An Experiment on the Impact of the Number of Candidates in Approval Voting

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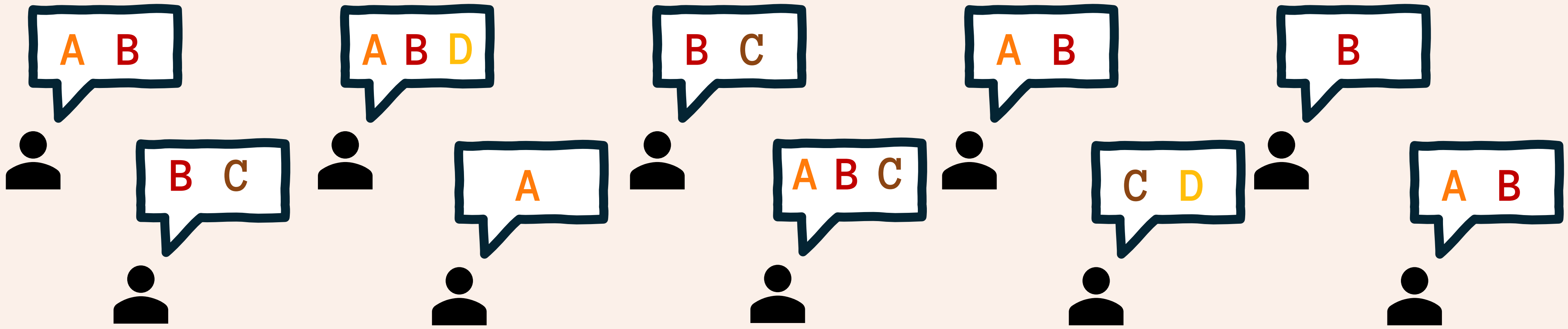
Context: Approval Voting

Set of **candidates**: A, B, C, D

Each **voter** indicates which candidates they **approve** of:



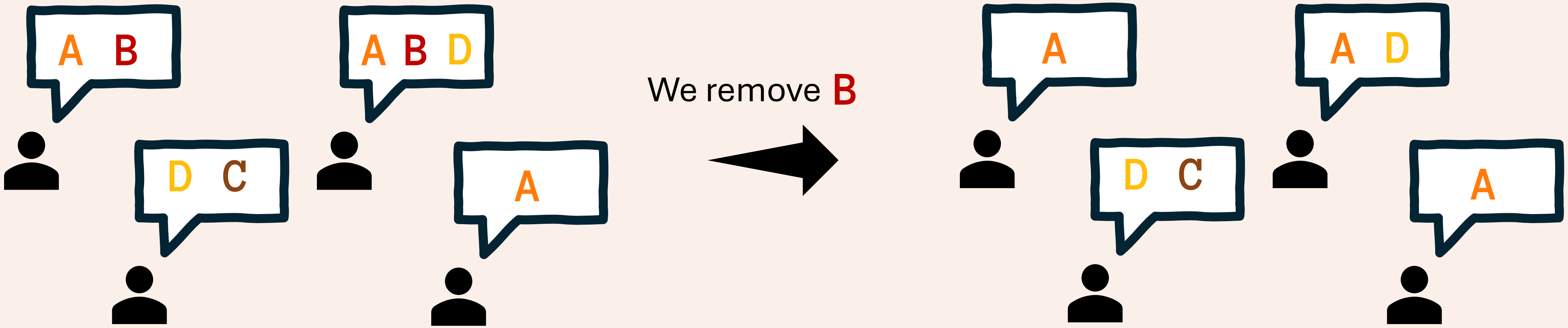
The candidate with the most votes is **the winner**.

Independence of Losers

A voting rule is **independent of losers** if removing a losing candidate from the election never changes the winner [Brandl and Peters, 2022].

Variant of the (in)famous **independence of irrelevant alternatives** axiom.

Prevent a losing alternative from **spoiling** the election by running.



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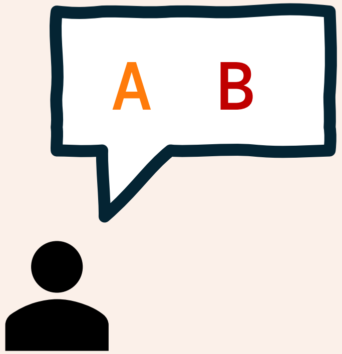
Theorem [Brandl and Peters, 2022] : AV satisfies independence of losers.

But in practice...

Scenario I

4 candidates:

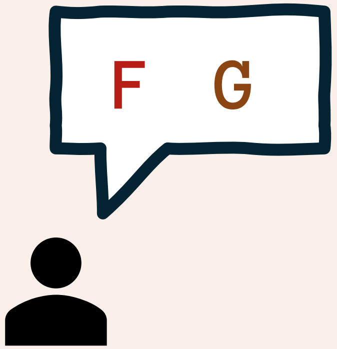
A,B,C,D



Scenario II

4 candidates:

E,F,G,H



Scenario III

8 candidates:

A,B,C,D,E,F,G,H

Why? Cognitive limits, strategic voting...

But in practice...

- This might affect the **legitimacy** of the winner.

Scenario I

4 candidates:
A, B, C, D

A wins with **60%** approval.

Scenario II

8 candidates:
A, B, C, D, E, F, G, H

A wins with **40%** approval.

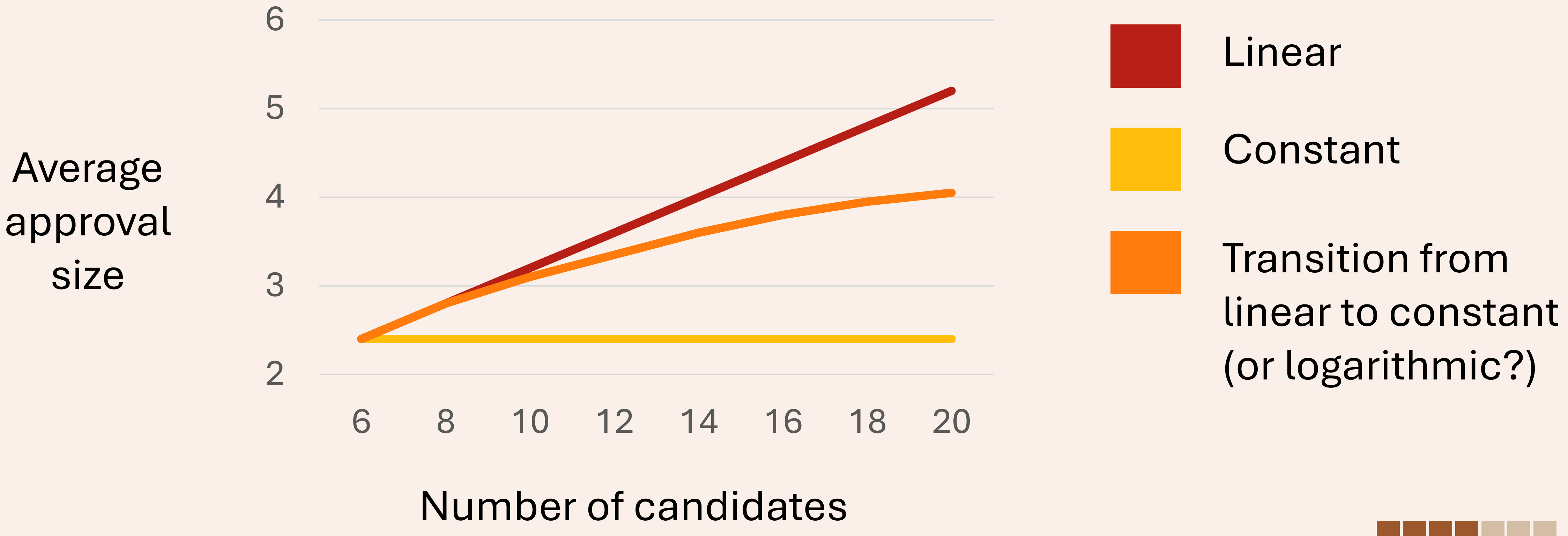
The central question

How does the **average number of approved candidates** depend on the **total number of candidates** in the election?

But... depends on the **context**, on the kind and **popularity of candidates**,...

Different **hypotheses**

Very much context-dependent, but we can probably observe **trends** across different contexts.



Some Related Work

- **Experiments on AV** with different number of candidates [Igerhseim et al 2016, Koc 1988].
- Literature on **framing effect** in voting and in other domains.
- Literature on **information overload** in voting [Le Lec et al 2022].
- Literature on **strategic voting** [Sanver and Laslier 2010].

[1] Model and Hypotheses

Definitions

- **n voters** $V = \{1, \dots, n\}$ and **M candidates** $C = \{c_1, \dots, c_M\}$.
- For every subset $S \subseteq C$, voters **have approval preferences** over candidates $A_i(S) \subseteq S$ for all $i \in V$.
- For $m \leq M$, we denote

$$f(m) = \frac{1}{\binom{n}{m}} \sum_{S \subseteq C, |S| = m} \left(\frac{1}{n} \sum_{i \in V} |A_i(S)| \right)$$

the **average number of candidates** approved when m candidates are running.

Tested **hypothesis**

Let us assume that candidates provide **utilities** to voters $u_i(c) \in [0,1]$.

1. Voter i always approve their $\boldsymbol{k_i}$ **favorite candidates** from some $k_i \in N$.

2. Voter i always approve all candidates **above some threshold** $\boldsymbol{\alpha_i \in [0,1]}$**.**

3. Voter i always approve their k_i favorite candidates from some $k_i \in N$, **plus all candidates with utility** $\boldsymbol{\alpha_i \in [0,1]}$ **as high** as one of them.

4. Voter i **vote strategically**, according to the classical strategic model for approval voting.

⚠ These hypothesis are independent

Hypothesis

Let us assume that candidates provide **utilities** to voters $u_i(c) \in [0,1]$.

1. Voter i always approve their $\boldsymbol{k_i}$ **favorite candidates** from some $k_i \in N$.

Ballot:

$$A_i = S_{\text{top}-k_i}$$

Constant function:

$$f(m) = k$$

Hypothesis

Let us assume that candidates provide **utilities** to voters $u_i(c) \in [0,1]$.

❷ Voter i always approve all candidates **above some threshold $\boldsymbol{\alpha_i} \in [\mathbf{0}, \mathbf{1}]$.**

Ballot:

$$A_i = \{x \in S \mid u_i(x) \geq \alpha_i\}$$

Linear function:

$$f(m) = \alpha m$$

Hypothesis

Let us assume that candidates provide **utilities** to voters $u_i(c) \in [0,1]$.

3. Voter i always approve their k_i favorite candidates from some $k_i \in N$, **plus all candidates that are almost $\alpha_i \in [0,1]$ as good** as one of them.

Ballot:

$$A_i = \{x \in S \mid u_i(x) \geq \min_{y \in S_{\text{top}-k_i}} \alpha_i \cdot u_i(y)\}$$

Affine function

$$f(m) = k + (1 - \alpha) \cdot m$$

Hypothesis

Let us assume that candidates provide **utilities** to voters $u_i(c) \in [0,1]$.

4. Voter i **vote strategically**, according to the classical strategic model for approval voting (they approve all candidates on top of their ranking up to the first one that has a chance to win).

Ballot:

$$A_i = \{x \in S \mid u_i(x) \geq \min_{y \in S^*} u_i(y)\}$$

(S^* is the set of expected potential winners.)

Logarithmic function?

$$f(m) = k + \alpha \cdot \log(m)?$$

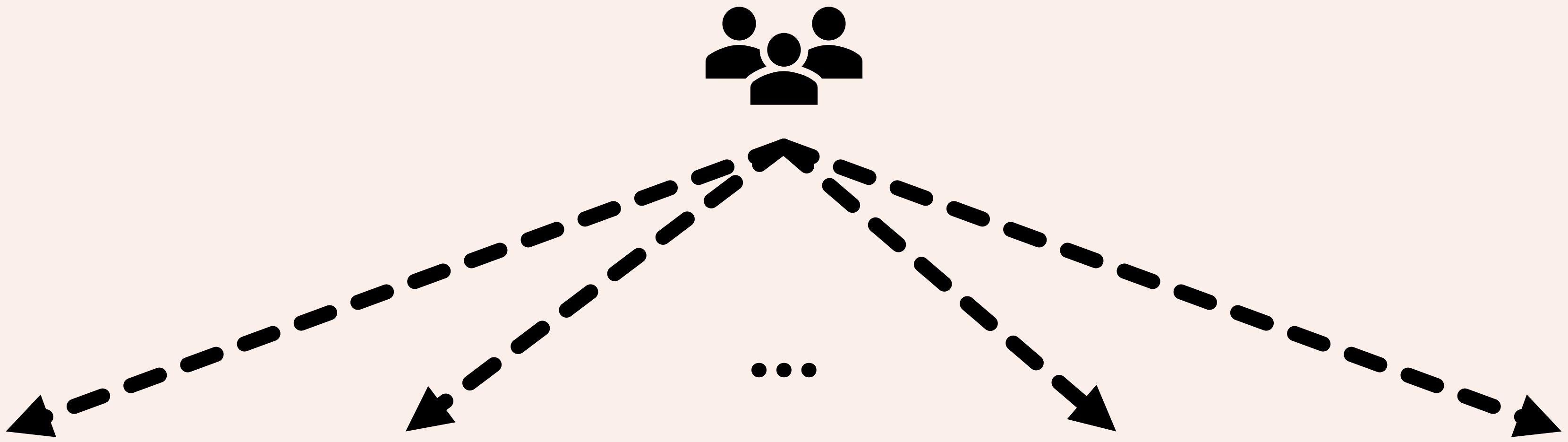
Remarks

- **Several of these hypotheses might be true**, and it might be possible that some voters vote according to one and other voters according to some other one.

- There might **be other valid hypotheses** behind voters' voting behaviors. In particular, all the ones considered here assume **independence** of candidates' utilities (a strong assumption in practice).

[2] Experimental Design

Experimental **design**



1 candidate (do you approve x ?)

2 candidates

8 candidates

9 candidates

Candidates are **selected at random** for each voter.

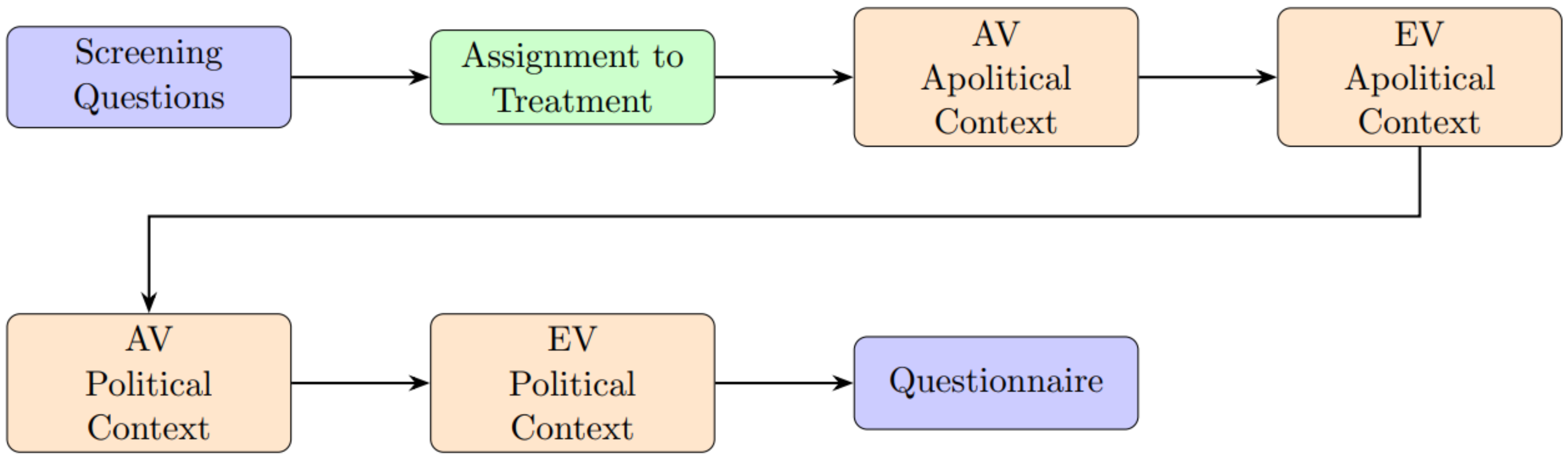
Several **contexts**

- **Food** (not an important vote)

- **Politics** (important vote)

Experiment **design**

Figure 1: Experimental Sequence



- **EV** with notes between -1 and 6 to **distinguish candidates** into popular, polarizing, unpopular and medium candidates [Darmann et al., 2017].

The central **question**

How does the **average number of approved candidates** depend on the **total number of candidates** in the election?

Regressions

$$y_i = \alpha + \sum_{2 \leq m \leq 9} \beta_m T_m^i + \epsilon_i$$

T_m^i control variable $=$ 1 if voter i saw m candidates, 0 otherwise.

$$y_i = \alpha + \beta m_i + \epsilon_i$$

To test the **linear/affine** hypothesis.

$$y_i = \alpha + \beta \log(m_i) + \epsilon_i$$

To test the **logarithmic** hypothesis.

Additional questions

- Does the behavior regarding average approval size vary **for different contexts?**
- Does the behavior regarding average approval size vary **for different categories of people (based on gender, age, etc.)?**
- Does the probability to be approved when there are more candidates changes differently **for different types of candidates?**
- How does the number of candidates impact **the time** taken by participants to vote?

[3] Preliminary Results

Literature

Comparing voting methods: 2016 US presidential election [Igersheim et al 2016].

Table 1
Candidate scores under three voting rules.

	Plurality		Approval	
	short set	long set	short set	long set
Clinton	47.73	31.38	50.12	39.78
	(1.63)	(1.50)	(1.62)	(1.51)
Trump	40.52	27.78	42.01	33.73
	(1.57)	(1.25)	(1.60)	(1.44)
Sanders		19.98		39.25
		(1.08)		(1.47)
Cruz		9.73		21.46
		(0.89)		(1.36)
Johnson	8.25	4.54	20.65	12.16
	(0.93)	(0.62)	(1.26)	(1.02)
Bloomberg		4.41		11.65
		(0.50)		(0.91)
McMullin		1.73		7.60
		(0.40)		(0.89)
Stein	3.51	0.26	11.52	5.06
	(0.65)	(0.15)	(1.18)	(0.72)
Castle		0.19		2.18
		(0.13)		(0.45)

4 candidates in the short set.

9 candidates in the long set.

An experiment in Dagstuhl

- **A vote on the cake for the next day**

Group A	50%	**AV-6** →	**EV-12** →	**AV-12**
Group B	50%	**AV-12** →	**EV-12** →	**AV-6**

⚠ The set of 6 was **always the same.**

An experiment in Dagstuhl

How many candidates **out of the first 6** are approved in average?

Group A

When seeing 6	When seeing 12
$\widehat{m_6} = 2.7$	$\widehat{m_6} = 2.4\ (\widehat{m_{12}} = 4.6)$

Group B

When seeing 12	When seeing 6
$\widehat{m_6} = 1.7(\widehat{m_{12}} = 3.9)$	$\widehat{m_6} = 1.9$

An experiment in Dagstuhl

How many candidates **out of the first 6** are approved in average?

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Group B

When seeing 12	When seeing 6
$\widehat{m_6} = 1.7 (\widehat{m_{12}} = 3.9)$	$\widehat{m_6} = 1.9$

Framing effect: having more alternatives changes the probability to approve one of the first 6.

An experiment in Dagstuhl

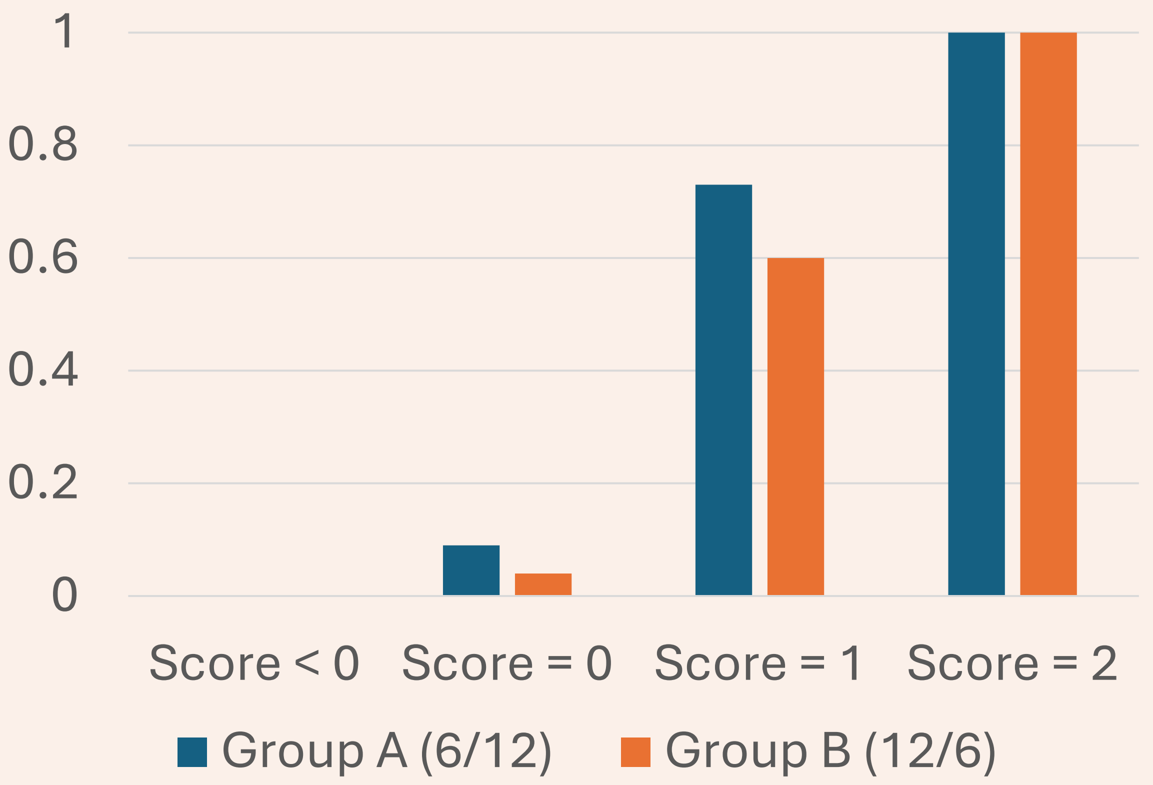

Fig. Approval rate depending on the score (on the first 6 candidates).

An experiment in Dagstuhl

How many candidates **out of the first 6** are approved in average?

Group A

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$\widehat{m_6} = 2.7$	$\widehat{m_6} = 2.4\ (\widehat{m_{12}} = 4.6)$

Slightly less

Group B

When seeing 12	When seeing 6
$\widehat{m_6} = 1.7(\widehat{m_{12}} = 3.9)$	$\widehat{m_6} = 1.9$

Slightly more

Carry-over effect: voters are being consistent between their votes.

An experiment in Dagstuhl

Group A (6->12) participant

$A_6 =$    

All of these are evaluated **with score $\geq$ 0** by the voter, and everything else with score < 0.

$A_{12} =$       

All of these are evaluated **with score $\geq$ 1** by the voter, and everything else with score < 1.

[4] Discussion

What is the impact?

It can change the winner

Depending on the hypothesis selected, changing the number of candidates can change the winner.

It can change the score

The winner might win with 55% of approvals in one case and 35% in the other: in a political election, **it would be more complicated to make the result accepted** (e.g., in a primary in which we want a clear winner).

Thanks for your attention!