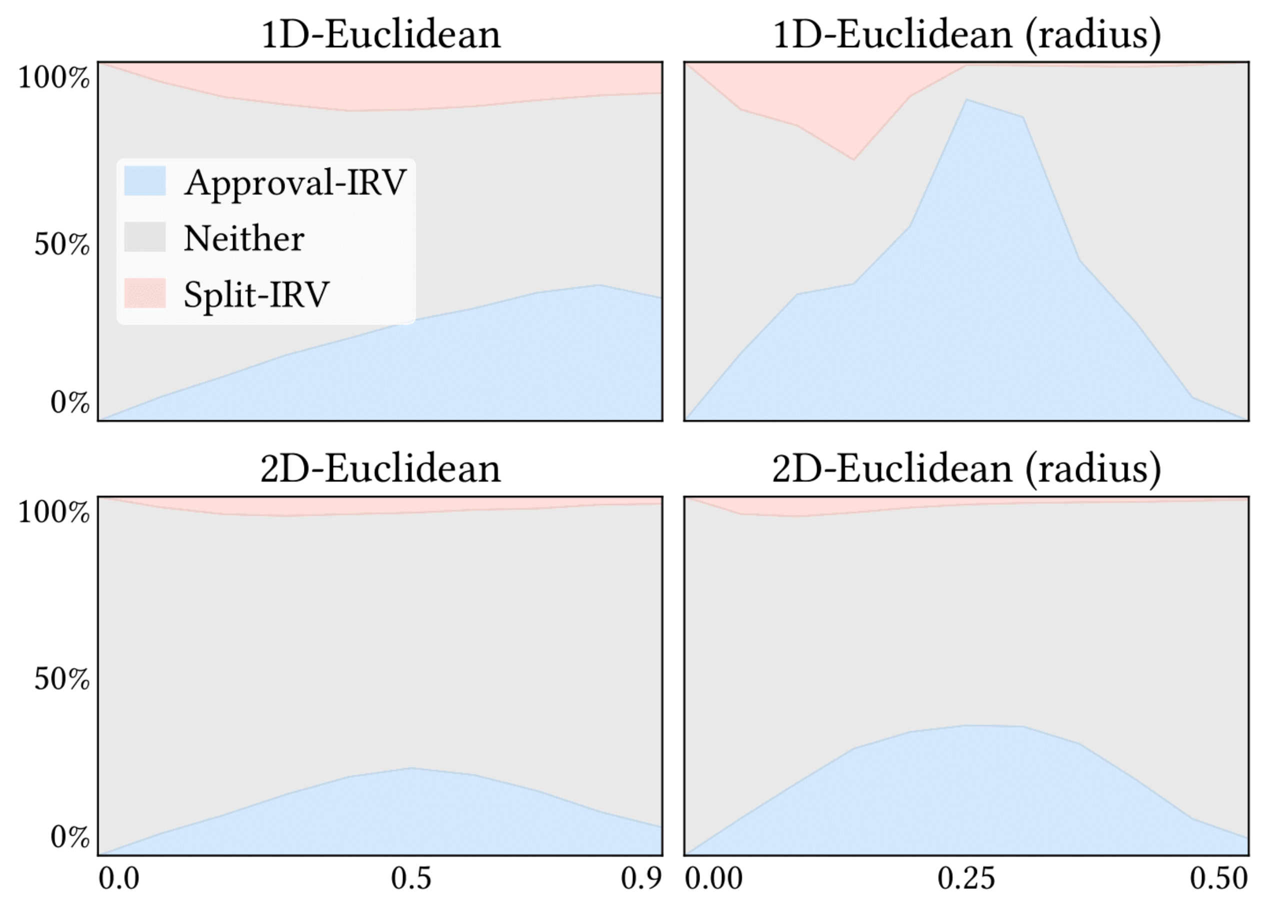
Results: Utilitarian perspective



- The **cost** of a candidate is the sum of its distance to all the voters.
- **The lower the cost, the better**

Results: Utilitarian perspective

Figure. The frequency of returning the candidate with the lowest cost for each rule.



Conclusion

Conclusion

Approval-IRV is the only rule that satisfies the generalization in the weak order setting of **desirable axioms satisfied by IRV.**

Approval-IRV is the only generalization of IRV to the weak order setting that **satisfies a weak monotonicity property.**

Empirically, Split-IRV will return the IRV winner more often while Approval-IRV will look for a more consensual candidate (because of its “approval” part).

In the Euclidean setting, Approval-IRV **often return better winners** than Split-IRV from a utilitarian perspective.

Also in the paper: Generalization of STV, the multi-winner versions of IRV.

Thanks for your attention!