Approximate Clones with Ordinal Preferences

Théo Delemazure

The Spoiler Effect

Russia 2021 legislative election

District 216 (St Petersburg)

Sergey Solovyov

“United Russia” (Poutine)

Boris Vishnevskiy

“Yabloko” (Social-liberal)

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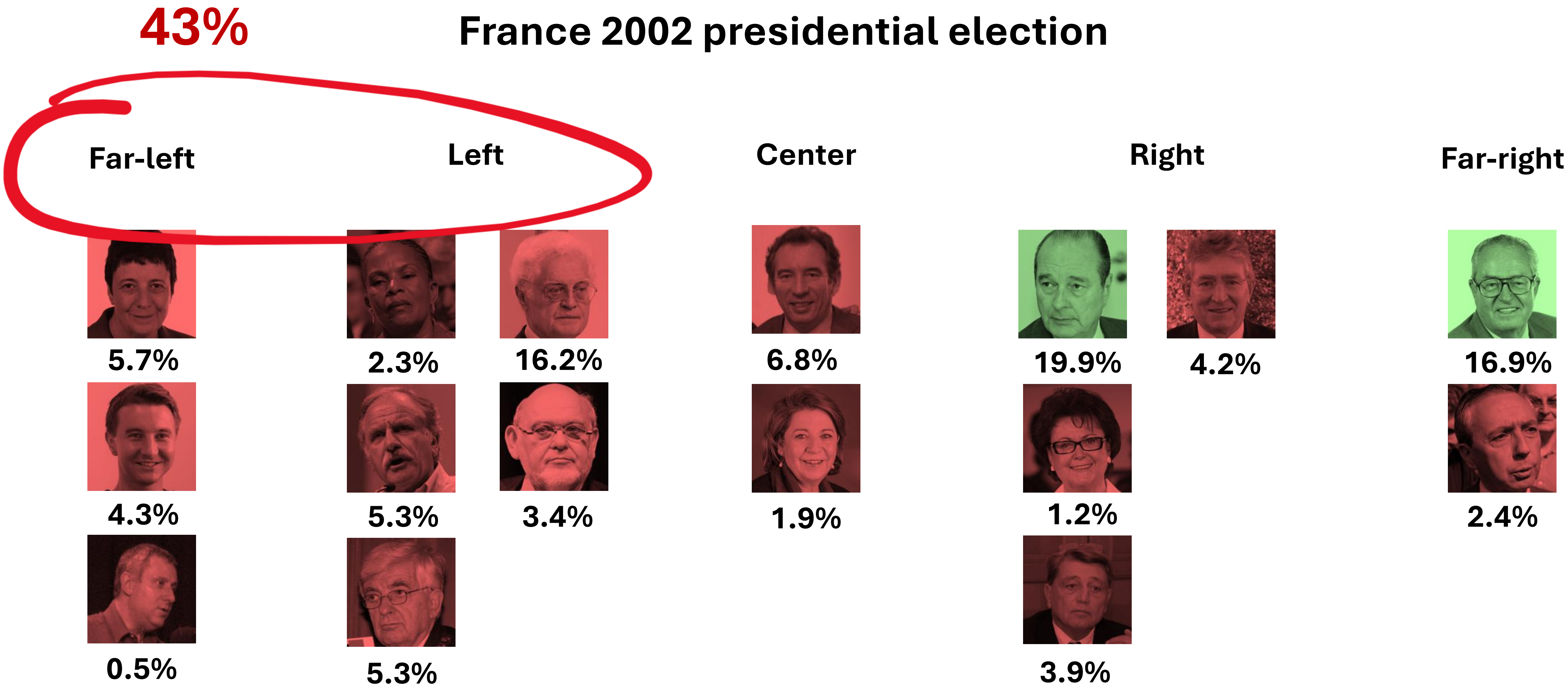
Boris Vishnevskiy
"The Greens"

Boris Vishnevskiy
Independent

The Spoiler Effect

France 2002 presidential election



Clones

$$\boldsymbol{A} \succ \boldsymbol{B} \succ C \succ D \succ E$$
$$\boldsymbol{B} \succ \boldsymbol{A} \succ E \succ D \succ C$$
$$C \succ \boldsymbol{B} \succ \boldsymbol{A} \succ E \succ D$$
$$D \succ C \succ \boldsymbol{A} \succ \boldsymbol{B} \succ E$$
$$E \succ \boldsymbol{B} \succ \boldsymbol{A} \succ D \succ C$$
$$\boldsymbol{A} \succ \boldsymbol{B} \succ D \succ E \succ C$$

$\boldsymbol{A}$ and $\boldsymbol{B}$ are clones if all voters rank them adjacently

Independence of Clones

$A \succ B \succ C \succ D \succ E$

$B \succ A \succ E \succ D \succ C$

$C \succ B \succ A \succ E \succ D$

$D \succ C \succ A \succ B \succ E$

$E \succ B \succ A \succ D \succ C$

$A \succ B \succ D \succ E \succ C$

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$A \succ E \succ D \succ C$

$C \succ A \succ E \succ D$

$D \succ C \succ A \succ E$

$E \succ A \succ D \succ C$

$A \succ D \succ E \succ C$

A or B is the winner $\Leftrightarrow$ A is the winner

D is the winner $\Leftrightarrow$ D is the winner

Independence of Clones

Instant Runoff Voting (used in Ireland, Australia, some US states, etc.) is independent of clones.

Also **Ranked Pairs** and **Schulze's method.**

Approximate Clones

$$\boldsymbol{A} \succ \boldsymbol{B} \succ C \succ D \succ E$$
$$\boldsymbol{B} \succ E \succ D \succ \boldsymbol{A} \succ C$$
$$C \succ \boldsymbol{B} \succ \boldsymbol{A} \succ E \succ D$$
$$D \succ C \succ \boldsymbol{A} \succ \boldsymbol{B} \succ E$$
$$E \succ \boldsymbol{B} \succ \boldsymbol{A} \succ D \succ C$$
$$\boldsymbol{A} \succ \boldsymbol{B} \succ D \succ E \succ C$$

A and $\boldsymbol{B}$ are approximate clones

Questions

1. When are candidates **“approximate clones”?**

2. Are there voting rules that are **independent of *approximate* clones** (in theory and in practice)**?**

3. Do clones and approximate clones **actually exist** in real-world datasets?

Approximate Clones

Model, and perfect clones

- Voters $V = \{1, \dots, n\}$, candidates $C = \{c_1, \dots, c_m\}$.
- Preference profile $P = (\succ_1, \dots, \succ_n)$.

Perfect Clones

Two candidates x and y are perfect clones if for every voter $i \in V$, there is no $z \in C$ such that $x \succ z \succ y$ or $y \succ z \succ x$.

We focus on pairs of candidates, but our negative results can be extended to larger sets of clones.

Approximate clones: α-deletion clones

α-deletion Clones

Two candidates x and y are α-deletion clones if we can remove at most $\alpha \cdot n$ voters from the profile and obtain perfect clones.

"MaxClones" in Janeczko et al. (2024)
"Independent Clones" in Faliszewski et al. (2025)

$$\boldsymbol{A} \succ \boldsymbol{B} \succ C \succ D \succ E$$
$$\boldsymbol{B} \succ E \succ D \succ \boldsymbol{A} \succ C$$
$$C \succ \boldsymbol{B} \succ \boldsymbol{A} \succ E \succ D$$
$$D \succ C \succ \boldsymbol{A} \succ \boldsymbol{B} \succ E$$

In this profile, $\boldsymbol{A}$ and $\boldsymbol{B}$ are ¼-deletion clones.

Approximate clones: α-deletion clones

Profile 1

60% $A \succ B \succ C \succ D \succ E$

40% $B \succ E \succ A \succ D \succ C$

Profile 2

70% $A \succ B \succ C \succ D \succ E$

30% $B \succ E \succ D \succ C \succ A$

In which profile A and B are closer to be clones?

Approximate clones: β-swap clones

β-swap Clones
Two candidates x and y are β-swap clones if we can perform at most $\beta \cdot n$ swaps of adjacent candidates and obtain perfect clones.

$$\boldsymbol{A} \succ \boldsymbol{B} \succ C \succ D \succ E$$
$$\boldsymbol{B} \succ E \succ D \succ \boldsymbol{A} \succ C$$
$$C \succ \boldsymbol{B} \succ \boldsymbol{A} \succ E \succ D$$
$$D \succ C \succ \boldsymbol{A} \succ \boldsymbol{B} \succ E$$

In this profile, $\boldsymbol{A}$ and $\boldsymbol{B}$ are $2/4$-swap clones.

Independence of Clones

Independence of Clones

Independence of Clones (Tideman, 1987)

A rule f is independent of clones if for every profile P in which a and a' are clones, we have:

1. For all $z \neq a, a'$, we have $z \in f(P)$ if and only if $z \in f(P_{-a'})$,
2. We have $f(P) \cap \{a, a'\} \neq \emptyset$ if and only if $a \in f(P_{-a'})$.

IRV, Ranked Pairs and Schulze's method ***satisfy*** Independence of Clones.

Positional Scoring Rules ***fail*** Independence of Clones.

Independence of Clones

Independence of Approximate Clones

A rule f is independent of α-deletion clones if for every profile P in which a and a' are α-deletion clones, we have:

1. For all $z \neq a, a'$, we have $z \in f(P)$ if and only if $z \in f(P_{-a'})$,
2. We have $f(P) \cap \{a, a'\} \neq \emptyset$ if and only if $a \in f(P_{-a'})$.

Independence of Clones

Independence of Approximate Clones

A rule f is independent of α-deletion clones if for every profile P in which a and a' are α-deletion clones, we have:

1. For all $z \neq a, a'$, we have $z \in f(P)$ if and only if z $\in f(P_{-a'})$,
2. We have $f(P) \cap \{a, a'\} \neq \emptyset$ if and only if $a \in f(P_{-a'})$.

All rules consistent with the majority rule when $m = 2$ ***fail*** Independence of Approximate Clones (for any $\alpha > 0$).

Independence of Clones

Weak Independence of Approximate Clones

A rule f is weakly independent of α-deletion clones if for every profile P in which a and a' are α-deletion clones, we have for either $P' = P_{-a}$ or $P' = P_{-a'}$

1. For all $z \neq a, a'$, we have $z \in f(P)$ if and only if $z \in f(P')$,
2. We have $f(P) \cap \{a, a'\} \neq \emptyset$ if and only if $a \in f(P')$.

Independence of Clones

Weak Independence of Approximate Clones

A rule f is weakly independent of α-deletion clones if for every profile P in which a and a' are α-deletion clones, we have for either $P' = P_{-a}$ or $P' = P_{-a'}$

1. For all $z \neq a, a'$, we have $z \in f(P)$ if and only if $z \in f(P')$,
2. We have $f(P) \cap \{a, a'\} \neq \emptyset$ if and only if $a \in f(P')$.

When $m \geq 4$, IRV, Ranked Pairs and Schulze's method all ***fail*** Weak Independence of Approximate Clones (for any $\alpha > 0$).

Independence of Clones

Weak Independence of Approximate Clones

A rule f is weakly independent of α-deletion clones if for every profile P in which a and a' are α-deletion clones, we have for either $P' = P_{-a}$ or $P' = P_{-a'}$
1. For all $z \neq a, a'$, we have $z \in f(P)$ if and only if z ∈ $f(P')$,
2. We have $f(P) \cap \{a, a'\} \neq \emptyset$ if and only if a ∈ $f(P')$.

When $m = 3$, IRV ***satisfy*** Weak Independence of $1/3$-deletion Clones, and Ranked Pairs and Schulze satisfy it for any α.

Note that when $m = 3$, α and β are the same.

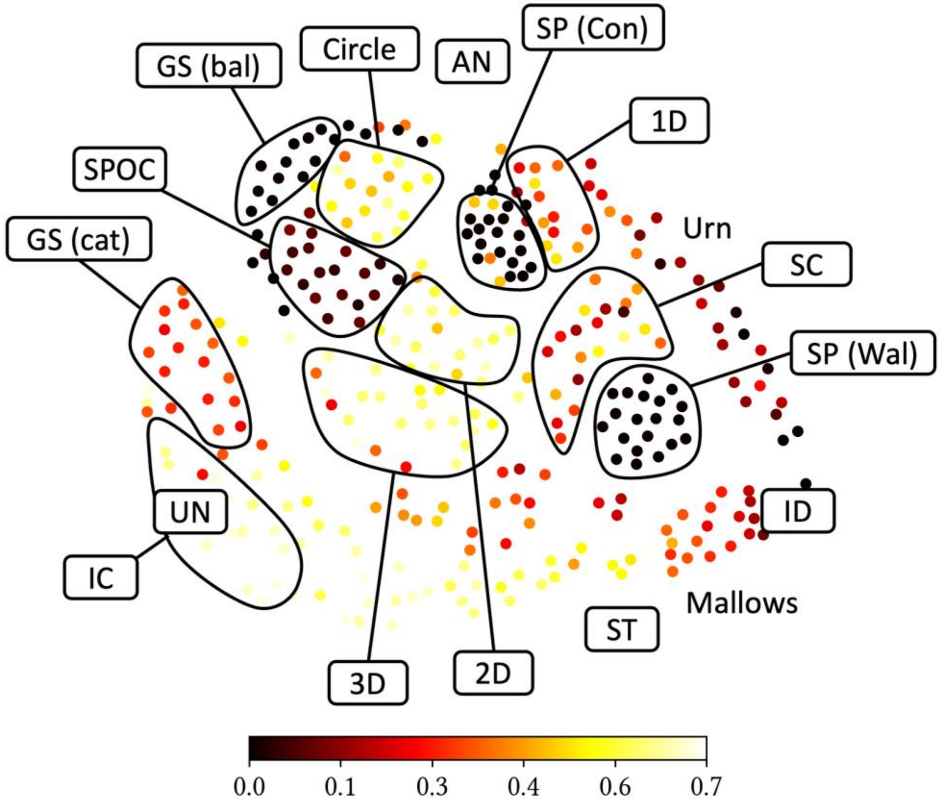
Empirical Analysis

Two questions

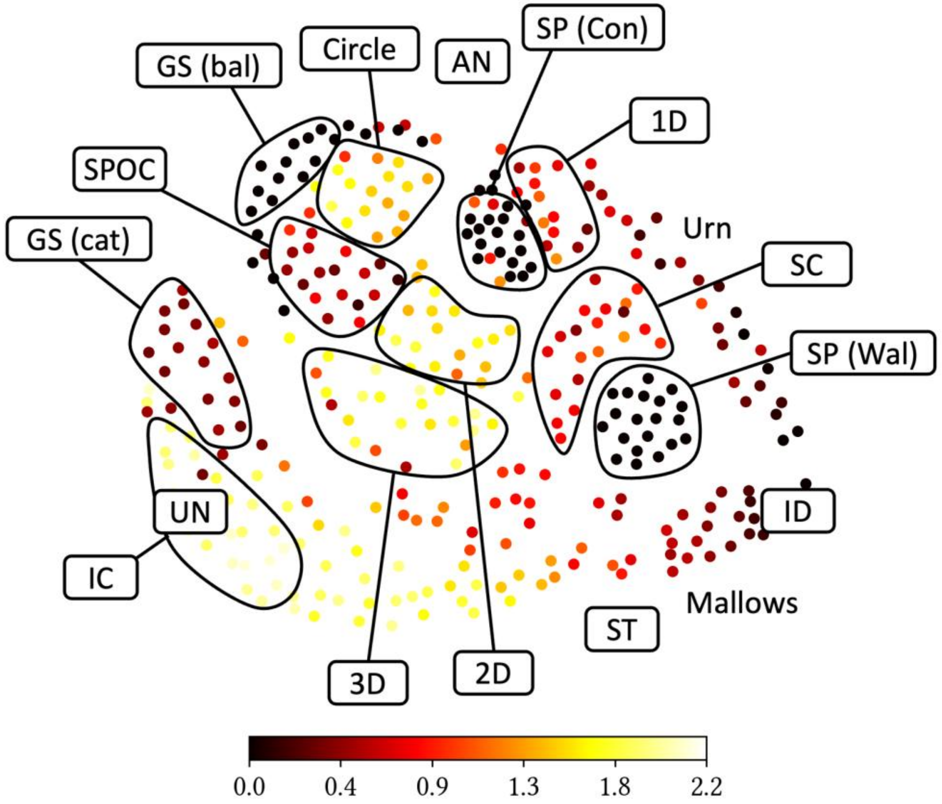
1. Do clones and approximate clones **actually exist** in real-world datasets?

2. Are voting rules independent of *approximate* clones **in practice?**

Synthetic Data (map of elections)



$\alpha_{\min}$



$\beta_{\min}$

French Presidential Elections

Context: French Presidential Elections between 2007 and 2022.

Official rule: Plurality with Runoff.

Proposed rule: IRV.

Candidates: between 10 and 12.

Datasets: 5

Dataset	Candidate 1	Candidate 2	α	β
2007-*in situ*	Besancenot	Buffet	0.54	1.54
2017-*in situ*	Cheminade	Lassalle	0.47	1.14
2017-*in situ*	Arthaud	Poutou	0.49	1.34
2017-*in situ*	Macron	Hamon	0.60	1.73
2012-*online*	Arthaud	Poutou	0.34	0.56
2017-*online*	Arthaud	Poutou	0.43	0.87
2017-*online*	Mélenchon	Hamon	0.57	1.45
2022-*online*	Arthaud	Poutou	0.50	0.97
2022-*online*	Zemmour	Le Pen	0.51	1.32

Pairs of candidates with $\alpha \leq 0.6$

Scotland Elections

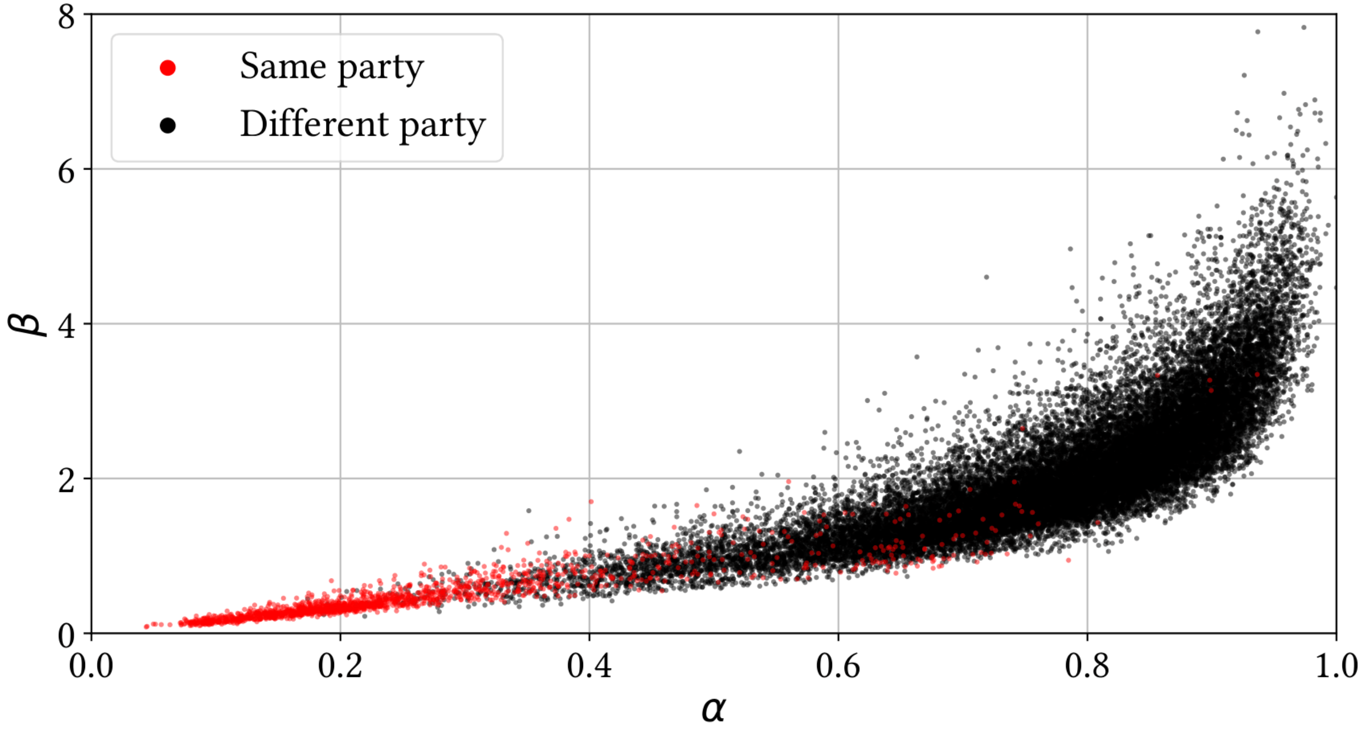
Context: Local Committee Elections in Scotland.

Official rule: IRV.

Candidates: Between 3 and 14.

Datasets: 1 070.

Particularity: Often several candidates from the same party.



Habermas Machine

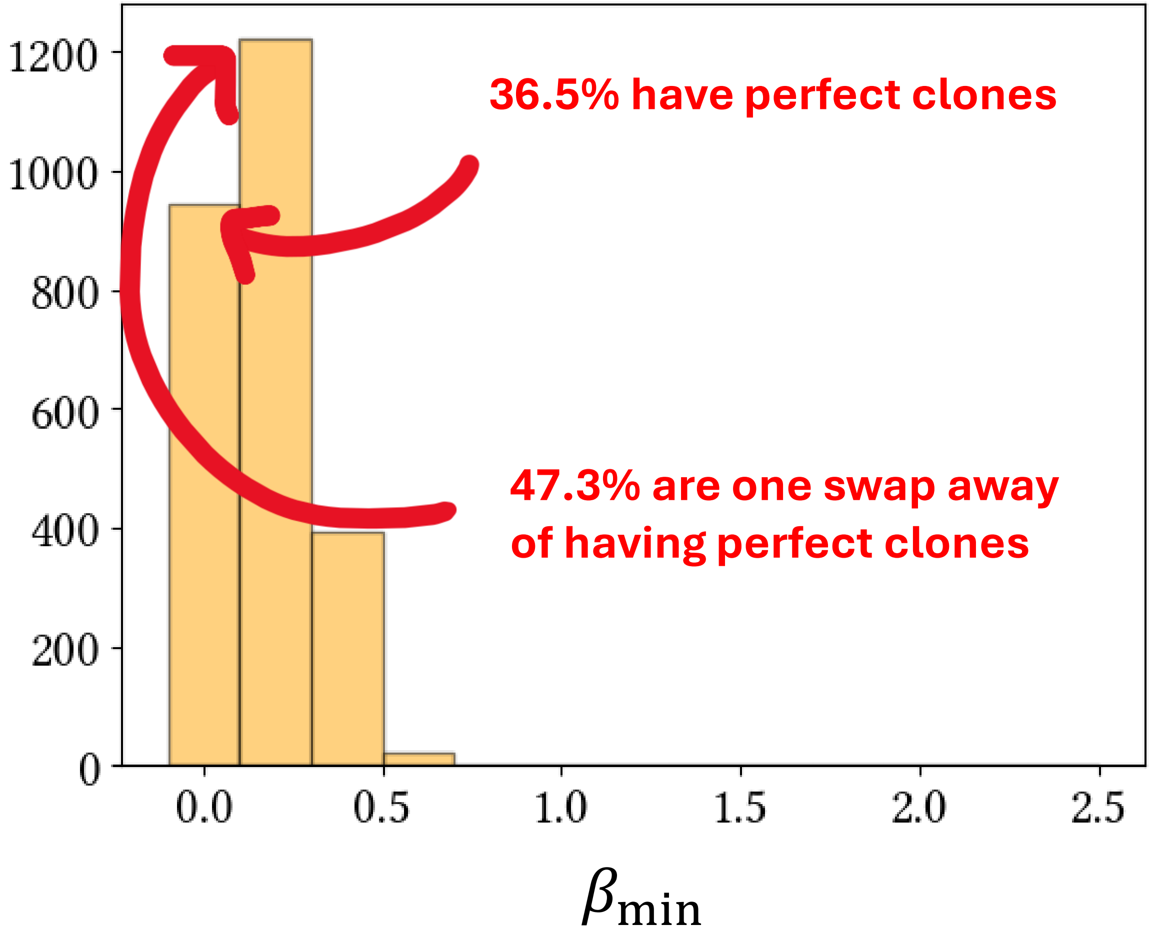
Context: Mini-jury (5 voters) deliberation with AI statements.

Official rule: Schulze.

Candidates: 4.

Datasets: 2 581.

Particularity: Statements can be very similar.



Independence in practice

Independent of Perfect Clones

IRV

Ranked Pairs

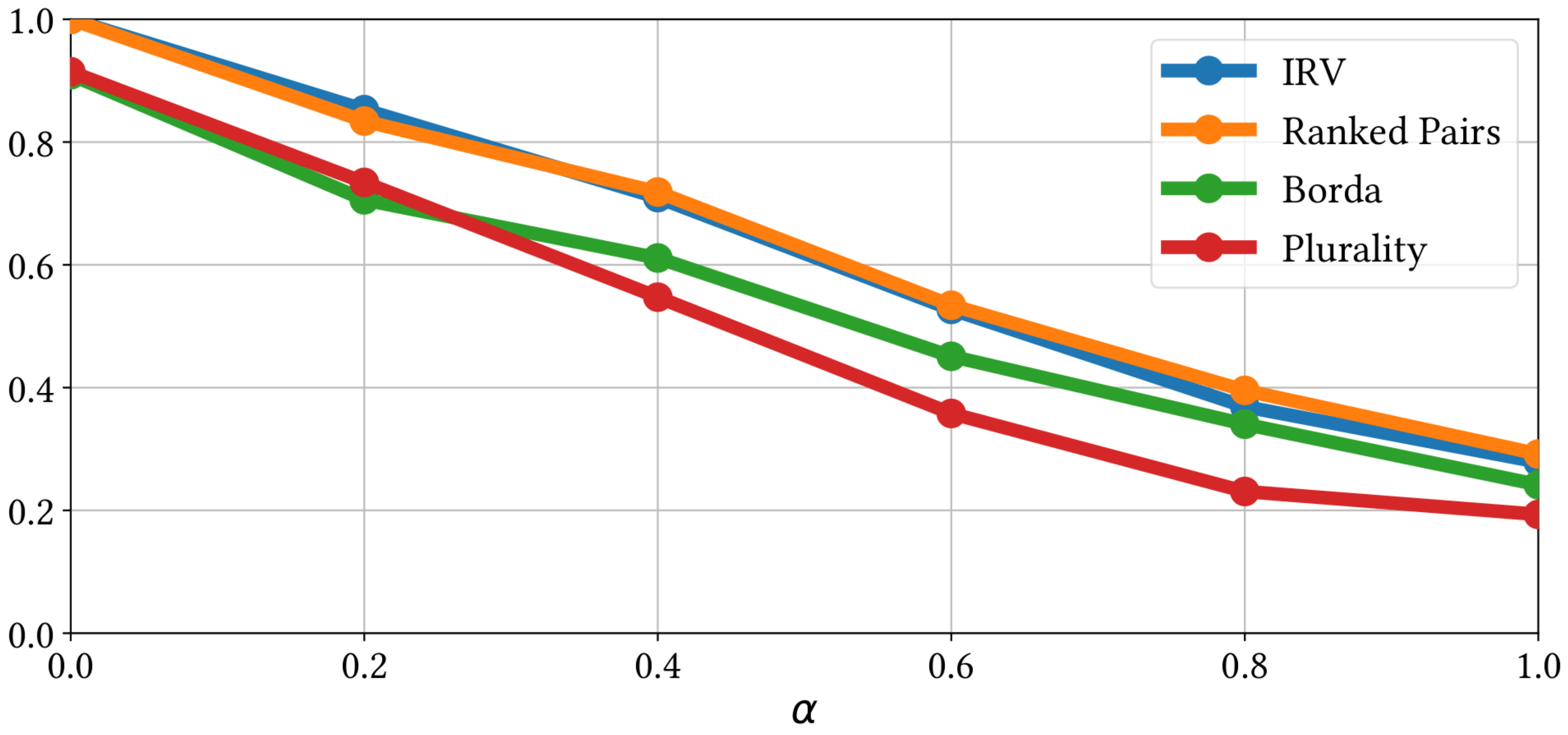
Not Independent of Perfect Clones

Borda

Plurality

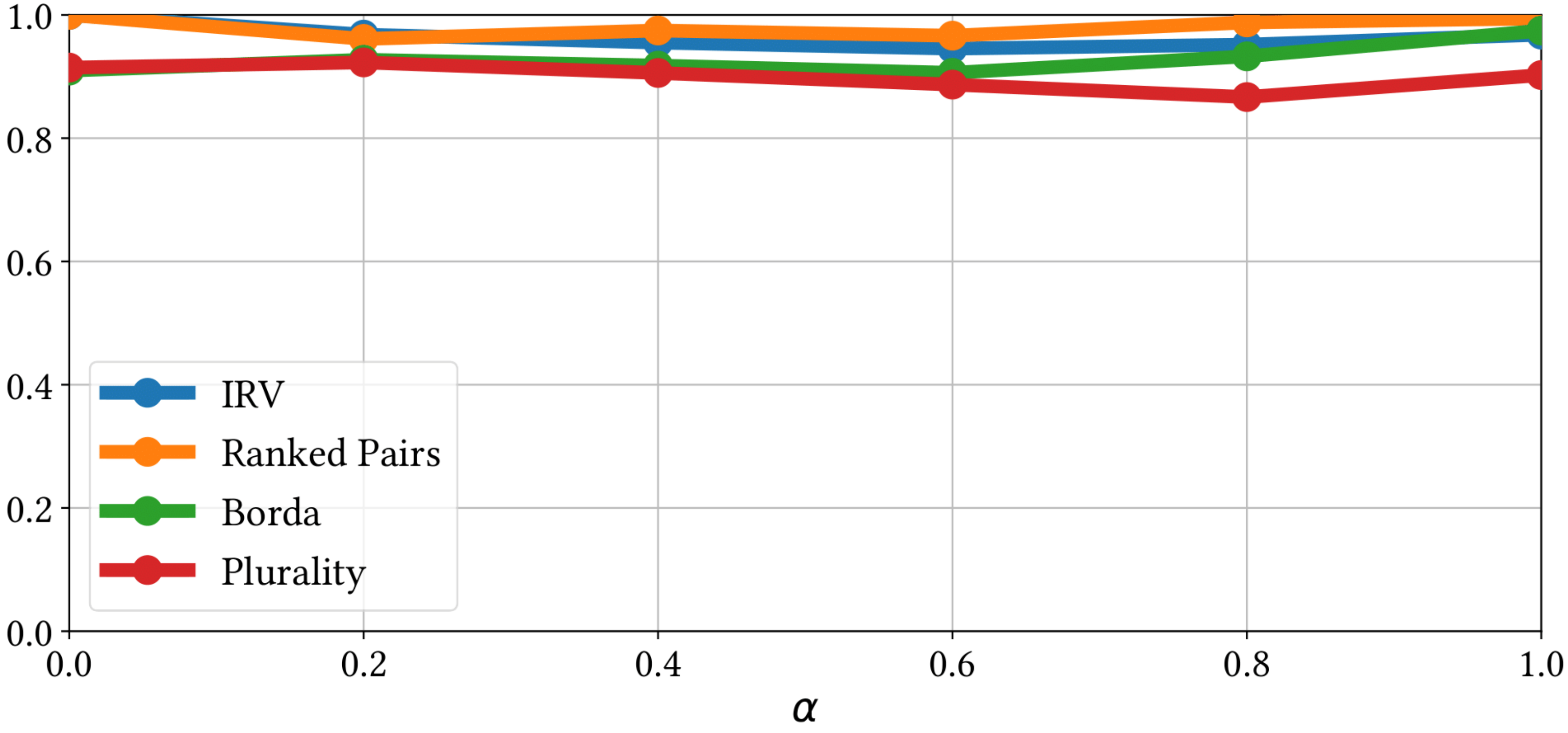
Independence in practice

% of pairs for which independence is satisfied, depending on their proximity. (Habermas dataset)



Independence in practice

% of pairs for which **weak** independence is satisfied, depending on their proximity. (Habermas dataset)



Conclusion

Conclusion

1. We discussed and compared **two notions of approximate clones**.

2. In the worst case and for $m \geq 4$, traditional rules **do not satisfy** weak independence of approximate clones (but more positive results for $m = 3$).

3. In practice, it is more frequent to **observe approximate clones** than perfect clones.

4. In practice, the strong independence seems to be **a more relevant axiom** than the weak version.