

# Friends or Foes?

# Inferring Latent Candidate Structures from Preference Rankings

*Théo Delemazure – ILLC, University of Amsterdam*

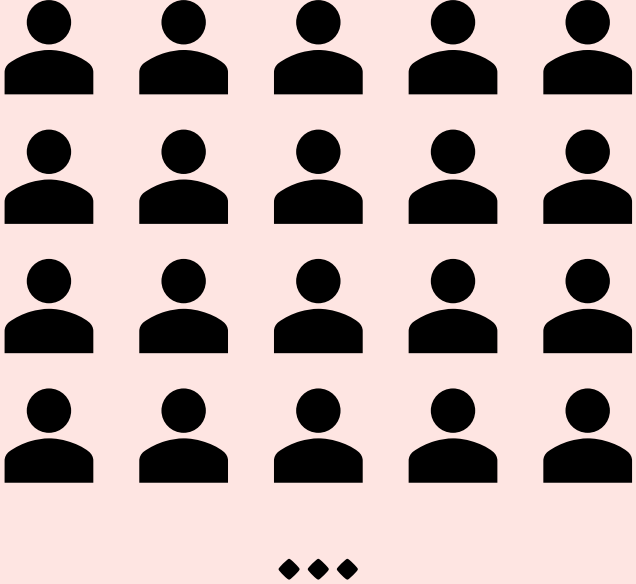


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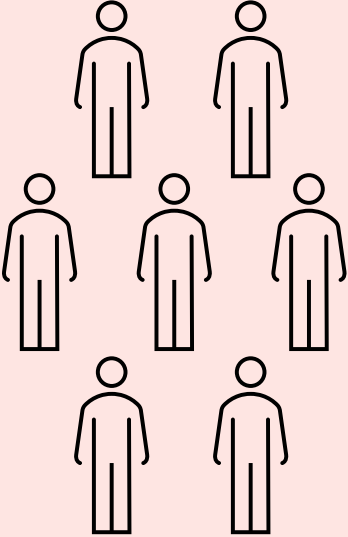
# **Learning From Preferences**

# Computational Social Choice Setting

## Voters

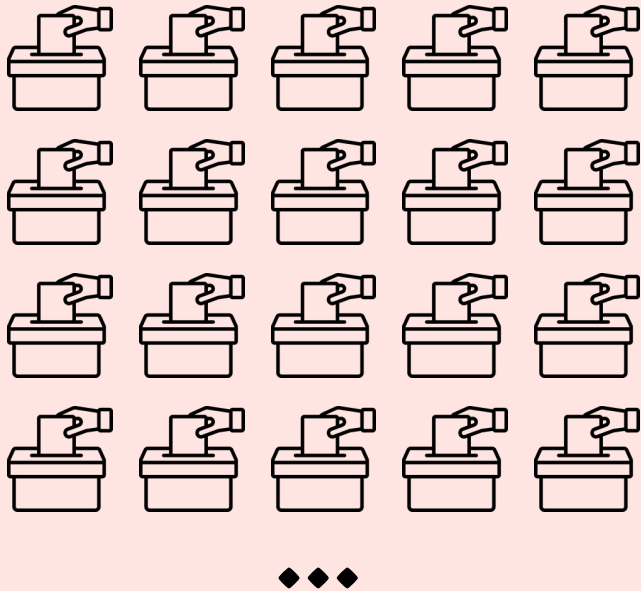


## Candidates

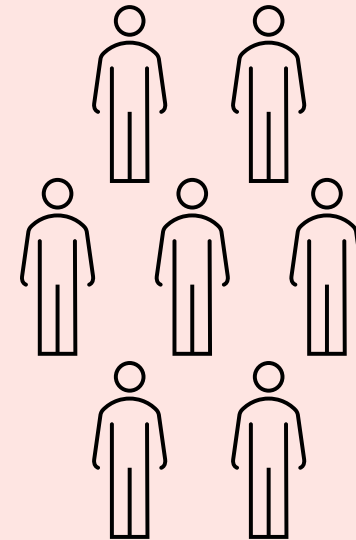


# Computational Social Choice Setting

**Voters** give their preferences over candidates

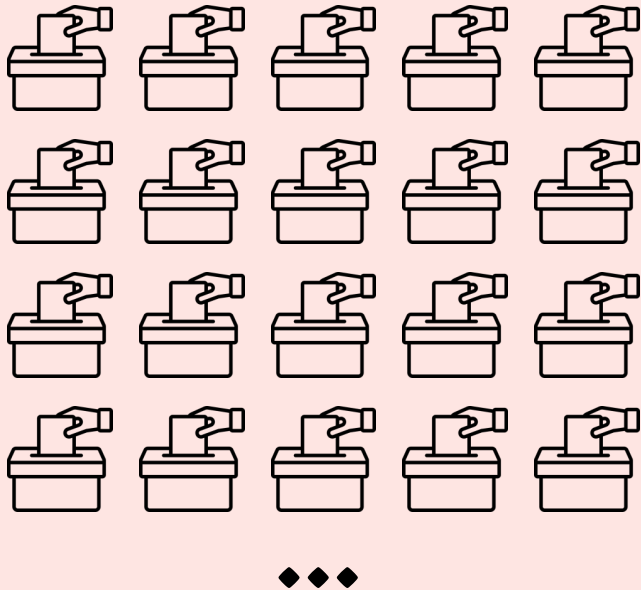


**Candidates**



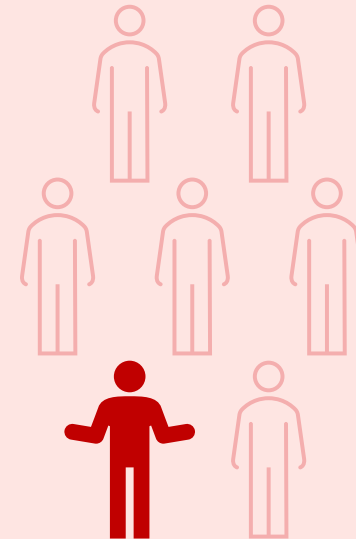
# Computational Social Choice Setting

**Voters** give their preferences over candidates



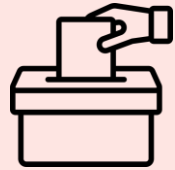
A **winner** is selected

**Voting rule**

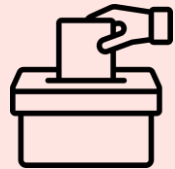


# Computational Social Choice Setting

Voters' preferences are **rankings** of the candidates



$A \succ B \succ C \succ D \succ E$



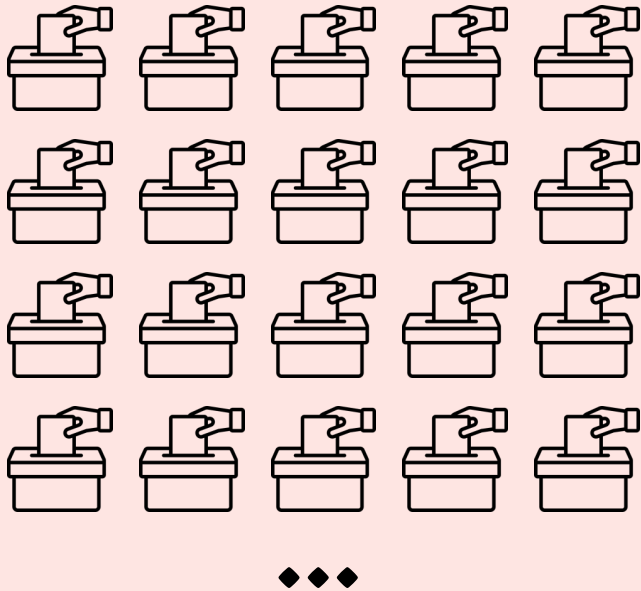
$C \succ D \succ B \succ A \succ E$



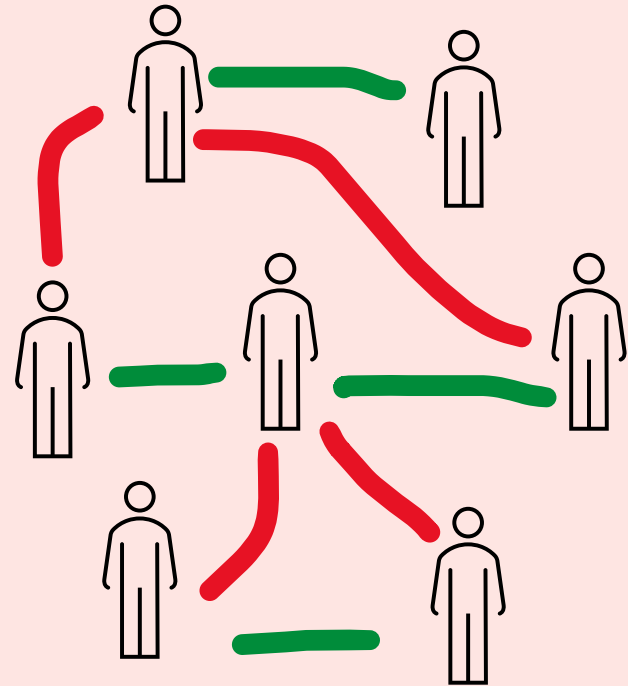
$D \succ E \succ C \succ B \succ A$

# Learning **Structures** from Preferences

**Voters** give their preferences over candidates



**(dis)similarities** between candidates



# Different **Levels** of Structure

At the **full profile** level:

50%  $A \succ B \succ C \succ D \succ E$

50%  $E \succ D \succ C \succ B \succ A$

*“This society is polarized as a whole”*

[Can et al, 2015] Measuring polarization in preferences. *Mathematical Social Sciences*.

[Faliszewski et al, 2023] Diversity, agreement, and polarization in elections. *IJCAI 2023*.

[Hashemi and Endriss, 2014] Measuring diversity of preferences in a group. *ECAI 2014*.

# Different **Levels** of Structure

At the **candidate/proposal** level:

50%  $A \succ B \succ C \succ D \succ E$

50%  $B \succ C \succ D \succ E \succ A$

*“**A** is very polarizing in the society”*

**[Colley et al, 2023]** Measuring and controlling divisiveness in rank aggregation. *IJCAI 2023*.

**[Endriss, 2025]** On the Difficulty of Measuring Divisiveness of Proposals under Ranked Preferences. *Working paper*.

# Different **Levels** of Structure

At the **candidate pairs** level:

50%  $A \succ C \succ D \succ E \succ B$

50%  $B \succ C \succ D \succ E \succ A$

*“**A** and **B** are inducing a lot of conflict in the society”*

[Delemazure et al., 2024] Selecting the Most Conflicting Pair of Candidates. *IJCAI 2024*.

# Types of Interactions

*A* > *C* > *D* > *E* > *B*

*A* > *D* > *C* > *E* > *B*

*A* > *E* > *D* > *C* > *B*

*B* > *E* > *D* > *C* > *A*

*B* > *D* > *C* > *E* > *A*

*B* > *C* > *D* > *E* > *A*

*A* > *B* > *C* > *D* > *E*

*D* > *B* > *A* > *C* > *E*

*E* > *D* > *B* > *A* > *C*

*B* > *A* > *E* > *D* > *C*

*D* > *C* > *E* > *B* > *A*

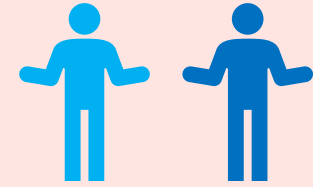
*C* > *B* > *A* > *D* > *E*



**Question:** how to **measure** how conflicting/similar two candidates are based on ranked preferences of voters?

**Question:** how to **find the pairs** of candidates that causes the most conflict/that are the most similar?

# Methodology



We define measures that assign a **conflict/similarity score** to pairs of candidates

$$S(A, B) = 0.8$$

How to compare these measures and decides which one should be used?

## **Axiomatic Analysis**

*Which measures satisfy which desirable normative properties?*

## **Experimental Analysis**

*How do the measures behave in practice on actual preference data?*

# Outline of the Talk

## 0 Introduction: Learning from Preferences

## 1 Finding the Most Divisive Pair of Candidates

- Measures
- Rules
- Axioms
- Experiments

## 2 Finding Approximate Clones

- Measures
- Axioms
- Experiments
- Link to Independence of Clones

### Selecting the Most Conflicting Pair of Candidates (IJCAI 2024)

Théo Delemazure, Łukasz Janeczko, Andrzej Kaczmarczyk, and Stanisław Szufa

arXiv:2401.20779v1 [cs.GT] 28 Jan 2026

#### Independence of Approximate Clones

THÉO DELEMAZURE, ILLC, University of Amsterdam, Netherlands  
Manuscript: January 2026

In an ordinal election, two candidates are said to be perfect clones if every voter ranks them adjacently. The independence of clones axiom then states that removing one of the two clones should not change the election outcome. This axiom has been extensively studied in social choice theory, and several voting rules are known to satisfy it (such as IRV, Ranked Pairs and Schulze). However, perfect clones are unlikely to occur in practice, especially for political elections with many voters.

In this work, we study different notions of approximate clones in ordinal elections. Informally, two candidates are approximate clones in a preference profile if they are close to being perfect clones. We discuss two measures to quantify this proximity, and we show under which conditions the voting rules that are known to be independent of clones are also independent of approximate clones. In particular, we show that for elections with at least four candidates, none of these rules are independent of approximate clones in the general case. However, we find a more positive result for the case of three candidates. Finally, we conduct an empirical study of approximate clones and independence of approximate clones based on three real-world datasets: votes in local Scottish elections, votes in mini-jury deliberations, and votes of judges in figure skating competitions. We find that approximate clones are common in some contexts, and that the closest two candidates are to being perfect clones, the less likely their removal is to change the election outcome, especially for voting rules that are independent of perfect clones.

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Author's Contact Information: Théo Delemazure, ILLC, University of Amsterdam, Netherlands, theo.delemazure@uva.nl

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### Selecting the Most Conflicting Pair of Candidates

Théo Delemazure<sup>1</sup>, Łukasz Janeczko<sup>2</sup>

Andrzej Kaczmarczyk<sup>2</sup>, Stanisław Szufa<sup>1,2</sup>

<sup>1</sup>CNRS, LAMSADE, Université Paris Dauphine - PSL, Paris, France

<sup>2</sup>AGH University, Kraków, Poland

theo.delemazure@dauphine.eu, ljanecc@agh.edu.pl,  
andrzej.kaczmarczyk@agh.edu.pl, s.szufa@gmail.com

#### Abstract

We study committee elections from a perspective of finding the most conflicting candidates, that is, candidates that imply the largest amount of conflict, as per voter preferences. By proposing basic axioms to capture this objective, we show that some of the prominent multi-candidate voting rules meet them. Consequently, we design committee voting rules compliant with our desiderata, introducing *conflictual voting rules*. A subsequent deeper analysis sheds more light on how they operate. Our investigation identifies various aspects of conflict, for which we come up with relevant axioms and quantitative measures, which may be of independent interest. We support our theoretical study with experiments on both real-life and synthetic data.

#### 1 Introduction

Where a collective decision over a set of options based on a number of opinions has to be reached, conflict is inevitable. Reflected by differences in the opinions, it usually comes from different perspectives of the opinions (e.g., in the case of human opinions, a beginner investor would likely have completely different opinion on various asset classes than their professional counterpart). However, conflict might also be option-based and stem from diverse, sometimes even contradicting, inherent qualities of the options (e.g., potentially high-return assets typically have high risk levels). We are interested in how to identify these conflicting options, based on the preferences. Since the options might represent multiple entities (e.g., sports players, societal issues, or marketing strategies), answering this question has numerous natural applications that include selecting competitors to organize engaging sport events (e.g., boxing matches), controversial topics to organize interesting political debates, disputable issues for socially-relevant deliberations, or conflicting ideas for boosting the creativity with passionate discussions.

A somewhat different application of identifying conflicting options is learning new insights about the options or the opinions' perspectives. Imagine a space agency that validates procedures (options) for landing on the Moon using a collection of complex simulations. Each simulation assesses the quality of each procedure and ranks the procedures (expressing an opinion) from the ones that are most likely to the one that are the least likely to succeed. The existence of two significantly conflicting procedures can then offer additional insights. It might suggest that there is some (possibly unknown) feature that impact only some of the simulations and is ignored by the rest. Alternatively, the two procedures may differ in some operational detail that is a crucial success factor for some of the simulated scenarios. In both cases, a careful inspection of the procedures would help to recognize the source of the conflict and thus contribute to advancing the explainability of the simulations or the knowledge about the procedures.

To provide a big picture of our approach, we use a particularly illustrative application, which is finding polarizing issues. Having a collection of *ordinal preferences of voters* expressing their view on the importance of the issues, we aim at identifying two issues that are the most conflicting.

1

### Independence of Approximate Clones (MPREF-2025) Théo Delemazure

**- 1 -**

**What does it mean for two  
candidates to induce conflict?**

# Formal model

*We have:*

- Set of **voters**  $V = \{1, 2, \dots, n\}$  and **candidates**  $C = \{c_1, \dots, c_m\}$ .
- A preference **profile**  $P = (\succ_1, \dots, \succ_n)$  of rankings of voters over candidates.

*We want:*

- **Conflict measures**  $\sigma$  that given a profile  $P$  associate to each pair of candidates  $a, b \in C$  a conflict value  $\sigma_P(a, b) \in [0, 1]$ .
- **Conflict rules**  $f$  that give for each profile  $P$  the most conflicting pair(s) of candidates.

# Extra notations

**Rank** of candidate  $A$  in ranking of voter  $i$ :  $\text{rank}_i(A)$ .

**Distance** between  $A$  and  $B$  in ranking of voter  $i$ :

$$d_i(A, B) = \text{rank}_i(A) - \text{rank}_i(B)$$

## Example

$$A \succ_i B \succ_i C \succ_i D \succ_i E \succ_i F$$

$$\text{rank}_i(A) = 1$$

$$\text{rank}_i(D) = 4$$

$$d_i(A, D) = -3$$

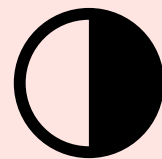
$$d_i(D, A) = +3$$

# Notions of Conflict

$A \succ B \succ C \succ D$   
 $A \succ B \succ C \succ D$   
 $A \succ B \succ C \succ D$   
 $A \succ B \succ C \succ D$   
 $D \succ C \succ B \succ A$   
 $D \succ C \succ B \succ A$   
 $D \succ C \succ B \succ A$   
 $D \succ C \succ B \succ A$



**Conflict Intensity:** **A** and **D** are inducing more conflict if they are distant in voters' preferences.



**Conflict Partitioning:** **A** and **D** are inducing more conflict if they split the voters more evenly.

# Notions of Conflict



## Conflict **intensity**

$$\beta_P(A, B) = \frac{1}{n(m-1)} \sum_{i \in V} |d_i(A, B)| \in [0, 1]$$



## Conflict **partitioning**

Number of voters preferring  $A$  to  $B$ .

$$\alpha_P(A, B) = \frac{2}{n} \min(N^{A>B}, N^{B>A}) \in [0, 1]$$

# Trade-off between them

*A* > *C* > *D* > *B*

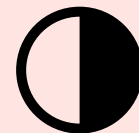
*A* > *C* > *D* > *B*

*A* > *D* > *C* > *B*

*B* > *D* > *C* > *A*



**A** and **B** have maximal conflict intensity



**C** and **D** have maximal conflict partitioning

# A third notion: **Conflict balance**

*A* > *C* > *E* > *F* > *G* > *D* > *B*

*A* > *C* > *E* > *F* > *G* > *D* > *B*

*E* > *D* > *B* > *F* > *A* > *C* > *G*

*E* > *D* > *B* > *F* > *A* > *C* > *G*



**(A,B)** and **(C,D)** have **the same value** of conflict intensity and conflict partitioning. However, **(C,D)** is more balanced.

# A third notion: **Conflict balance**



**Conflict balance**

$$\gamma_P(A, B) \in [0,1]$$

$$\phi_P(A, B) \in [0,1]$$

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# **Finding the most conflicting pair of candidates**

# Conflict Rules from Measures

A simple way to create conflict rules is to select the pair that maximizes **some increasing function** of the conflict measures:

$$\operatorname{argmax}_{A,B \in C^2} f(\alpha_P, \beta_P, \gamma_P, \phi_P)$$

**Rule:**  $p$ -MaxPolarization Rule

The  $p$ -MaxPolar rule with  $p > 0$  selects the pairs maximizing  $\alpha_P \cdot (\beta_P)^p$ .

# Conflict Rules

**Rule:**  $p$ -MaxPolarization

**Rule:** MaxSumConflict

**Rule:** MaxNashConflict

**Rule:** MaxSwapConflict

**Question:** What are the behavioral differences between these rules? Which one should we use?

# Fundamental **Axioms**

## **Axiom:** Reverse Stability

The outcome should not change if all rankings are reversed.



$D \succ B \succ C \succ A$

$A \succ B \succ C \succ D$

$B \succ C \succ D \succ A$

$B \succ A \succ C \succ D$



$D \prec B \prec C \prec A$

$A \prec B \prec C \prec D$

$B \prec C \prec D \prec A$

$B \prec A \prec C \prec D$

# Matching Domination

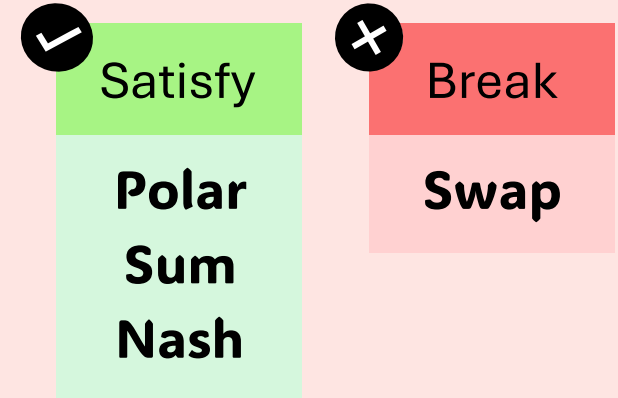
## Axiom: Matching Domination

If a pair  $(a, b)$  matching-dominates a pair  $(x, y)$ , then the pair  $(x, y)$  should **not** be selected.

$A \succ X \succ Y \succ B$   
 $A \succ X \succ Y \succ B$   
 $A \succ X \succ Y \succ B$   
 $Y \succ B \succ X \succ A$   
 $B \succ Y \succ A \succ X$



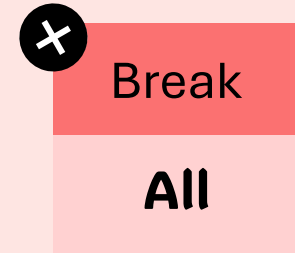
$(A, B)$  dominates  $(X, Y)$ .  
Thus,  $(X, Y)$  should not be selected.



# The Paradox of Monotonicity

## Axiom: Conflict Monotonicity

If a pair  $(a, b)$  is selected in a profile of rankings, it should still be selected if we increase the conflict between  $a$  and  $b$  in the profile.



$A \succ B \succ C \succ D$   
 $B \succ A \succ D \succ C$

$(A, B)$  is selected.



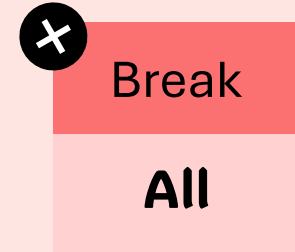
$A \succ B \succ C \succ D$   
 $B \succ D \succ C \succ A$

$(A, B)$  should be selected.

# The Paradox of Monotonicity

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If a pair  $(a, b)$  is selected in a profile of rankings, it should still be selected if we increase the conflict between  $a$  and  $b$  in the profile.



$A \succ B \succ C \succ D$   
 $B \succ A \succ D \succ C$

$(A, B)$  is selected.



$A \succ B \succ C \succ D$   
 $B \succ D \succ C \succ A$

$(A, D)$  is selected.

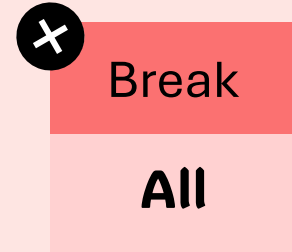
# The Paradox of **Monotonicity**

**Impossibility Theorem:** No rule can satisfy (1) conflict consistency, (2) matching domination and (3) conflict monotonicity.

# (Uniform) Reinforcement

## Axiom: Reinforcement

If a pair  $(c, d)$  is selected in a profile  $P_1$  and another profile  $P_2$ , it should be selected in  $P_1 + P_2$ .



$A \succ C \succ D \succ B$

$A \succ D \succ C \succ B$

$(C, D)$  is selected.



$B \succ C \succ D \succ A$

$B \succ D \succ C \succ A$

$(C, D)$  is selected.



$A \succ C \succ D \succ B$

$A \succ D \succ C \succ B$

$B \succ C \succ D \succ A$

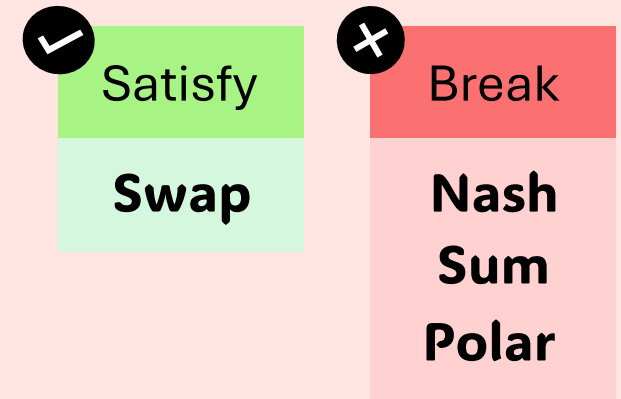
$B \succ D \succ C \succ A$

$(C, D)$  should be selected.

# (Uniform) Reinforcement

## **Axiom:** *Uniform Reinforcement*

If a pair  $(c, d)$  is selected in a profile  $P_1$ , it should be selected in  $P_1 + \text{UN}$ , where UN is the uniform profile.

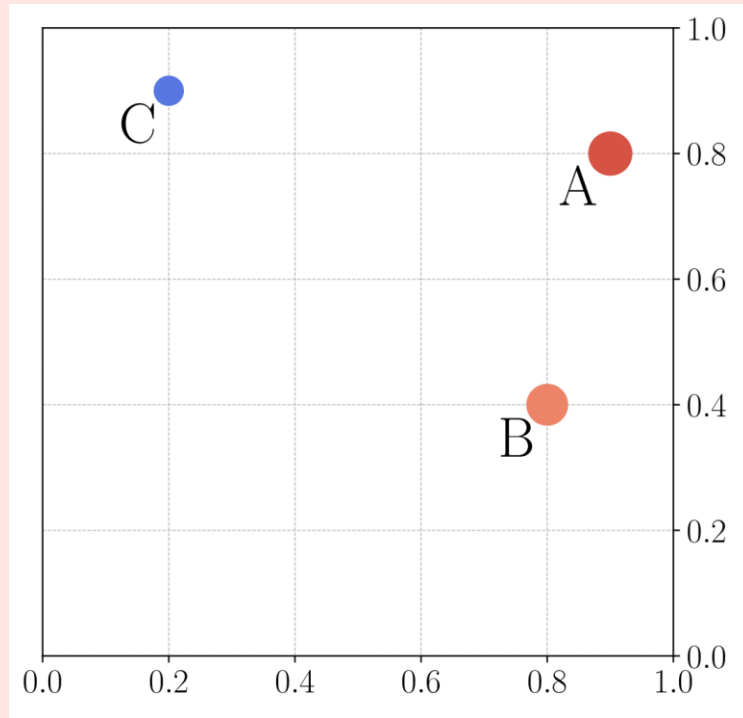


# Summary of the Axiomatic Analysis

	Polar	Sum	Nash	Swap
Conflict Consistency	✓	✓	✓	✓
Reverse Stability	✓	✓	✓	✓
Conflict Monotonicity				
Matching Domination	✓	✓	✓	
Balance Preference			✓	✓
Uniform Reinforcement				✓

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**Experiments**

# Experiments: Representing Pairs and measures



Higher  
**conflict  
intensity**



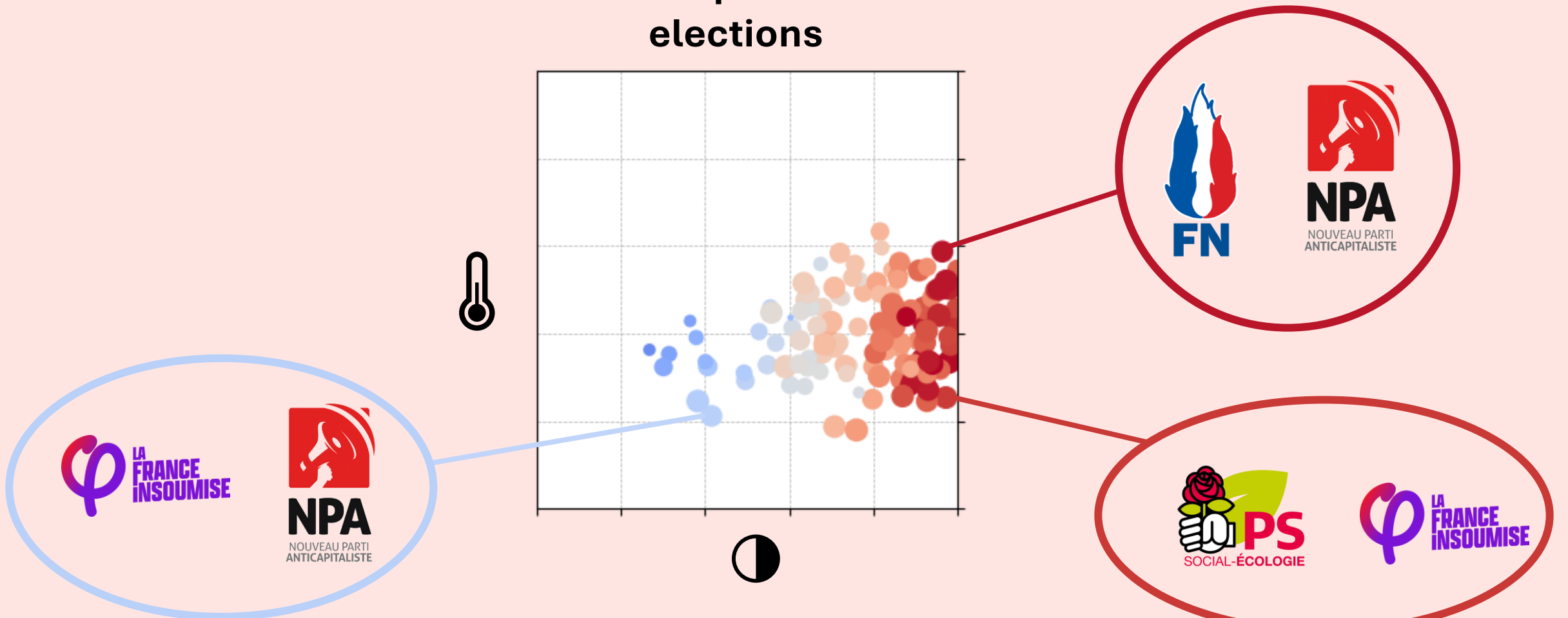
Higher  
**conflict  
balance**



Higher **conflict partitioning**

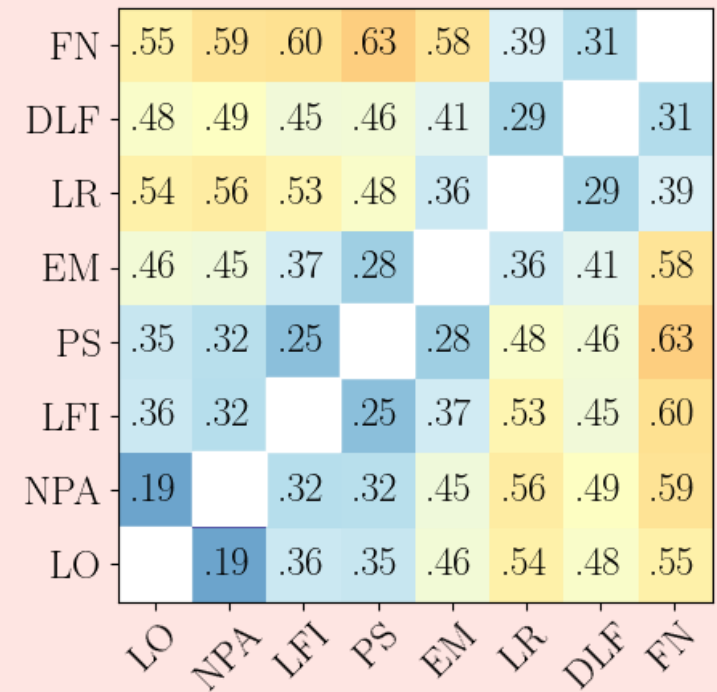
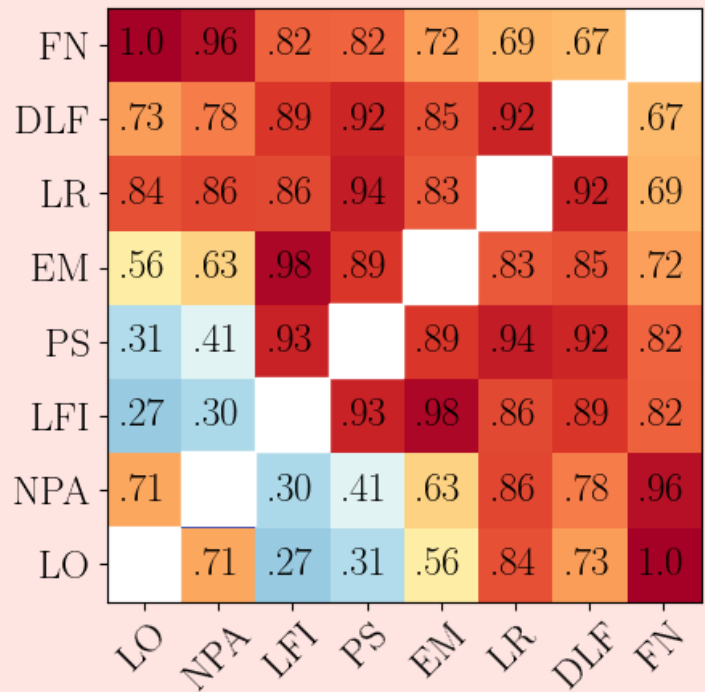
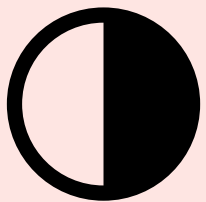
# Experiments: **Voter Autrement**

Preferences over candidates  
In **French presidential elections**



# Experiments: Voter Autrement

Left-right  
axis



# Experiments: **Voter Autrement**

	LO	NPA	LFI	PCF	SOC	ECO	EM	LR	DLF	RN	REC
2-MaxPolar	LO	NPA	LFI	PCF	SOC	ECO	EM	LR	DLF	RN	REC
MaxSwap	LO	NPA	LFI	PCF	SOC	ECO	EM	LR	DLF	RN	REC
MaxSum	LO	NPA	LFI	PCF	SOC	ECO	EM	LR	DLF	RN	REC
MaxNash	LO	NPA	LFI	PCF	SOC	ECO	EM	LR	DLF	RN	REC

# Summary of Conflict Rules

There are **different aspects of conflicts** and pairs of candidates can be the most conflictual in different ways.

We define **conflict rules** to select most conflicting pairs of candidates and showed how they relate to these aspects of conflict through **axiomatic and experimental analysis**.

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# Finding Approximate Clones

# Formal model

*We have:*

- Set of **voters**  $V = \{1, 2, \dots, n\}$  and **candidates**  $C = \{c_1, \dots, c_m\}$ .
- A preference **profile**  $P = (\succ_1, \dots, \succ_n)$  of rankings of voters over candidates.

*We want:*

- **Closeness measures**  $\sigma$  that given a profile  $P$  associate to each pair of candidates  $a, b \in C$  a closeness value  $\sigma_P(a, b) \geq 0$ .

# Perfect Clones

**A** > **B** > C > D > E

D > **B** > **A** > C > E

E > D > **B** > **A** > C

**B** > **A** > E > D > C

D > C > E > **B** > **A**

C > **B** > **A** > D > E

In this preference profile, **A** and **B** are perfect clones: they are always ranked next to each other.

# Approximate Clones

$A \succ B \succ C \succ D \succ E$

$D \succ B \succ A \succ C \succ E$

$E \succ D \succ B \succ A \succ C$

⊗  $B \succ E \succ A \succ D \succ C$

⊗  $D \succ B \succ C \succ E \succ A$

$C \succ B \succ A \succ D \succ E$

In practice, perfect clones **are very rare**. How to still quantify the closeness of a pair of candidates to being clones?

# Approximate Clones: the simplest measure

## $\alpha$ -deletion clones

Two candidates are  $\alpha$ -deletion clones if we can remove a  $\alpha$  portion of the profile such that they become perfect clones.

$A \succ B \succ C \succ D \succ E$

$D \succ B \succ A \succ C \succ E$

$E \succ D \succ B \succ A \succ C$

⊗  $B \succ E \succ A \succ D \succ C$

⊗  $D \succ B \succ C \succ E \succ A$

$C \succ B \succ A \succ D \succ E$

[Janeczko et al., 2024] Discovering Consistent Subelections. *AAMAS 2024*.

[Faliszewski et al, 2026] Identifying Imperfect Clones in Elections. *AAAI 2026*.

[Delemazure, 2026] Independence of Approximate Clones. *Working Paper 2026*.

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Two candidates are  $\alpha$ -deletion clones if we can remove a  $\alpha$  portion of the profile such that they become perfect clones.

$A \succ B \succ C \succ D \succ E$

$D \succ B \succ A \succ C \succ E$

$E \succ D \succ B \succ A \succ C$

⊗  $B \succ E \succ A \succ D \succ C$

⊗  $D \succ B \succ C \succ E \succ A$

$C \succ B \succ A \succ D \succ E$

$$\alpha = \frac{1}{3}$$

[Janeczko et al., 2024] Discovering Consistent Subelections. *AAMAS 2024*.

[Faliszewski et al, 2026] Identifying Imperfect Clones in Elections. *AAAI 2026*.

[Delemazure, 2026] Independence of Approximate Clones. *Working Paper 2026*.

# Candidates Closeness

**A** > **B** > **C** > **D** > **E** > **F** > **G**

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**A** > **D** > **E** > **C** > **F** > **G** > **B**

Pairs (**A**,**B**) and (**C**,**D**) have the same value of  $\alpha$ .

# Another approximate clone measure

## $\beta$ -swap clones

Two candidates are  $\beta$ -swap clones if we can perform an average\* of  $\beta$  swaps per voter such that they become perfect clones.

$A \succ B \succ C \succ D \succ E$

$D \succ B \succ A \succ C \succ E$

$E \succ D \succ B \succ A \succ C$

⊗  $B \succ E \succ A \succ D \succ C$

⊗  $D \succ B \succ C \succ E \succ A$

$C \succ B \succ A \succ D \succ E$

[Faliszewski et al, 2026] alternatively consider the *maximum* instead of the *average*.

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$$\beta = \frac{3}{6}$$

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# Link with **conflict intensity**

**$\beta$ -swap clones**

$$\beta = \frac{1}{n} \sum_{i \in V} |d_i(a, b) - 1|$$

**Conflict intensity**

$$\beta = \frac{1}{n(m-1)} \sum_{i \in V} |d_i(a, b)|$$

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What about **partitioning** and **balance**?

# Axioms: Monotonicity

## Axiom: Closeness Monotonicity

If a pair  $(a, b)$  is the closest to being clones in a profile of rankings, it should still be the closest if we make them closer in the profile



$A \succ C \succ D \succ B \succ E \quad \longrightarrow \quad A \succ B \succ C \succ D \succ E$

# Axioms: Reinforcement

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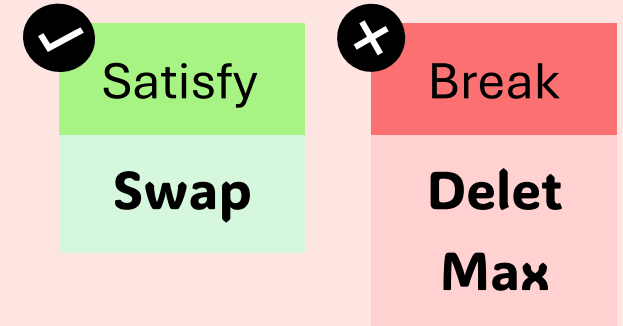
If a pair  $(a, b)$  is the closest to being clones in two profiles, it is also the closest in the union of these two profiles.



# Axioms: Matching-domination

## Axiom: Matching Domination

If a pair  $(a, b)$  matching-dominates a pair  $(x, y)$ , then the pair  $(x, y)$  should **not** be selected.

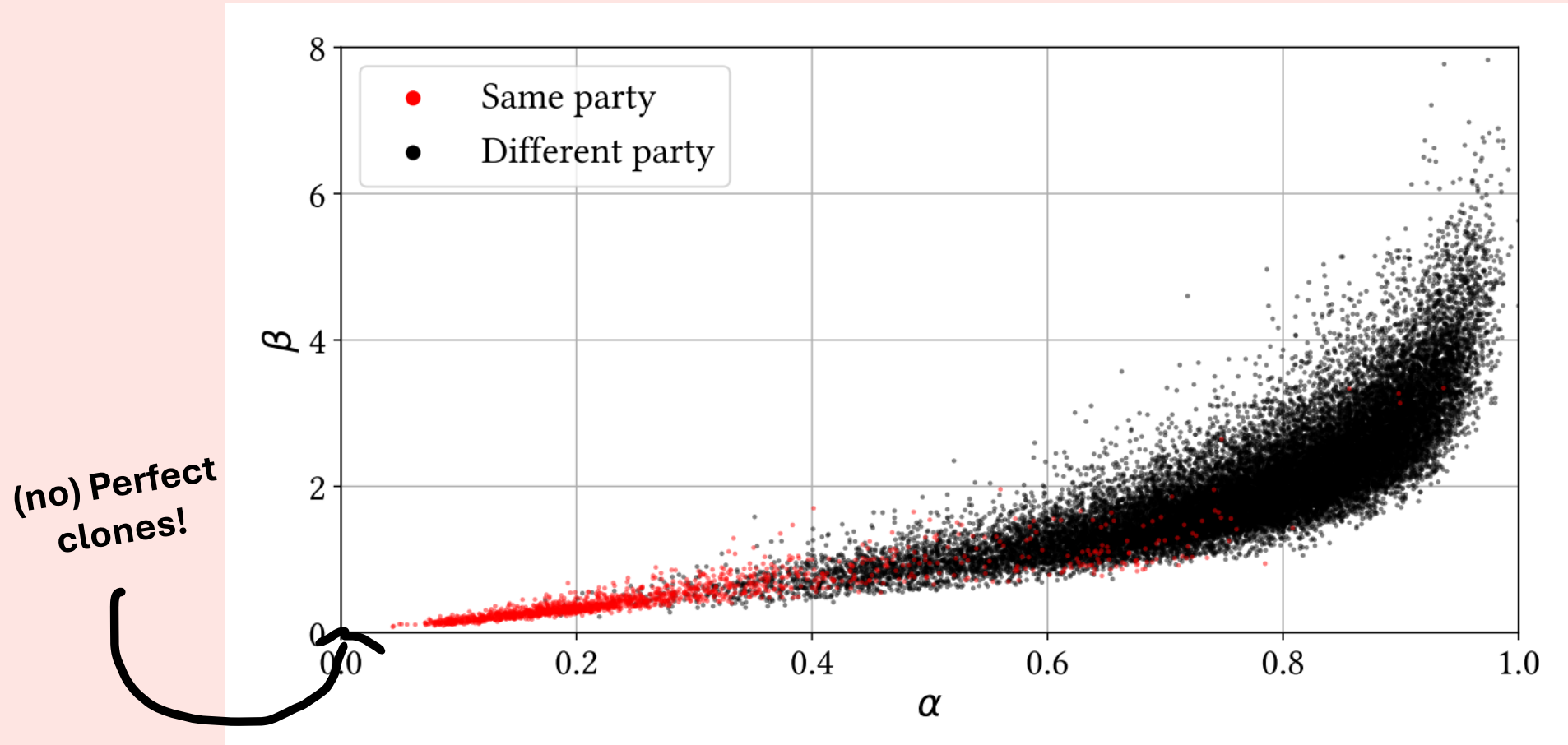


$A \succ B \succ X \succ Y \succ E$   
 $A \succ B \succ X \succ Y \succ E$   
 $A \succ B \succ X \succ Y \succ E$   
 $Y \succ B \succ E \succ A \succ X$



$(A, B)$  dominates  $(X, Y)$ .  
Thus,  $(X, Y)$  should not be selected.

# In real datasets: **Scottish elections**



**Fig.** Values of closeness measures of the pairs in the Scottish elections dataset.

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# Further works

# Related and **Further works**

**Bigger sets** instead of pairs.

**Computational complexity** of finding these sets of candidates.

[Faliszewski et al, 2026] Identifying Imperfect Clones in Elections. *AAAI 2026*.

**Other settings** (approval ballots, budget allocation...)

[Delemazure et al, 2026] Detecting Approximate Clones under Approval Voting. *AAMAS 2026*.

**Going from perfect clones to approximate clones**

[Delemazure, 2025] Independence of Approximate Clones. *MPREF 2025*.

# Thanks for your attention!

## Questions?

*Théo Delemazure – ILLC, University of Amsterdam*

