

Reallocating Wasted Votes in Proportional Parliamentary Elections with Thresholds

Théo Delemazure

*Joint work with Rupert Freeman, Jérôme Lang, Jean-François Laslier and Dominik Peters
(published at EC-2025)*

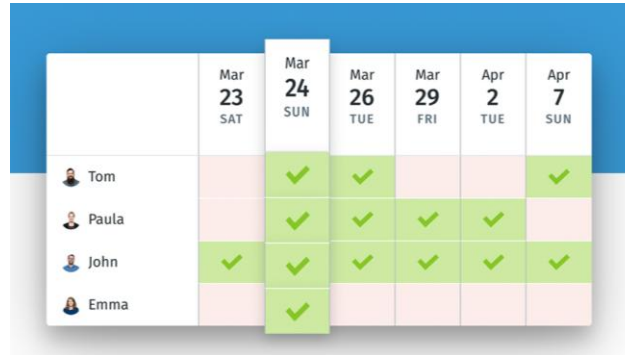
1. Computational Social Choice

Social Choice Theory:

Designing and analyzing methods for collective decision making



Political election



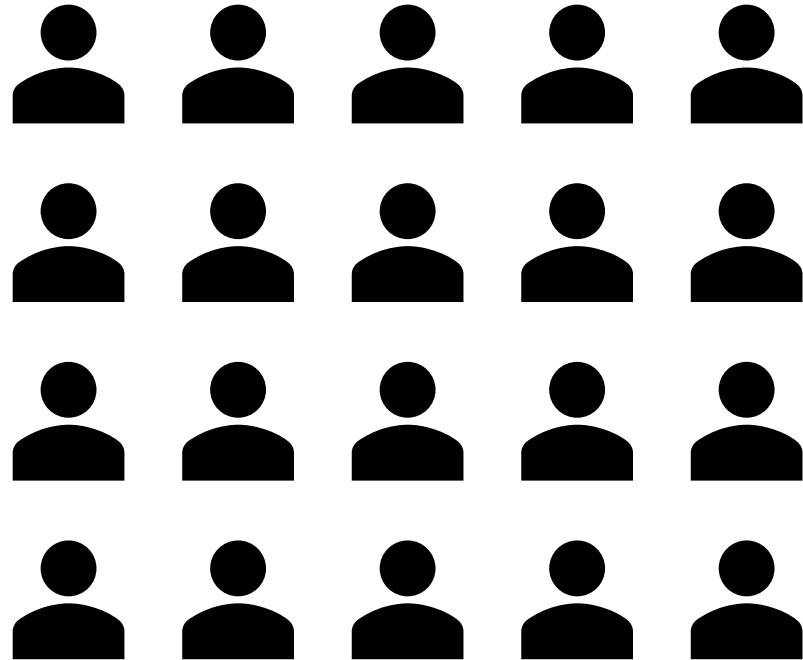
	Mar 23 SAT	Mar 24 SUN	Mar 26 TUE	Mar 29 FRI	Apr 2 TUE	Apr 7 SUN
Tom		✓	✓			✓
Paula		✓	✓	✓	✓	
John	✓	✓	✓	✓	✓	✓
Emma		✓				

Decide on a date

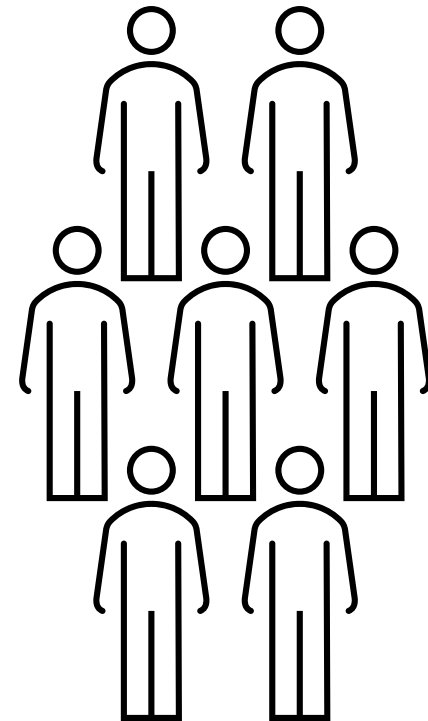


Jury decision

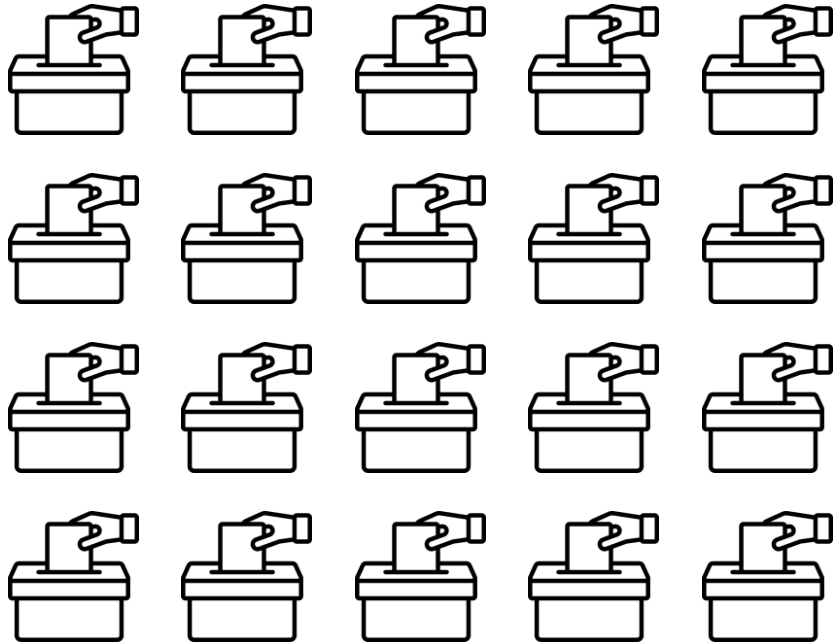
Voters



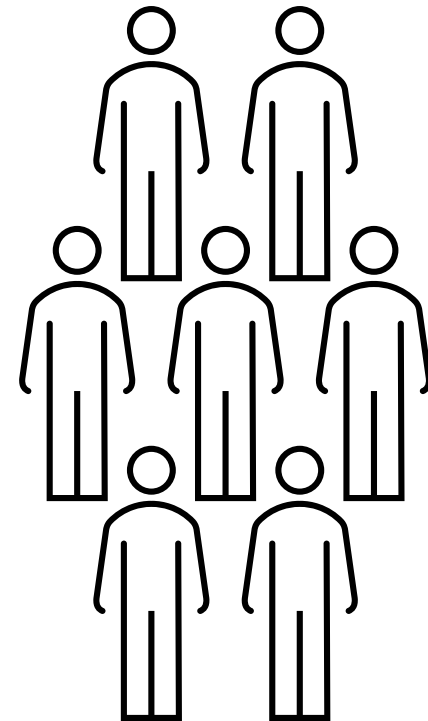
Candidates



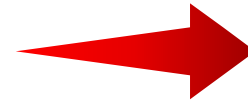
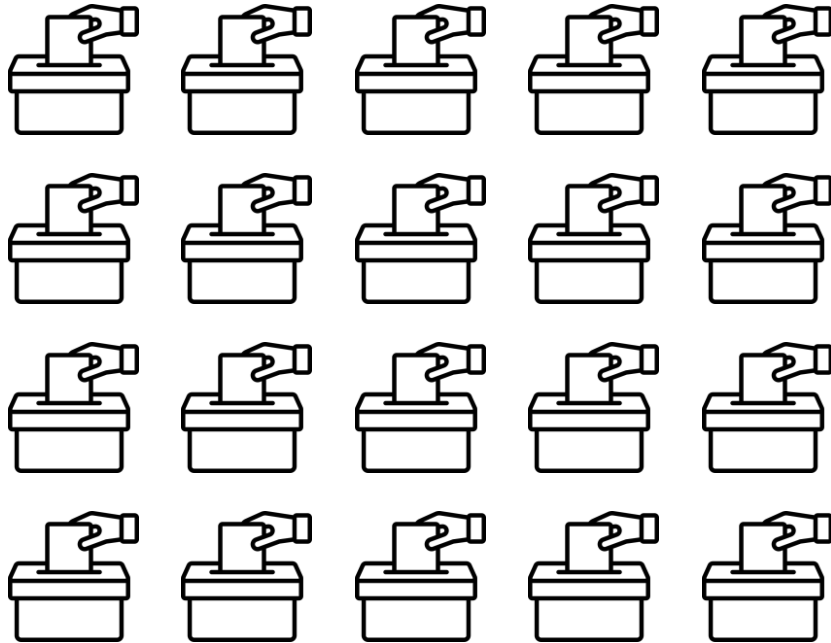
Voters give their preferences
over candidates



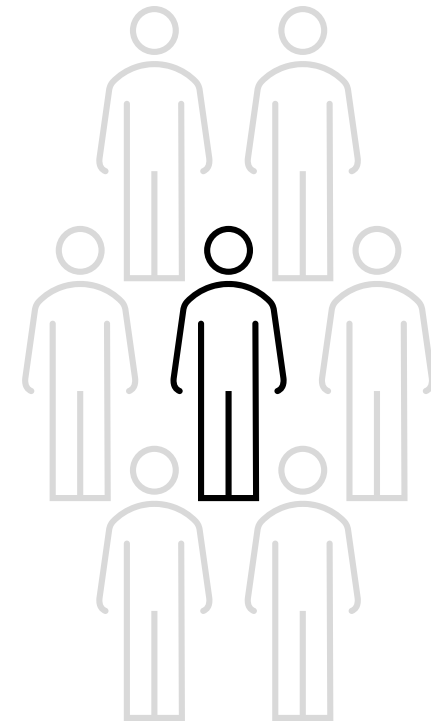
Candidates



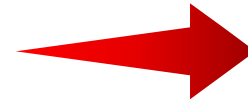
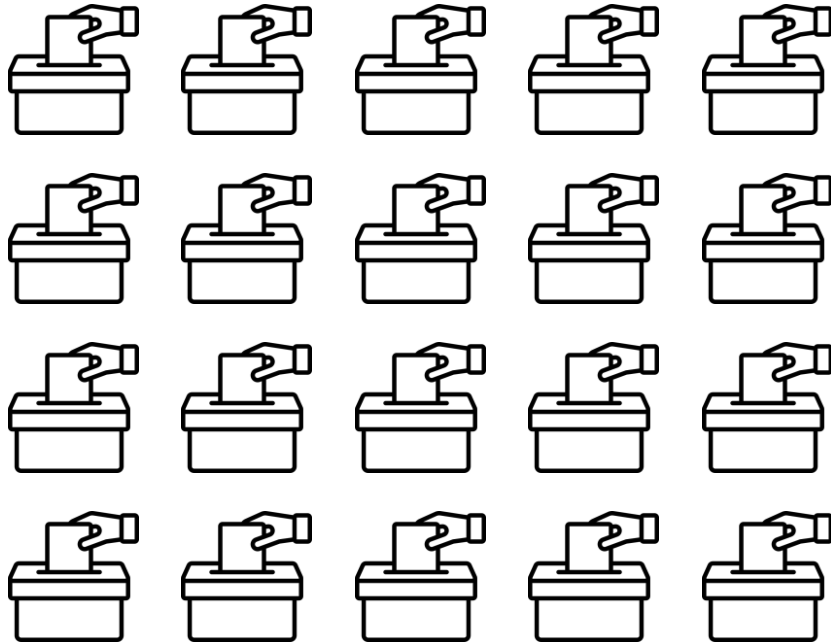
Voters give their preferences
over candidates



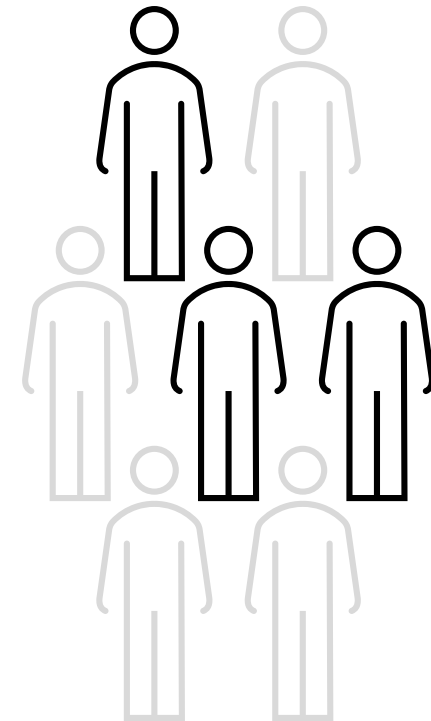
A **winner** is selected



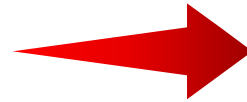
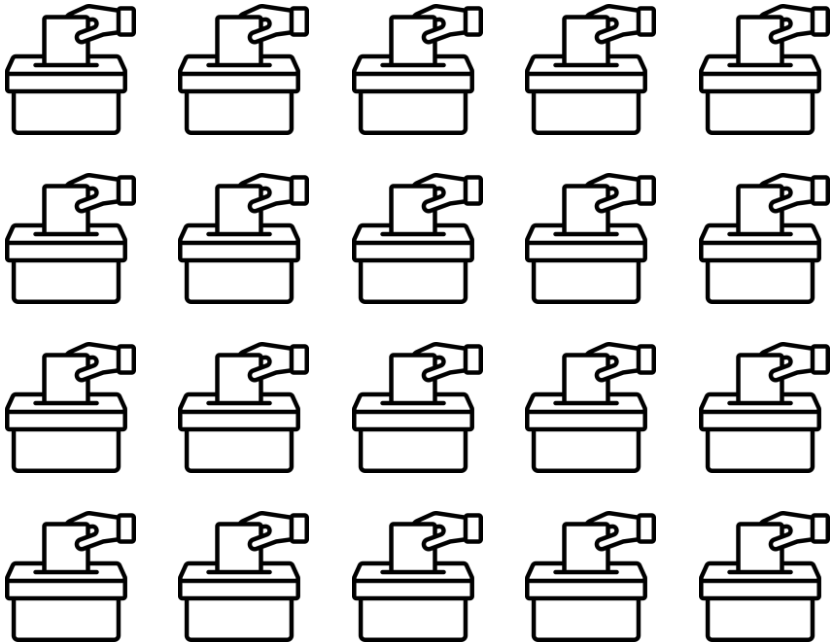
Voters give their preferences
over candidates



A **committee** is selected

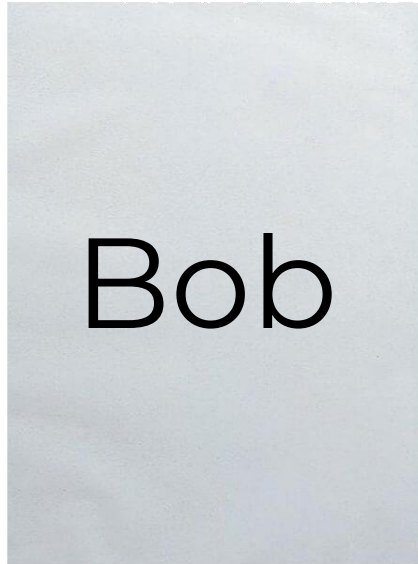


Voters give their preferences
over candidates



A **parliament** is selected





Uninominal
Ballots

A light gray rectangular ballot with the name "Bob" centered in a large, black, sans-serif font.

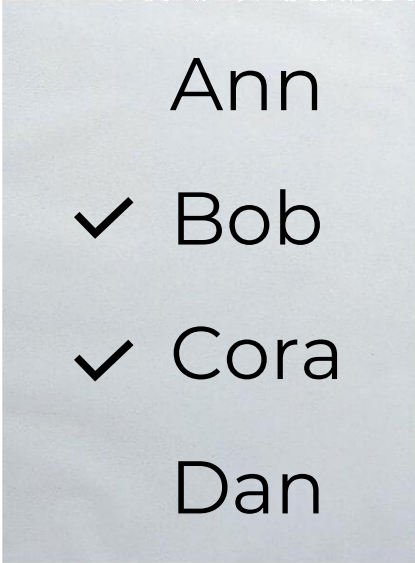
Bob

Uninominal
Ballots

A light gray rectangular ballot with a list of four candidates and their ranks, separated by a vertical line. The text is in a black, sans-serif font.

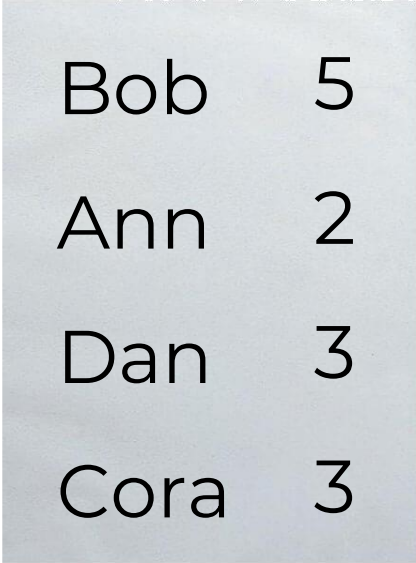
1 Bob
2 Ann
3 Dan
4 Cora

Rankings

A light gray rectangular ballot with a list of four candidates. The first two, "Ann" and "Bob", are preceded by a checkmark. The text is in a black, sans-serif font.

Ann
✓ Bob
✓ Cora
Dan

Approval
Ballots

A light gray rectangular ballot with a list of four candidates and their scores, separated by a vertical line. The text is in a black, sans-serif font.

Bob 5
Ann 2
Dan 3
Cora 3

Scores

2. Single-winner voting with rankings

We have:

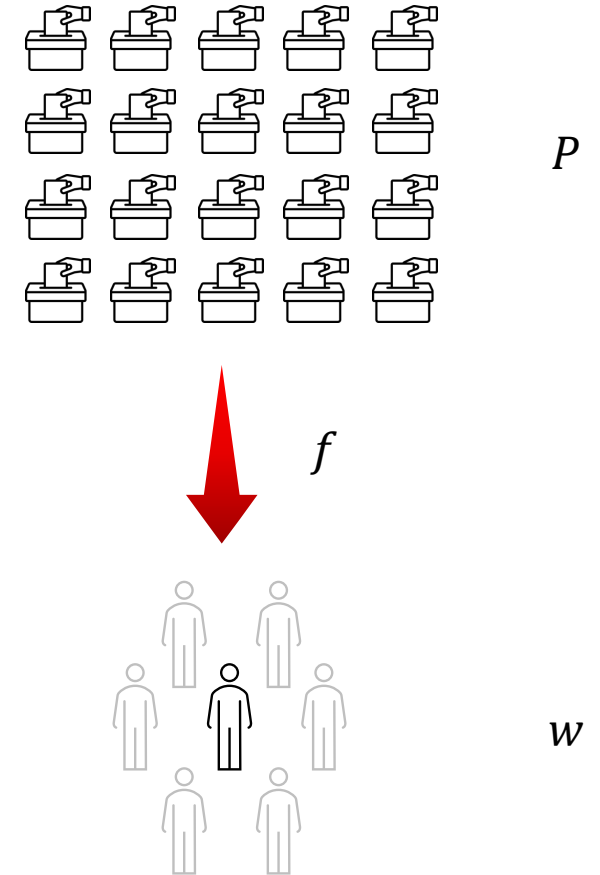
- A set of **voters** $V = \{1, 2, \dots, n\}$.
- A set of **candidates** $C = \{c_1, \dots, c_m\}$.
- A preference **profile** $P = (\succ_1, \dots, \succ_n) \in \mathcal{L}(C)^n$ of rankings of voters over candidates.

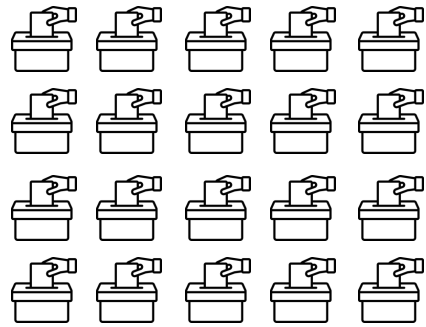
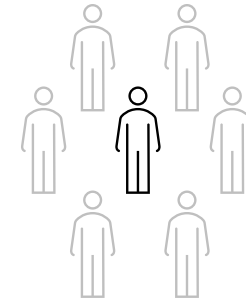
We want:

- A **winning candidate** $w \in C$.

For this, we use:

- A **voting rule** $f: \mathcal{L}(C)^n \rightarrow C$.



Profile P Rule f Winner w 

Question: which voting rule should we use?

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- 1 **Design** rules, analyze their complexity, and propose algorithms to compute them.
- 2 Check the **normative properties** (the *axioms*) satisfied by these rules.
- 3 **Run simulations** of the rules on real or synthetic preference data.

1 Design rules: *Plurality*

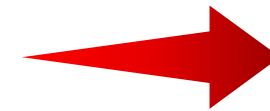
Plurality: The winner is the candidate that is ranked first by the most voters.

40% $A \succ B \succ D \succ C$

25% $B \succ C \succ D \succ A$

20% $C \succ B \succ D \succ A$

15% $D \succ C \succ A \succ B$



Winner: A

1 Design rules: *Veto*

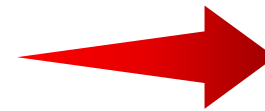
Veto: The winner is the candidate that is ranked last by the fewest voters.

40% $A \succ B \succ D \succ C$

25% $B \succ C \succ D \succ A$

20% $C \succ B \succ D \succ A$

15% $D \succ C \succ A \succ B$



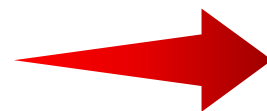
Winner: D

1

Design rules: *Borda*

Borda: Voters give $m - 1$ points to the first candidate, $m - 2$ to the second, and so on. The winner is the candidate with the highest score.

40%	$A \succ B \succ D \succ C$
25%	$B \succ C \succ D \succ A$
20%	$C \succ B \succ D \succ A$
15%	$D \succ C \succ A \succ B$
	+3 +2 +1

**Winner:** B

1

Design rules: The family of *Scoring rules*

1 Bob	→	1
2 Ann	→	0
3 Dan	→	0
4 Cora	→	0

Plurality

1 Bob	→	3
2 Ann	→	2
3 Dan	→	1
4 Cora	→	0

Borda

1 Bob	→	1
2 Ann	→	1
3 Dan	→	1
4 Cora	→	0

Veto

1 Design rules: *Instant Runoff Voting* (IRV/STV)

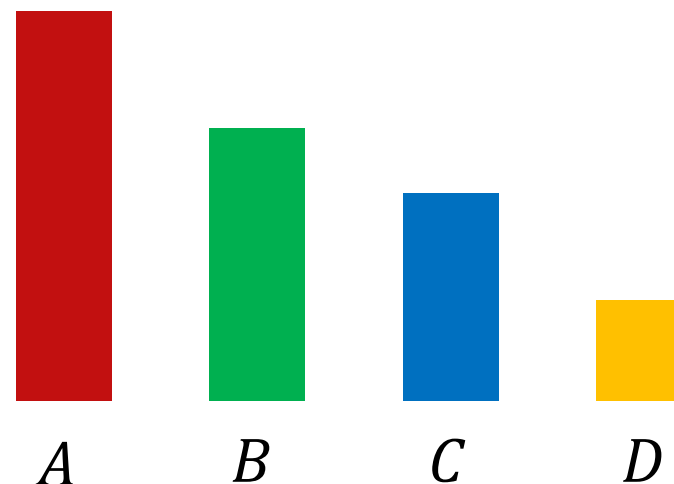
Instant Runoff Voting (IRV): Repeatedly eliminate the candidate with the fewest 1st-place votes until one candidate gets 50% of the votes.

40% **A** \succ B \succ D \succ C

25% **B** \succ C \succ D \succ A

20% **C** \succ B \succ D \succ A

15% **D** \succ C \succ A \succ B



1 Design rules: *Instant Runoff Voting* (IRV/STV)

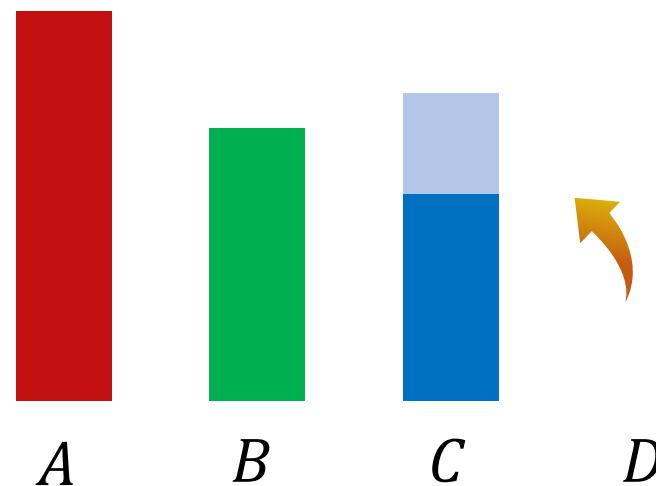
Instant Runoff Voting (IRV): Repeatedly eliminate the candidate with the fewest 1st-place votes until one candidate gets 50% of the votes.

40% **A** > B > C

25% **B** > C > A

20% **C** > B > A

15% **C** > A > B



1 Design rules: *Instant Runoff Voting* (IRV/STV)

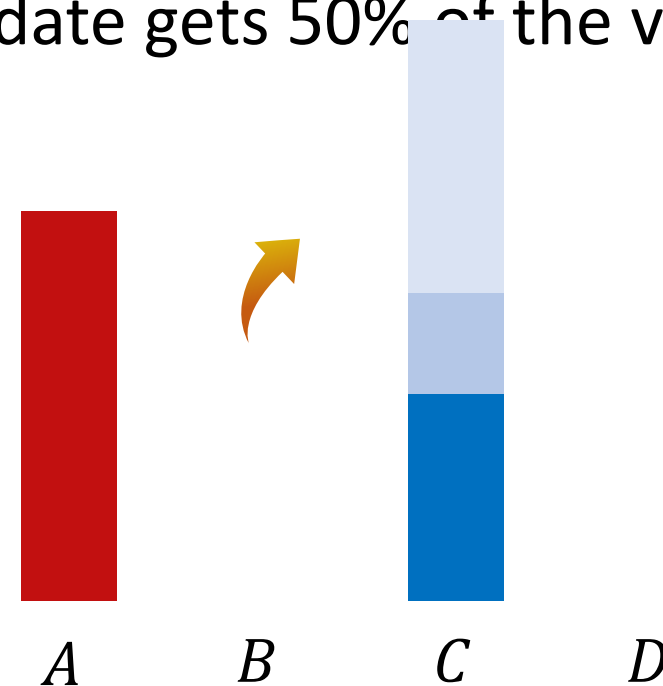
Instant Runoff Voting (IRV): Repeatedly eliminate the candidate with the fewest 1st-place votes until one candidate gets 50% of the votes.

40% **A** > C

25% **C** > A

20% **C** > A

15% **C** > A



2 Check the **normative properties** (the *axioms*).

Axiom: Reinforcement

If a candidate wins in a profile P_1 and in a profile P_2 , it also wins in the profile $P_1 + P_2$.

Characterization Theorem (*Smith and Young, 1973*) : **Scoring Rules** are the only voting rules that satisfy **reinforcement**, neutrality, and anonymity.

2 Check the **normative properties** (the *axioms*).

Axiom: Strategyproofness

A voter cannot obtain a better winner by misreporting their preferences.

4 × $A \succ B \succ D \succ C$

3 × $B \succ C \succ A \succ D$

2 × $C \succ B \succ D \succ A$

2 × $D \succ C \succ A \succ B$



4 × $A \succ B \succ D \succ C$

3 × $B \succ C \succ A \succ D$

2 × $B \succ C \succ D \succ A$

2 × $D \succ C \succ A \succ B$

The plurality winner is A .

The plurality winner is now B .

2 Check the **normative properties** (the *axioms*).

Axiom: Strategyproofness

A voter cannot obtain a better winner by misreporting their preferences.

Impossibility Theorem (*Gibbard and Satterthwaite, 1973*) : There exists no rule that satisfies **strategyproofness**, resoluteness, non-imposition and non-dictatorship.

3 Run simulations with the voting rules.

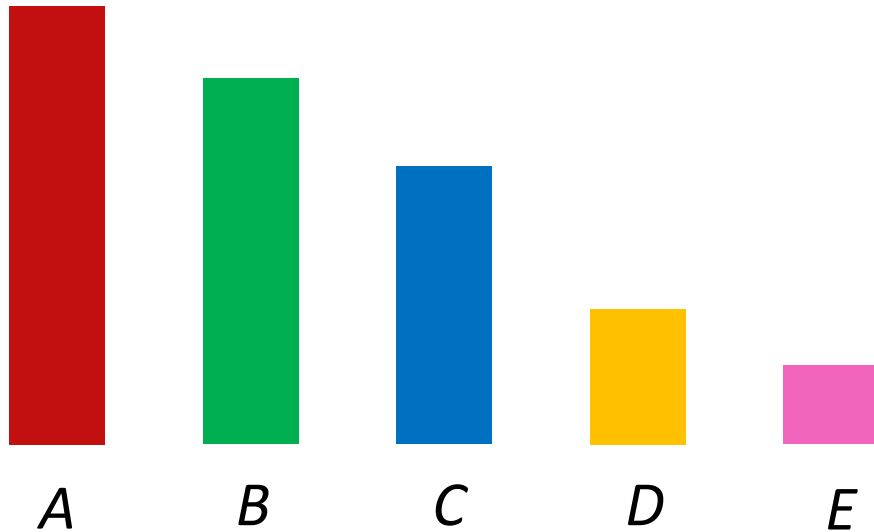
We need preference data for the simulations:

- ➔ Generate **synthetic data** from probabilistic models.
- ➔ Use data from **online libraries** of datasets (e.g., Preflib).
- ➔ Design **voting experiments** and collect data.

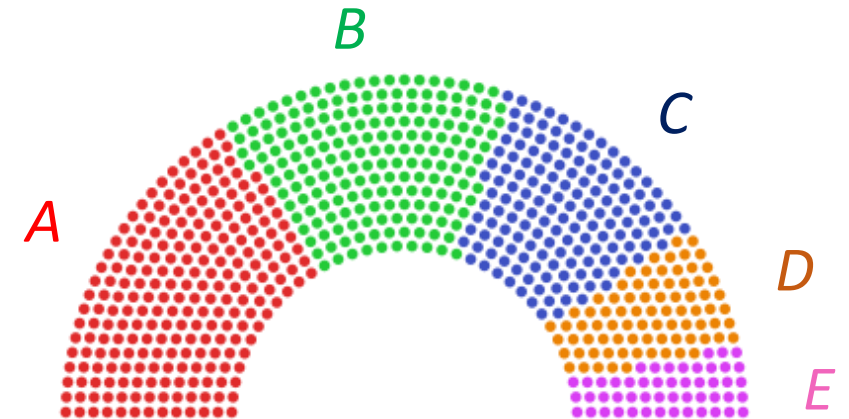
3. Proportional Parliamentary elections with Threshold



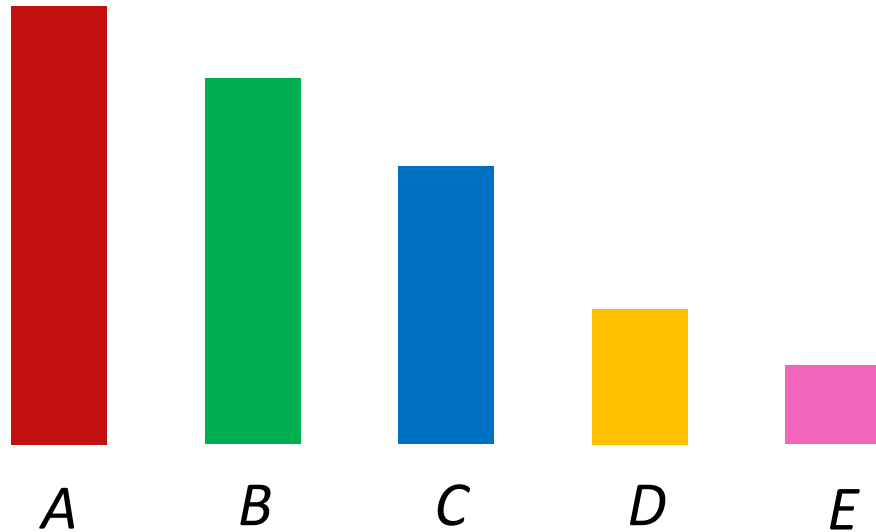
Voters vote for one party.



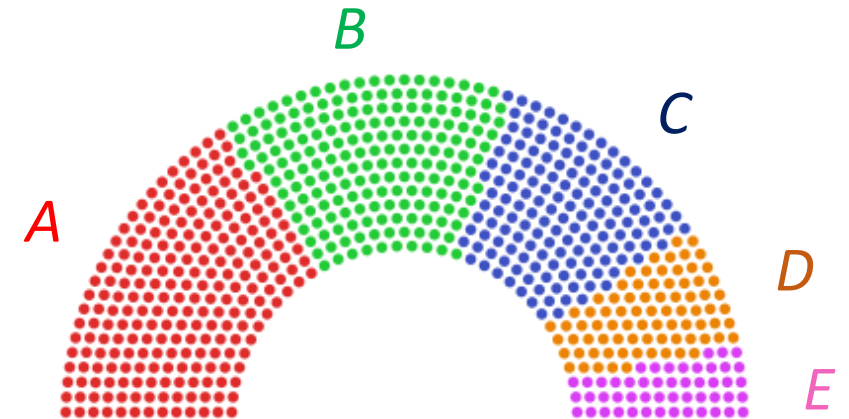
Seats are allocated to parties **proportionally to their scores.**



Voters vote for one party.



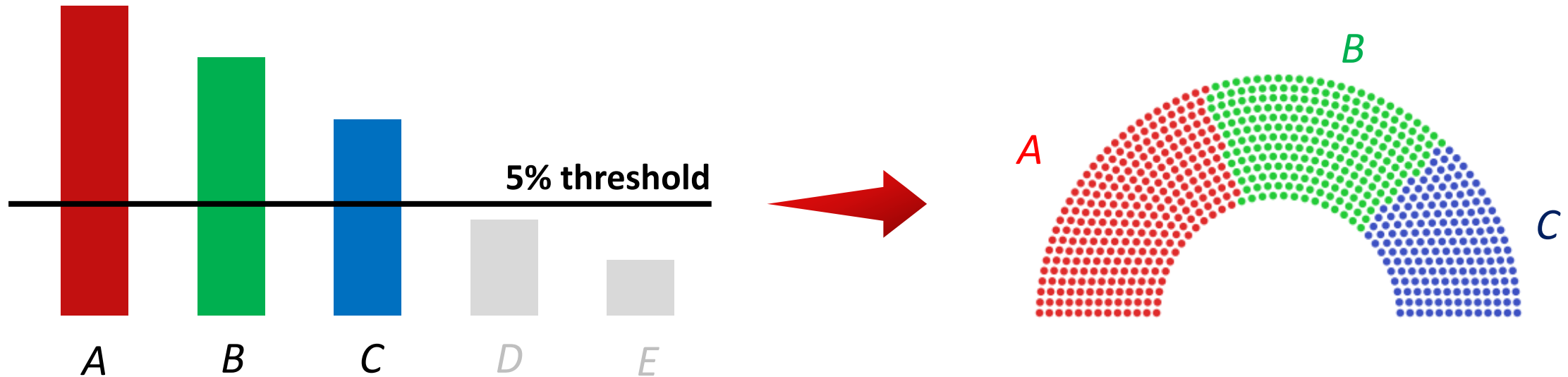
Seats are allocated to parties **proportionally to their scores.**



Problem: possible political fragmentation (many parties get a seat).

Many countries impose an **electoral threshold** to reduce political fragmentation.





- ➔ **Some votes are “lost”:** *D* and *E* supporters have no influence on the seat distribution.
- ➔ This incentivizes forms of **tactical voting**.



2019 election of the French
representative to the EU Parliament.

Threshold

5%

“Lost” votes


20%



2025 election of the *Bundestag*
members.

5%

14%

 increasing in
recent decades



2002 election of the *Turkish*
Parliament members.

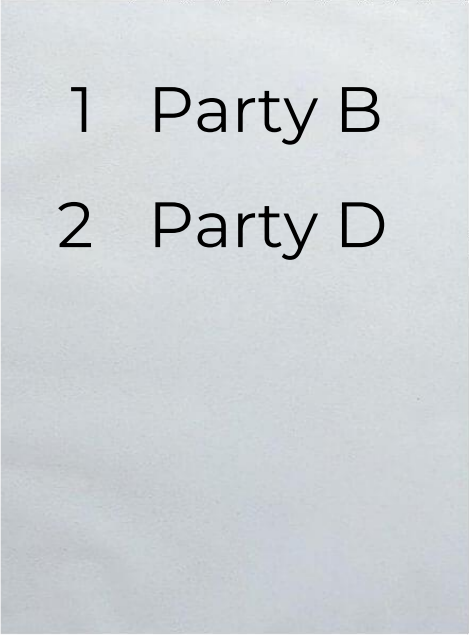
10%

46%

We could let voters **indicate a second choice** to be used in case their first choice does not reach the threshold.

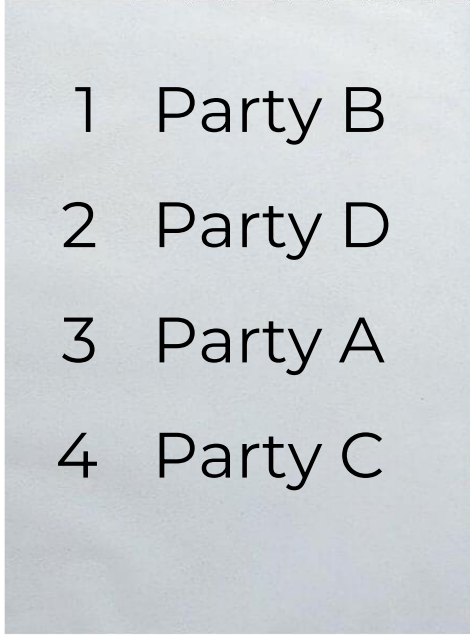


We could ask voters to rank **two parties**



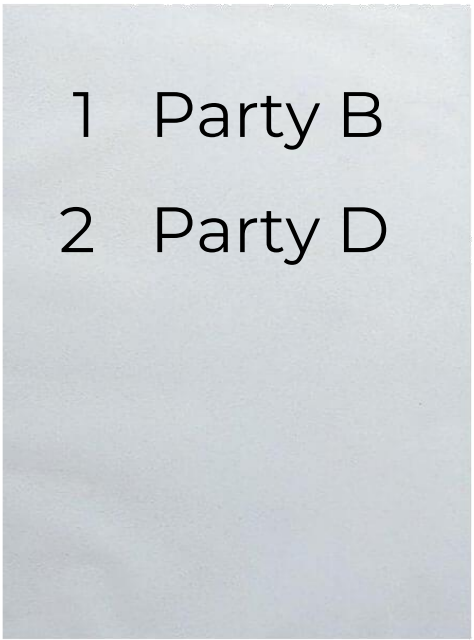
1 Party B
2 Party D

We could even ask for a **truncated ranking**



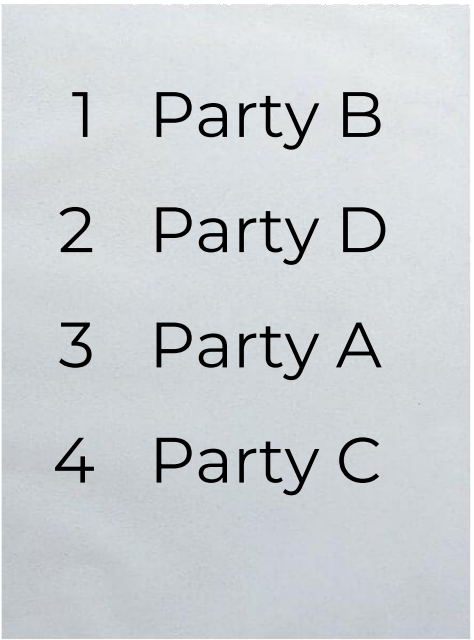
1 Party B
2 Party D
3 Party A
4 Party C

We could ask voters to rank **two parties**



1 Party B
2 Party D

We could even ask for a **truncated ranking**



1 Party B
2 Party D
3 Party A
4 Party C

Question: how to decide which parties are “above the threshold”?

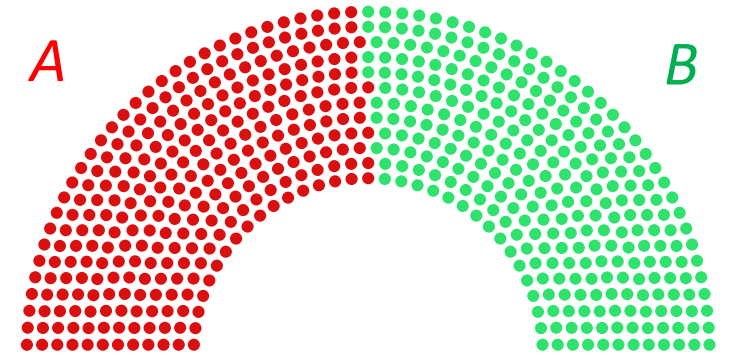
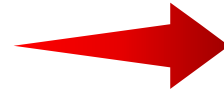
43% *A*

43% *B*

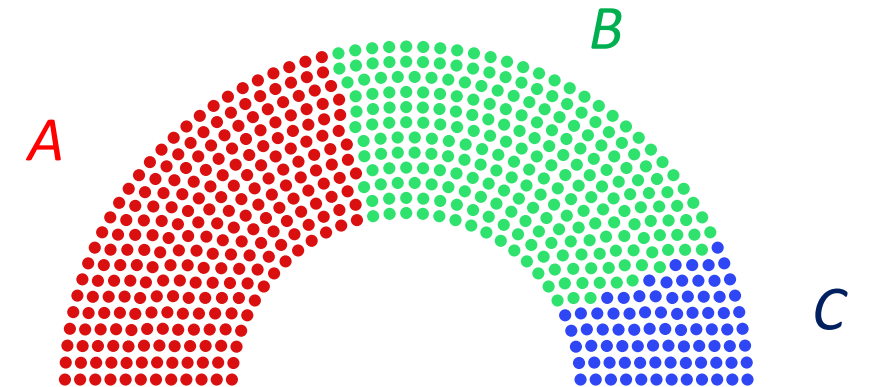
4% *C* \succ *D*

2% *D* \succ *C*

5% threshold



or



We have:

- Sets of **voters** $V = \{1, 2, \dots, n\}$ and **parties** $C = \{p_1, \dots, p_m\}$.
- A preference **profile** $P = (\succ_1, \dots, \succ_n)$ of truncated rankings of voters over parties.
- A given **threshold** τ (absolute number of voters).

We want:

- A set of selected parties $S \subseteq C$, called the **outcome**.
- Voters are **represented** by their most-preferred party in S (if any).
- An outcome S is **feasible** if every party represents at least τ voters.
- We assume that parties in S get a **number of seats proportional** to the **share** of voters they represent.

- Profile P :

6 × **A** \succ $B \succ D \succ C$

4 × $B \succ C \succ E \succ$ **A** $\succ D$

3 × $C \succ B$

3 × $D \succ E \succ B \succ$ **A** $\succ C$

2 × $C \succ$ **A** $\succ E$

- Threshold $\tau = 5$.

Outcome $\{A\}$ is feasible.

- Profile P :

6 × $A \succ B \succ D \succ C$

4 × $B \succ C \succ \textcolor{blue}{E} \succ A \succ D$

3 × $C \succ B$

3 × $D \succ \textcolor{blue}{E} \succ B \succ A \succ C$

2 × $C \succ A \succ \textcolor{blue}{E}$

- Threshold $\tau = 5$.

Outcome $\{A\}$ is feasible.

Outcome $\{E\}$ is feasible.

- Profile P :

6 × **A** \succ $B \succ D \succ C$

4 × $B \succ$ **C** $\succ E \succ A \succ D$

3 × **C** $\succ B$

3 × $D \succ E \succ B \succ$ **A** $\succ C$

2 × **C** $\succ A \succ E$

- Threshold $\tau = 5$.

Outcome $\{A\}$ is feasible.

Outcome $\{E\}$ is feasible.

Outcome $\{A, C\}$ is feasible.

- Profile P :

6 × **A** \succ $B \succ D \succ C$

4 × **B** $\succ C \succ E \succ A \succ D$

3 × **C** $\succ B$

3 × $D \succ E \succ$ **B** $\succ A \succ C$

2 × **C** $\succ A \succ E$

- Threshold $\tau = 5$.

Outcome $\{A\}$ is feasible.

Outcome $\{E\}$ is feasible.

Outcome $\{A, C\}$ is feasible.

Outcome $\{A, B, C\}$ is feasible.

- Profile P :

6 × $A \succ B \succ D \succ C$

4 × $B \succ C \succ E \succ A \succ D$

3 × $C \succ B$

3 × $D \succ E \succ B \succ A \succ C$

2 × $C \succ A \succ E$

- Threshold $\tau = 5$.

Outcome $\{A\}$ is feasible.

Outcome $\{E\}$ is feasible.

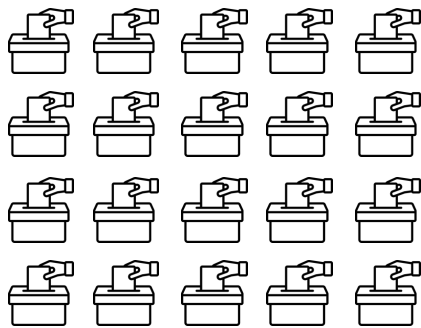
Outcome $\{A, C\}$ is feasible.

Outcome $\{A, B, C\}$ is feasible.

Outcome $\{B, D\}$ is not feasible.

- ! If $\tau > n/2$ and with full rankings, this corresponds to the **single-winner voting model** (if we additionally force a non-empty outcome).

(This is because when $\tau > n/2$, only one candidate can be part of the outcome since each candidate in the outcome needs to represent more than τ voters)

Profile P ⊕ Threshold τ Party Selection Rule f Feasible
Outcome

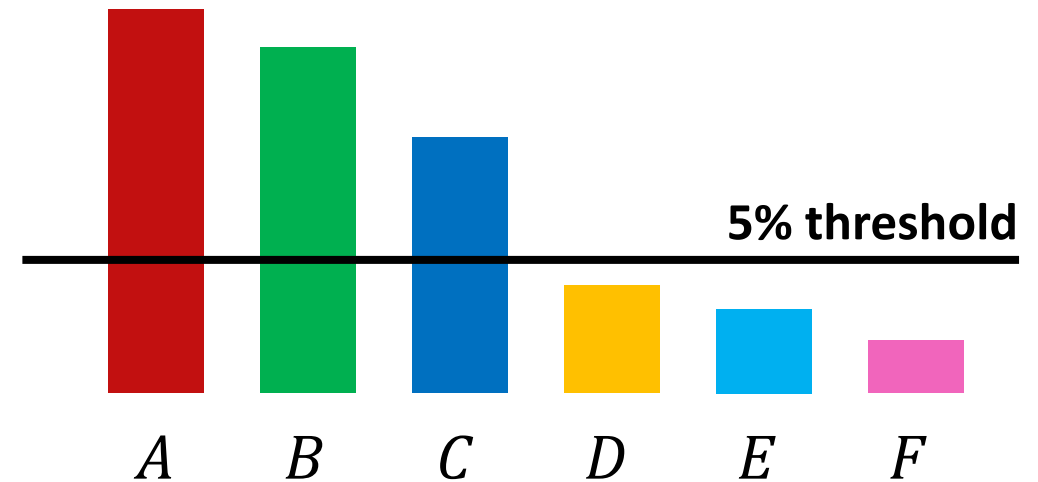
$$S \subseteq \mathcal{C}$$

4. Party Selection Rules

Rule: Direct Winners Only (DO)

The selected parties are all those which receive more first-place votes than required by the threshold.

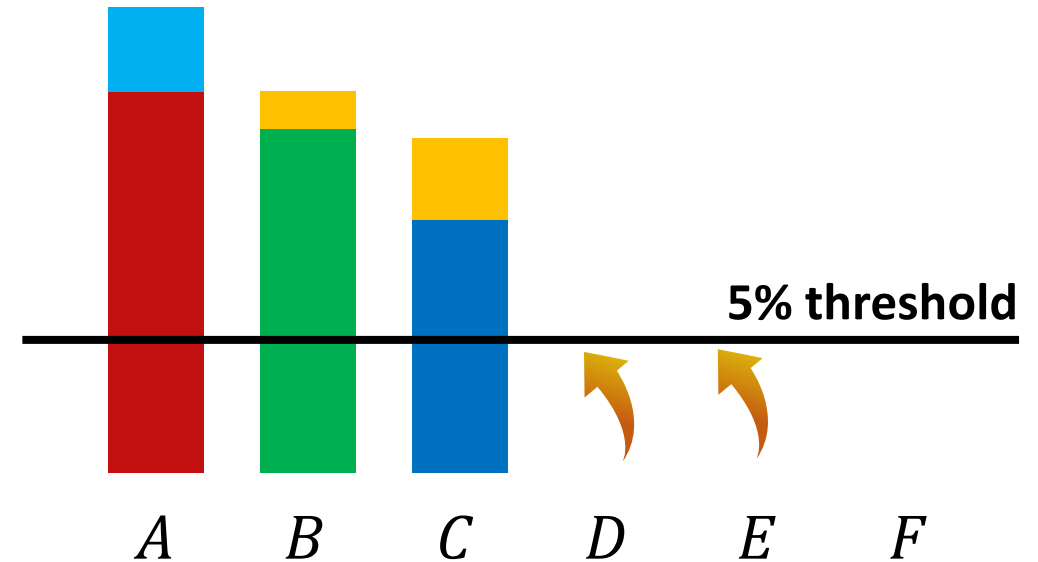
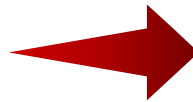
35% A > C	3% D > E > F > C
30% B	3% E > F > A
20% C > E > A	2% F > E > D
6% C > A > B	1% D > F > B



Rule: Direct Winners Only (DO)

The selected parties are all those which receive more first-place votes than required by the threshold.

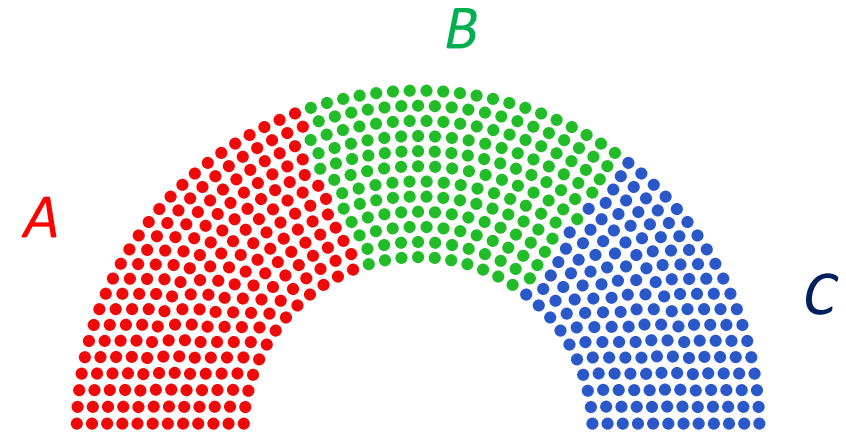
35% A > C	3% <i>D</i> > <i>E</i> > <i>F</i> > C
30% B	3% <i>E</i> > <i>F</i> > A
20% C > <i>E</i> > <i>A</i>	2% <i>F</i> > <i>E</i> > <i>D</i>
6% C > <i>A</i> > <i>B</i>	1% <i>D</i> > <i>F</i> > B



Rule: Direct Winners Only (DO)

The selected parties are all those which receive more first-place votes than required by the threshold.

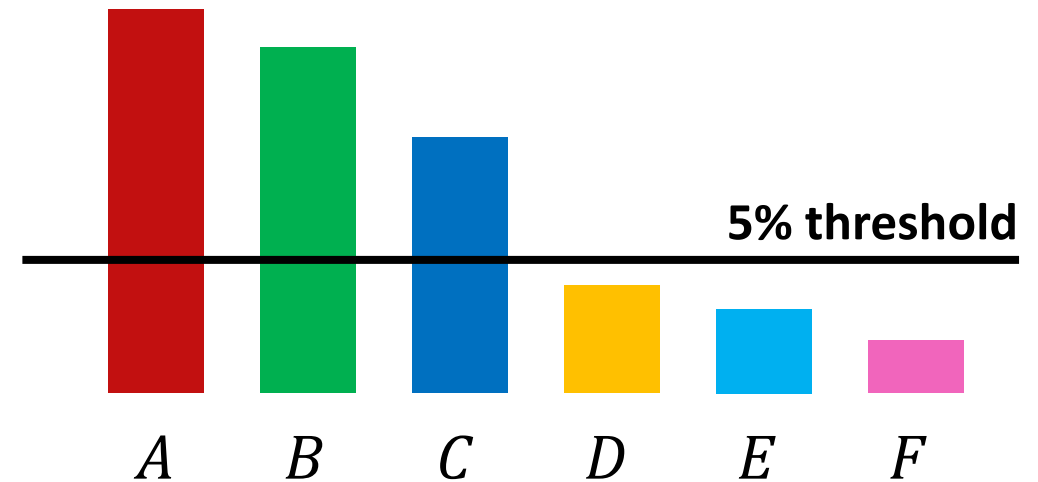
35% A > C	3% D > E > F > C
30% B	3% E > F > A
20% C > E > A	2% F > E > D
6% C > A > B	1% D > F > B



Rule: Single Transferable Vote (STV)

Parties that receive the fewest votes are successively eliminated until all parties receive more votes than required by the threshold.

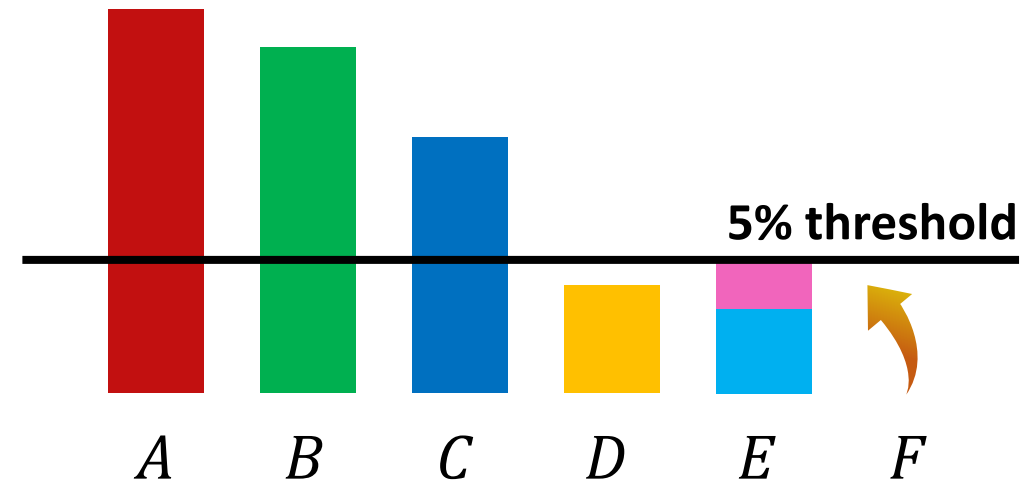
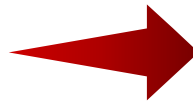
35% A > C	3% D > E > F > C
30% B	3% E > F > A
20% C > E > A	2% F > E > D
6% C > A > B	1% D > F > B



Rule: Single Transferable Vote (STV)

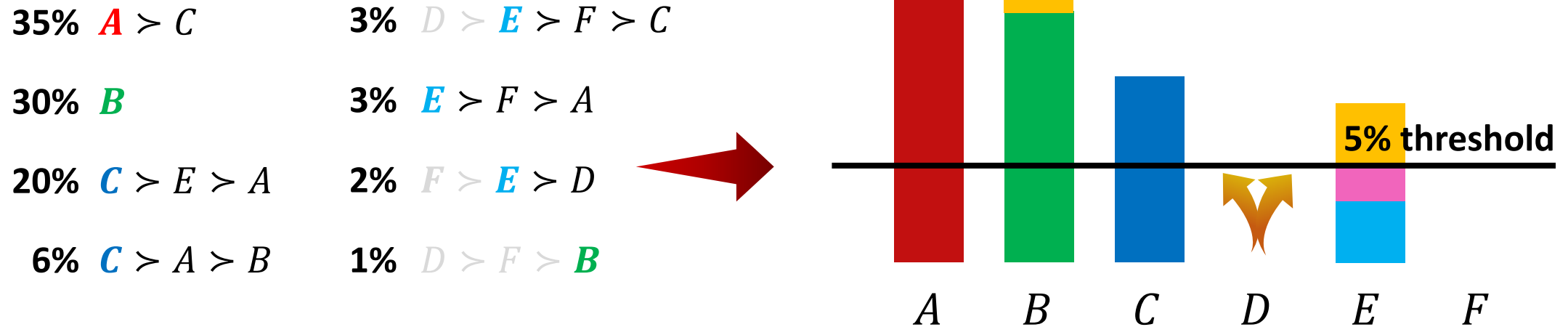
Parties that receive the fewest votes are successively eliminated until all parties receive more votes than required by the threshold.

35% A > C	3% D > E > F > C
30% B	3% E > F > A
20% C > E > A	2% F > E > D
6% C > A > B	1% D > F > B



Rule: Single Transferable Vote (STV)

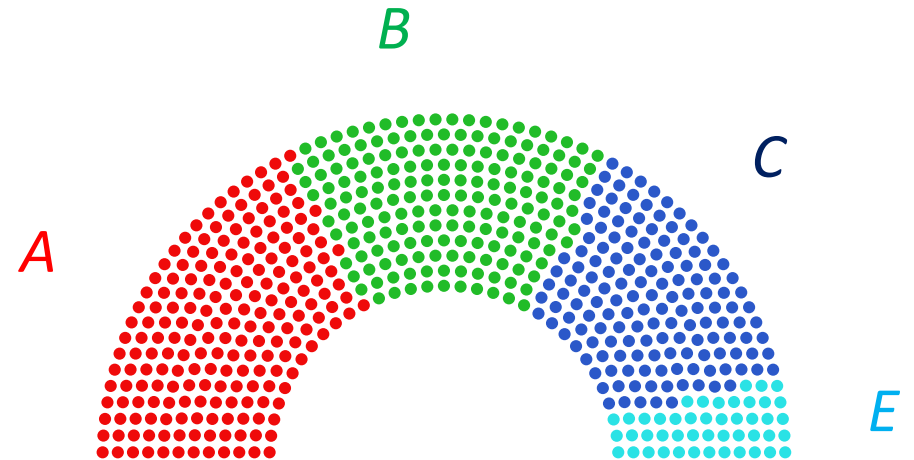
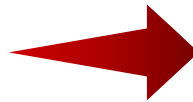
Parties that receive the fewest votes are successively eliminated until all parties receive more votes than required by the threshold.



Rule: Single Transferable Vote (STV)

Parties that receive the fewest votes are successively eliminated until all parties receive more votes than required by the threshold.

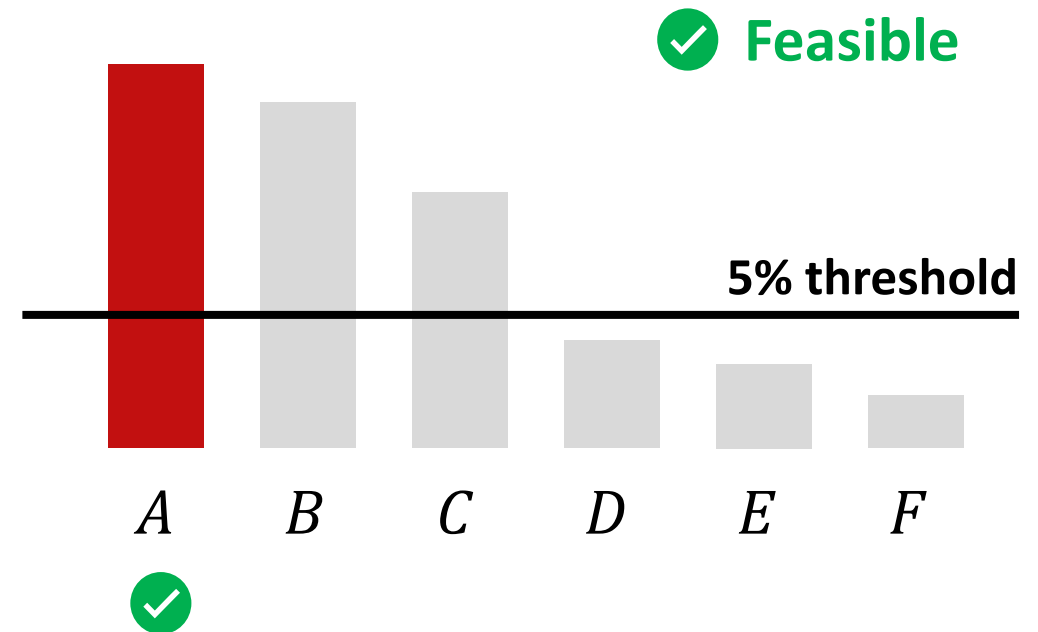
35% A > C	3% <i>D</i> > E > <i>F</i> > C
30% B	3% E > <i>F</i> > A
20% C > <i>E</i> > A	2% <i>F</i> > E > <i>D</i>
6% C > A > B	1% <i>D</i> > <i>F</i> > B



Rule: Greedy Plurality (GP)

Parties are considered in decreasing order of plurality score, and added to the outcome if it remains feasible.

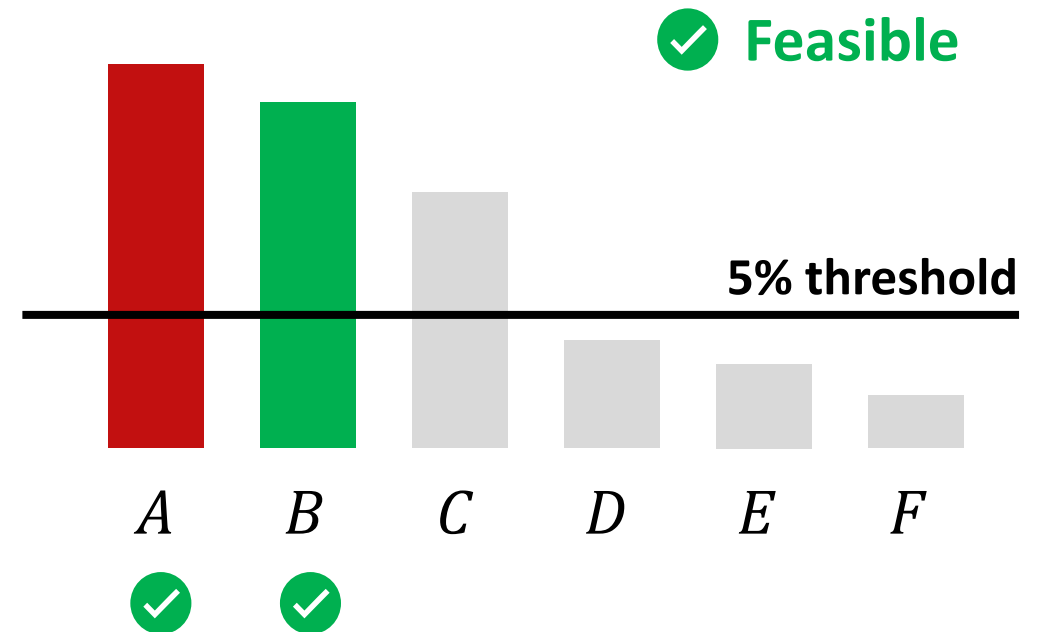
35% A > C	3% D > E > F > C
30% B	3% E > F > A
20% C > E > A	2% F > E > D
6% C > A > B	1% D > F > B



Rule: Greedy Plurality (GP)

Parties are considered in decreasing order of plurality score, and added to the outcome if it remains feasible.

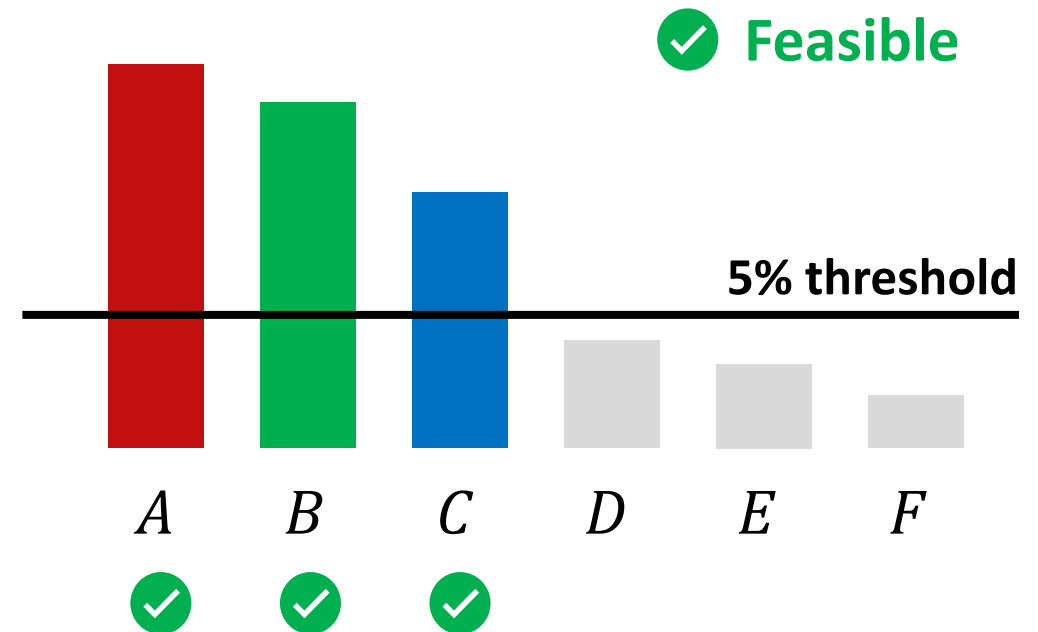
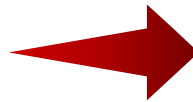
35% A > C	3% D > E > F > C
30% B	3% E > F > A
20% C > E > A	2% F > E > D
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Rule: Greedy Plurality (GP)

Parties are considered in decreasing order of plurality score, and added to the outcome if it remains feasible.

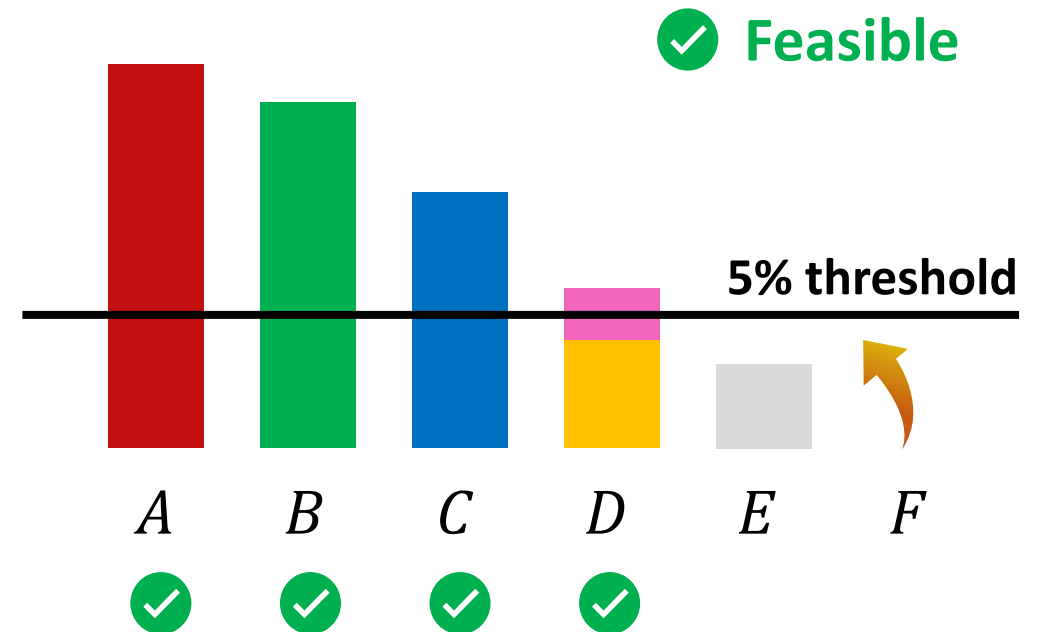
35% A > C	3% D > E > F > C
30% B	3% E > F > A
20% C > E > A	2% F > E > D
6% C > A > B	1% D > F > B



Rule: Greedy Plurality (GP)

Parties are considered in decreasing order of plurality score, and added to the outcome if it remains feasible.

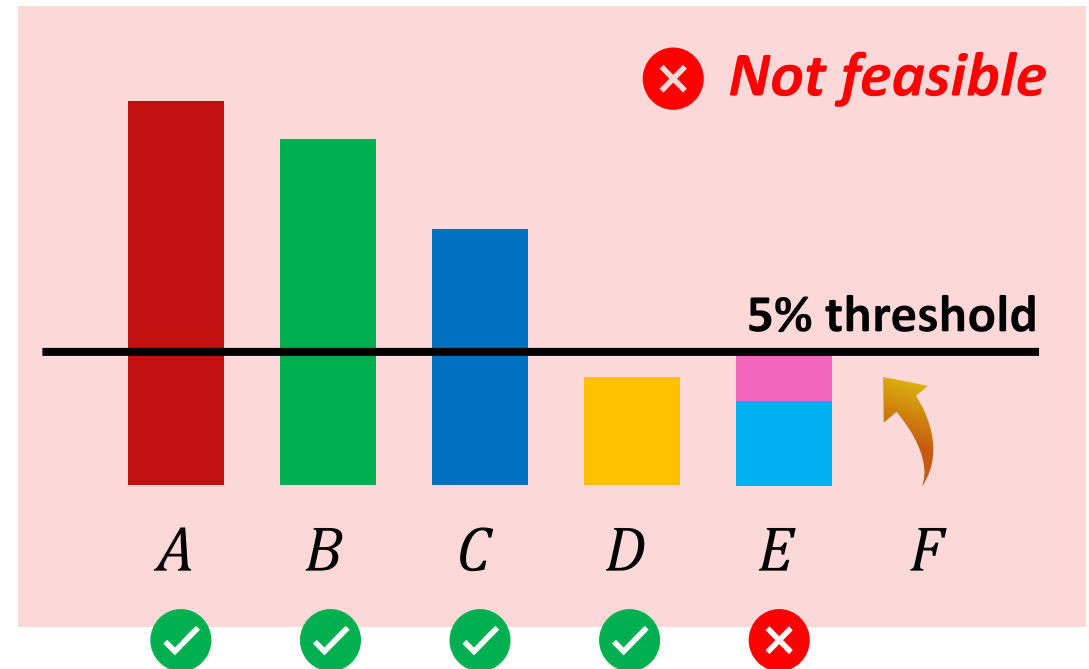
35% A > C	3% D > E > F > C
30% B	3% E > F > A
20% C > E > A	2% F > E > D
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Rule: Greedy Plurality (GP)

Parties are considered in decreasing order of plurality score, and added to the outcome if it remains feasible.

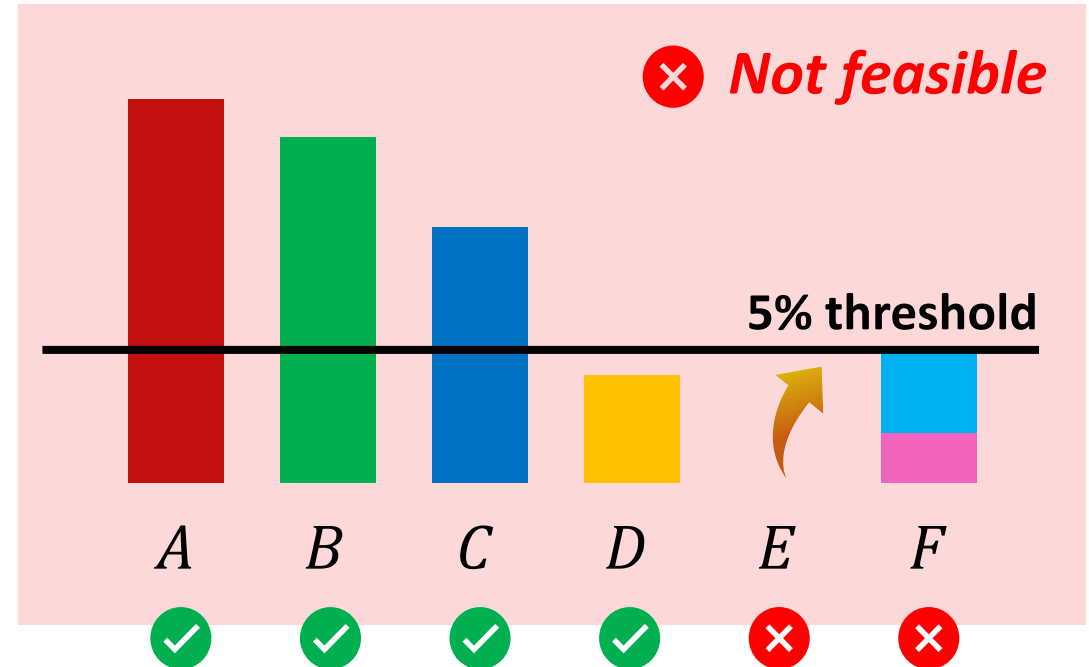
35% A > C	3% D > E > F > C
30% B	3% E > F > A
20% C > E > A	2% F > E > D
6% C > A > B	1% D > F > B



Rule: Greedy Plurality (GP)

Parties are considered in decreasing order of plurality score, and added to the outcome if it remains feasible.

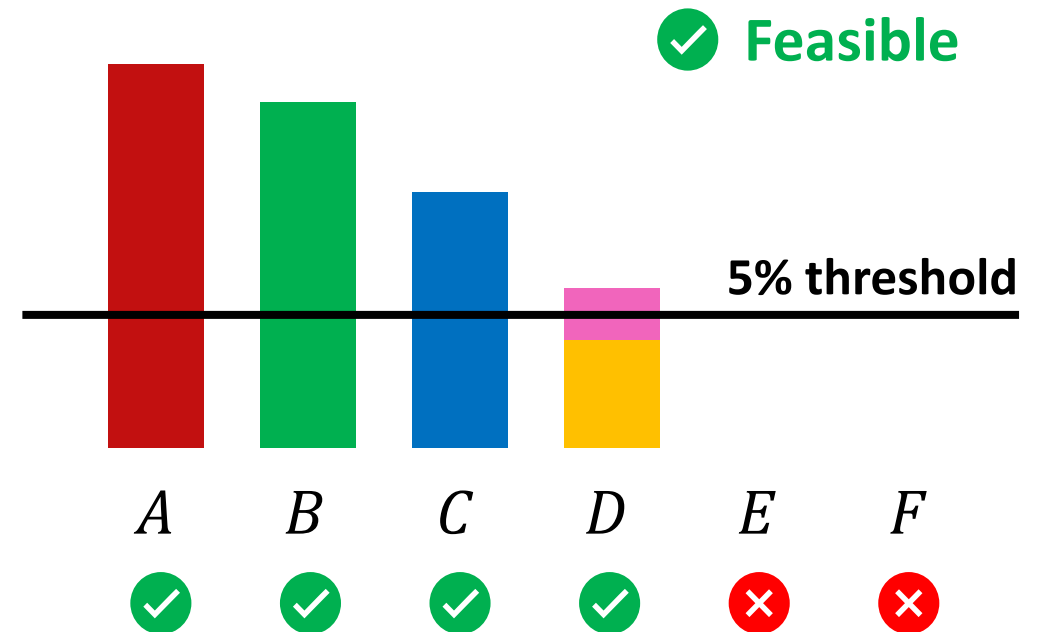
35% A > C	3% D > E > F > C
30% B	3% E > F > A
20% C > E > A	2% F > E > D
6% C > A > B	1% D > F > B



Rule: Greedy Plurality (GP)

Parties are considered in decreasing order of plurality score, and added to the outcome if it remains feasible.

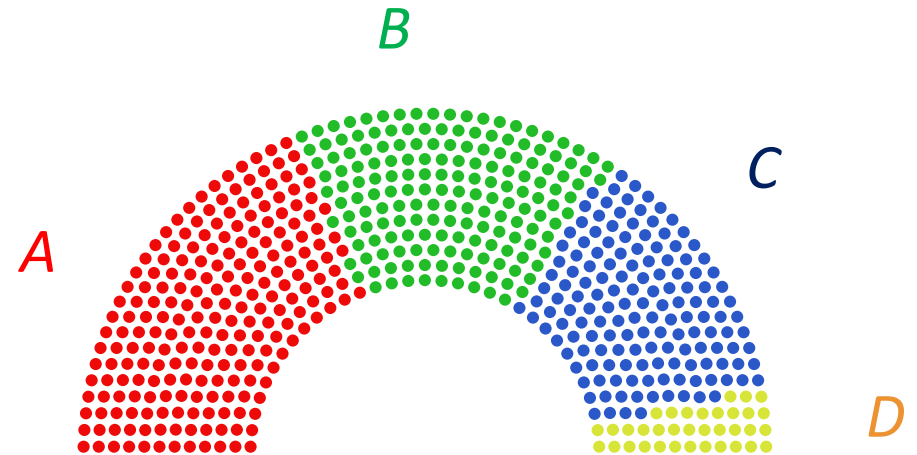
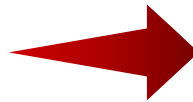
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Rule: Greedy Plurality (GP)

Parties are considered in decreasing order of plurality score, and added to the outcome if it remains feasible.

35% A > C	3% D > E > F > C
30% B	3% E > F > A
20% C > E > A	2% F > E > D
6% C > A > B	1% D > F > B



Rule: Maximum Representation (MaxR)

Return the feasible outcome that maximizes the number of voters that are **represented**.

Rule: Maximum Plurality (MaxP)

Return the feasible outcome that maximizes the number of voters that are **represented by their first choice**.

Theorem : The outcome of DO, STV and GP can be computed in **polynomial time**.

Theorem : The problem of computing the outcome of MaxR and MaxP is **NP-hard**.

(proof by reduction to the independent set problem)

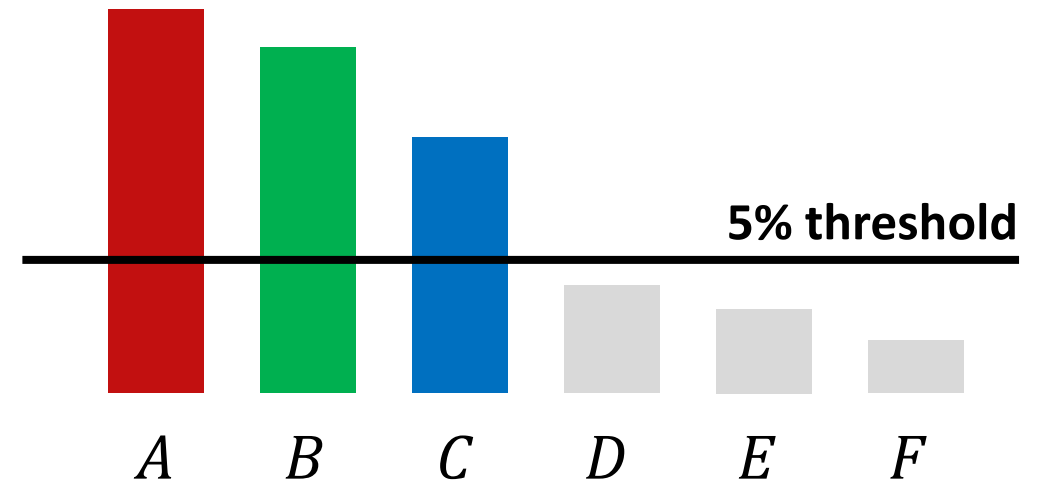
5. Axiomatic Analysis

Axiom: Inclusion of Direct Winners

If τ voters or more rank a party x on top of their rankings, this party should be selected.

- ✓ DO ✗ MaxP
- ✓ STV ✗ MaxR
- ✓ GP

35% $A \succ C$	4% $D \succ E \succ F$
30% B	3% $E \succ F \succ A$
26% $C \succ E \succ A$	2% $F \succ E \succ D$



Axiom: Representation of solid coalitions

If τ voters or more rank a set of parties T on top of their rankings, at least one of these parties should be selected.

Inspired by *Proportionality for Solid Coalitions* [Dummet 94]

- ✗ DO ✗ MaxP
- ✓ STV ✗ MaxR
- ✗ GP

35% $A \succ C$

4% $D \succ E \succ F$

30% B

3% $E \succ F \succ A$

26% $C \succ E \succ A$

2% $F \succ E \succ D$

= 5%



Party E or F
should be part of
the outcome.

Axiom: Threshold Monotonicity

DO



STV



GP



MaxP



MaxR

If a party is selected for threshold τ , then it is also selected for threshold $\tau' < \tau$.

Axiom: Ind. of Definitely Losing Parties

DO



STV



GP



MaxP



MaxR

Once some parties are losing at some threshold τ , then for all larger thresholds $\tau' > \tau$, the rule should behave as if none of the losing parties had been available.

Characterization Theorem : STV is the only party selection rule that satisfies inclusion of direct winners and independence of definitely losing parties.

Axiom: Reinforcement for Winning Parties

If a party is selected for profile P_1 with threshold τ_1 and for profile P_2 with threshold τ_2 , then it should be selected for profile $P_1 + P_2$ with threshold $\tau_1 + \tau_2$.



DO



MaxP



STV



MaxR



GP

Characterization Theorem : DO is the only party selection rule that satisfies inclusion of direct winners and reinforcement for winning parties.

Axiom: Representative-strategyproofness

Voters cannot cause a party to be selected that they prefer to all currently selected parties by misreporting their preferences.

Axiom: Share-strategyproofness

Voters cannot cause a party to be selected that they prefer to all currently selected parties by misreporting their preferences **OR** increase the share of voters represented by their most-preferred selected party.



No hope for strategyproofness in general.

(Gibbard-Satterthwaite impossibility result applies since single-winner voting is a special case of our model)

Say that a party is **(all with respect to a voter i)**...

- ...**Safe** if it is always selected no matter how i votes.
- ...**Risky** if it might or might not be selected depending on how i votes.
- ...**Out** if it is always not selected no matter how i votes.

$\ll \tau$ votes

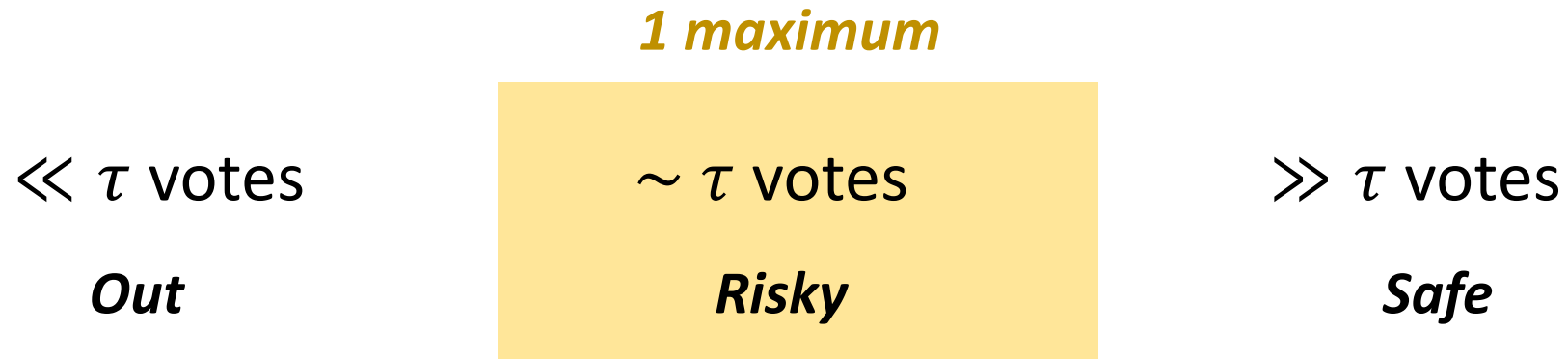
Out

$\sim \tau$ votes

Risky

$\gg \tau$ votes

Safe



Proposition : GP, MaxP and MaxR satisfy representative-strategyproofness when there is *at most one risky* party from the perspective of each voter. (DO and STV do not.)

Always one in top 2

$\ll \tau$ votes

Out

$\sim \tau$ votes

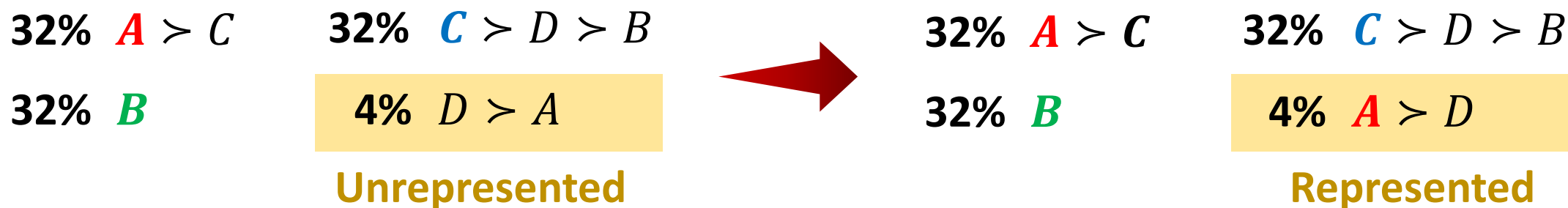
Risky

$\gg \tau$ votes

Safe

Proposition : **DO** satisfies share-strategyproofness when every voter has a *safe party as one of their two most-preferred* parties.
(GP, STV, MaxP and MaxR do not.)

In the (current) uninominal system, voters are incentivized to **vote for their favorite party among the ones that will be selected**:



Proposition : **DO** and **GP** satisfy share-strategyproofness under the restriction that voters can only misreport by promoting their most-preferred *selected* party into first place. (STV, MaxP and MaxR do not)

	DO	STV	GP	MaxP	MaxR
Set-maximality			✓	✓	✓
Inclusion of direct winners	✓	✓	✓		
Representation of solid coalitions		✓			
Threshold monotonicity	✓	✓			
Ind. of definitely losing parties		✓			
Ind. of clones		✓			✓
Reinforcement for winning parties	✓				
Monotonicity	✓				
Rep-SP (one risky party)			✓	✓	✓
Share-SP (safe 1 st or 2 nd)	✓				
Share-SP (rep. ranked 1 st)	✓		✓		

6. Empirical Analysis

To collect appropriate preference data, we **ran a voting experiment** during the 2024 election of the French representative to the EU parliament.

Candidate parties: 38

Threshold: 5%

Parties above the threshold: 7

Lost votes: 12.1%



- 1 Explanation of the issues caused by the threshold.
- 2 Presentation of the candidate lists.
- 3 **Vote with alternative voting methods.**
- 4 Questionnaire.

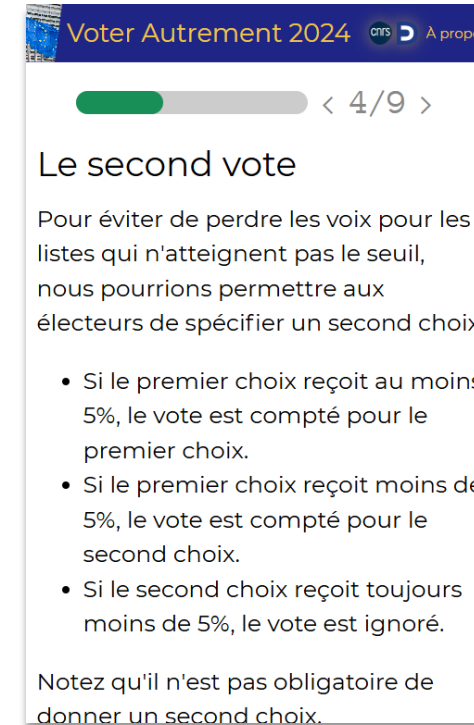


Fig. Screenshot of [the website of the experiment](#) conducted during the 2024 election of the French representative to the EU Parliament.

1 *Self-selected* sample

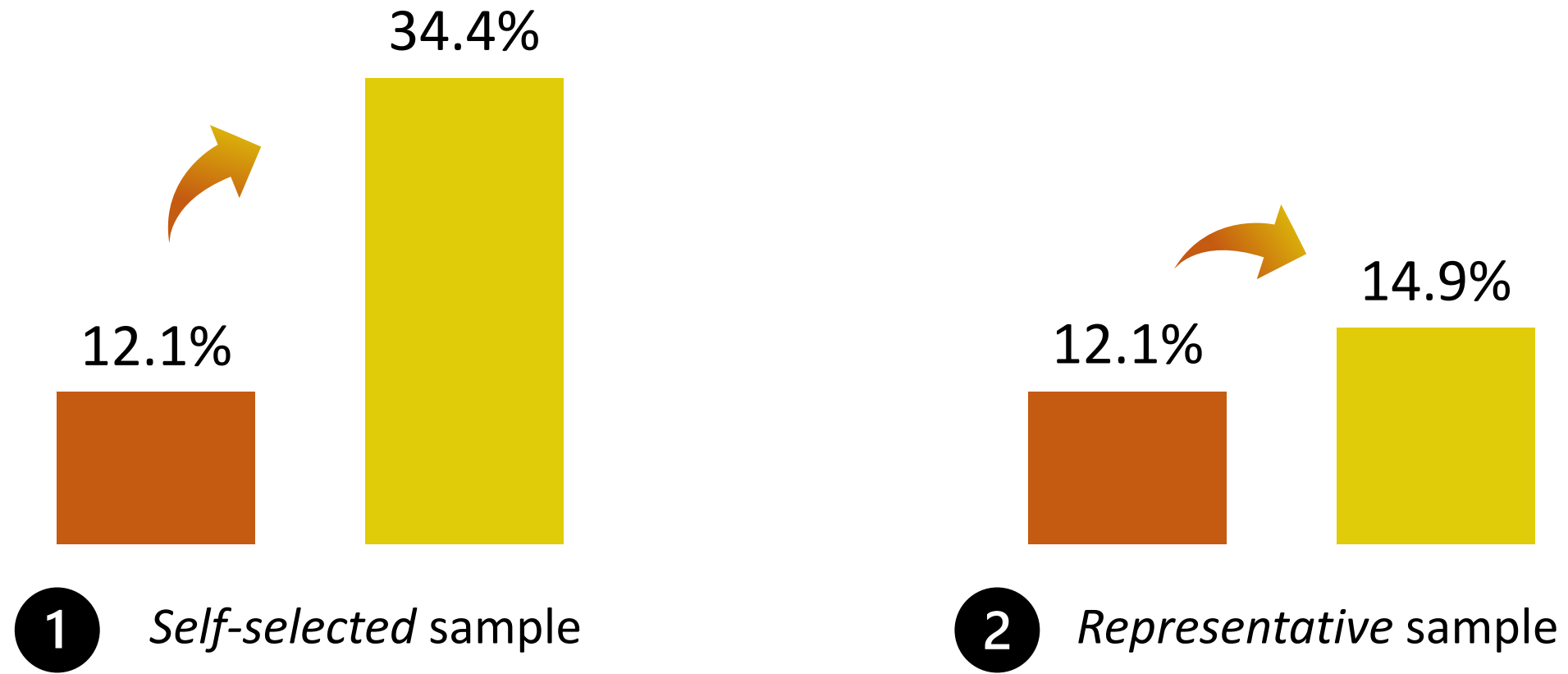
- 3 046 participants in a week.
- Recruited through social media, unpaid.
- Overrepresentation of left-wing, young and educated people.

2 *Representative* sample

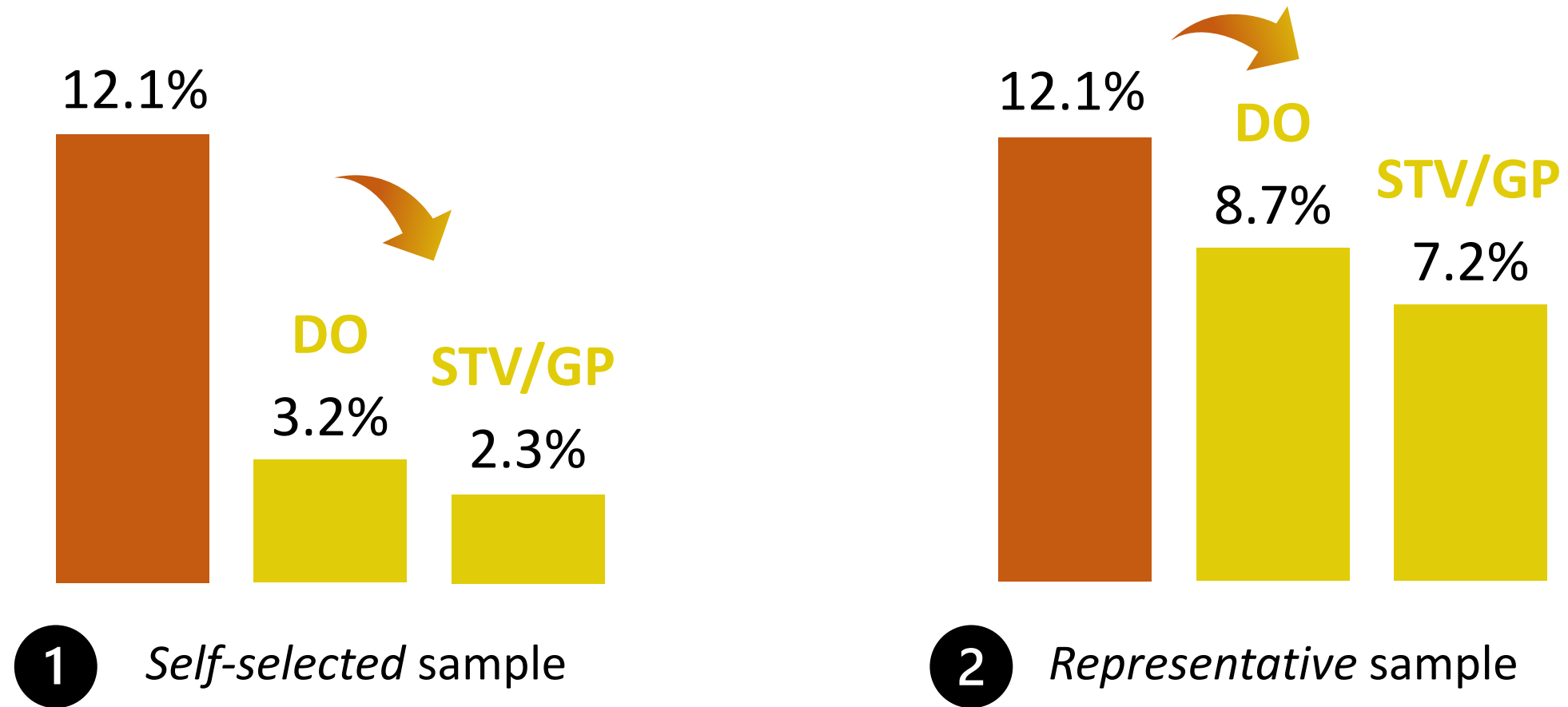
- 1 000 participants.
- Recruited via a polling institute and paid a fixed amount to participate.
- Representative of the French population.



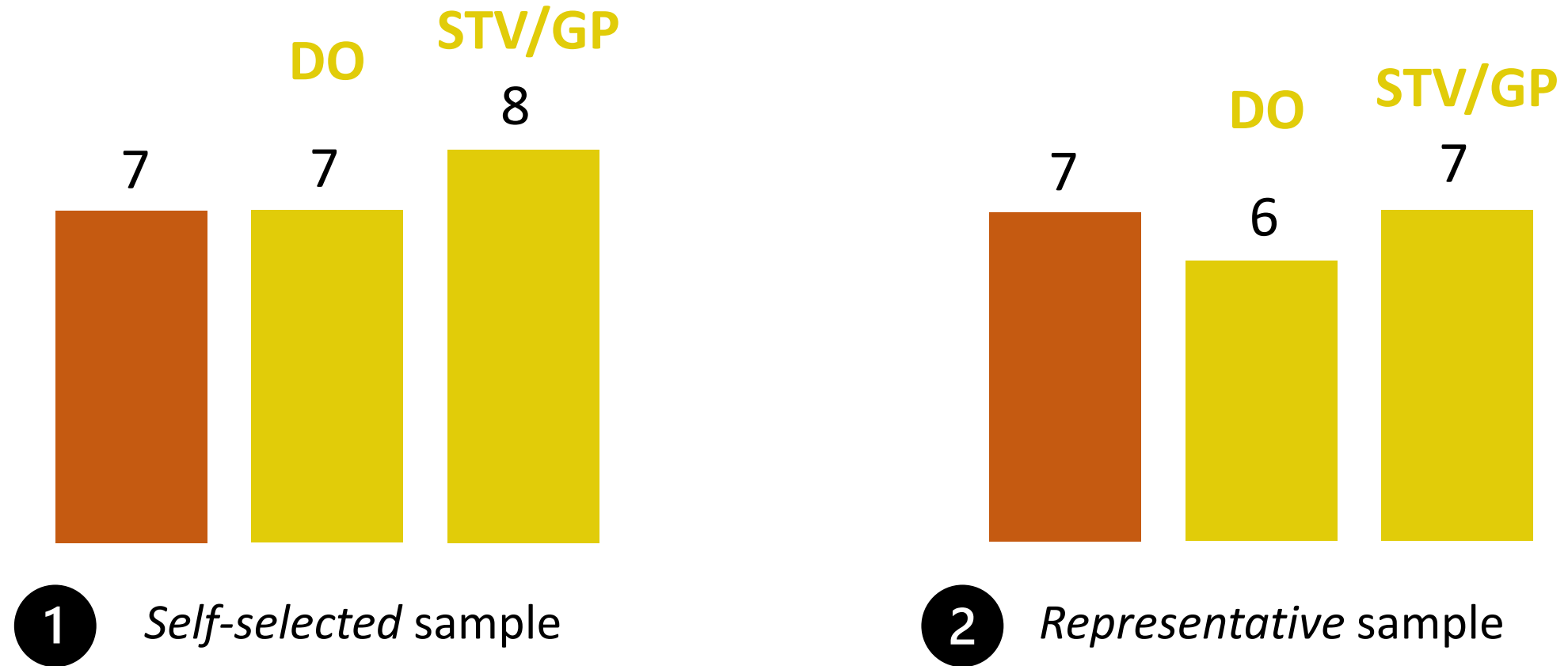
We assign weights to the voters to reduce the biases: weights are selected based on the vote of the participant at the actual election, to match the share of votes received by each party.



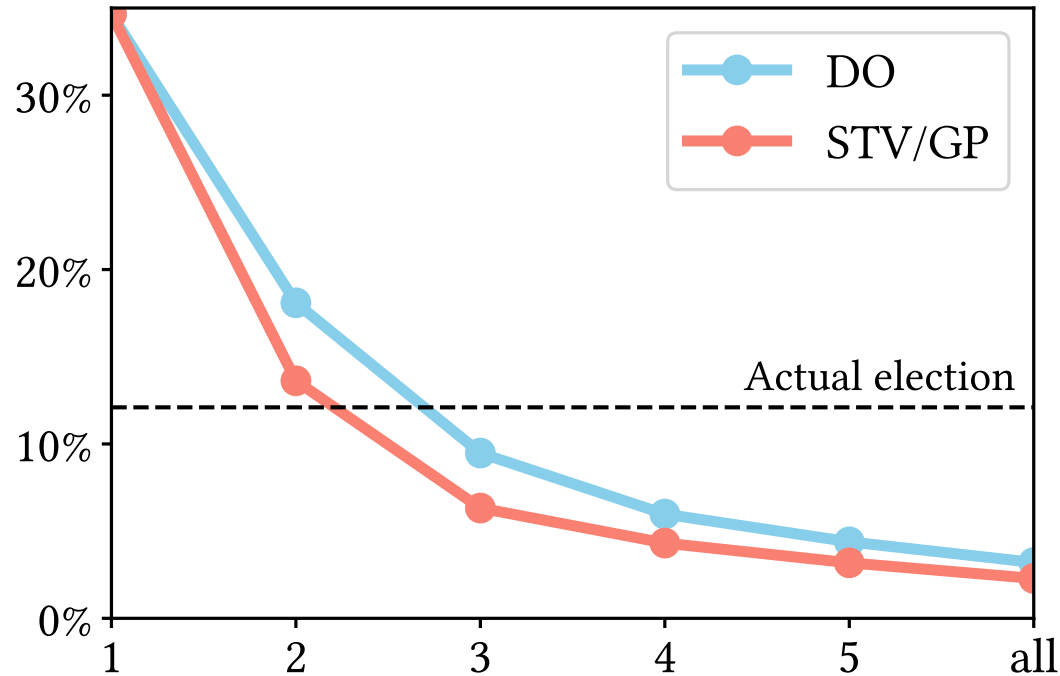
- Share of voters that vote for a “small” party in the actual election
- Share of voters that put a “small” party first in their ranking



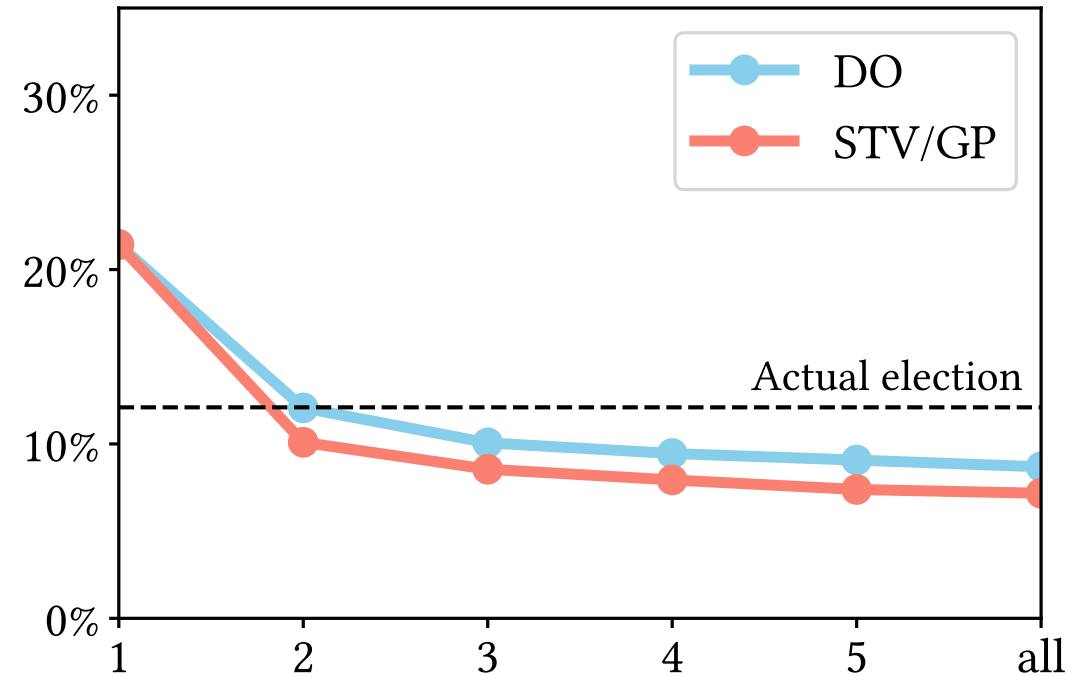
- Share of unrepresented voters in the actual election
- Share of unrepresented voters with ranking-based rules



- Number of parties receiving a seat in the actual election
- Number of parties receiving a seat with ranking-based rules

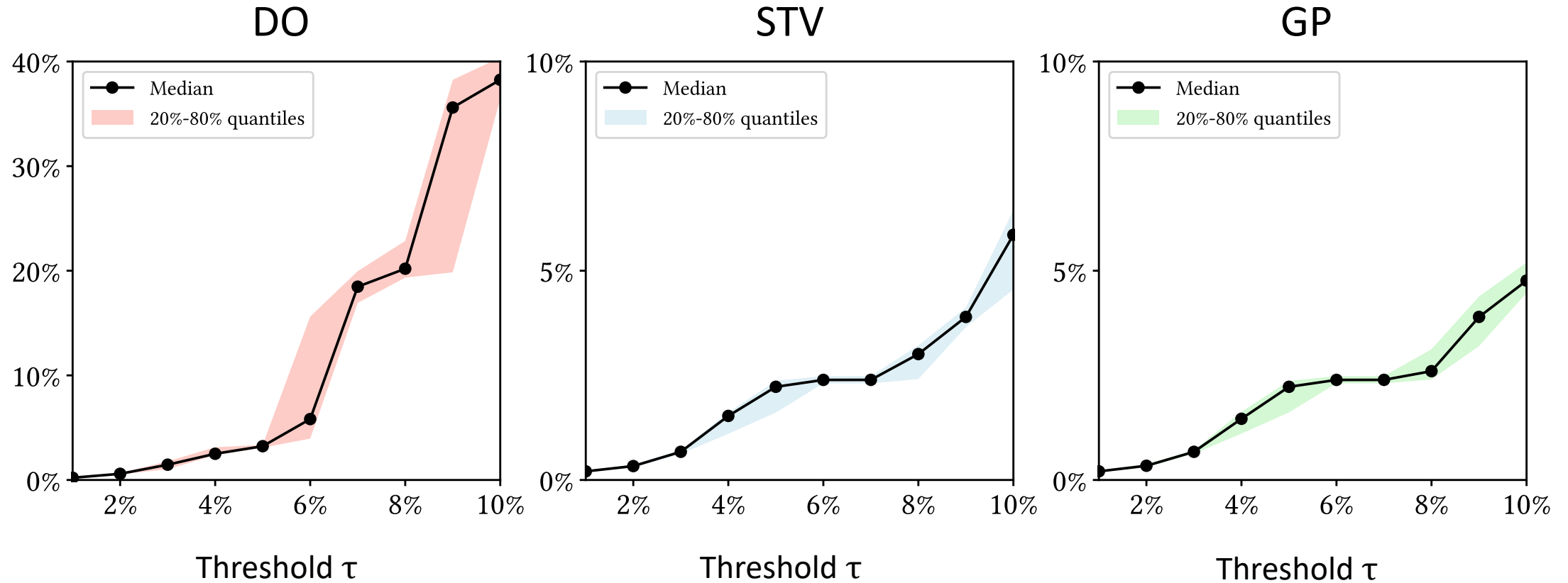


1 *Self-selected* sample



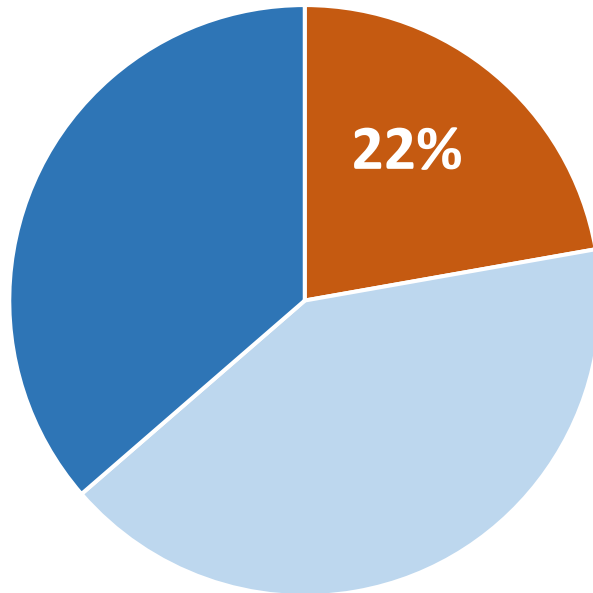
2 *Representative* sample

➤ Share of unrepresented voters if all rankings are truncated to rank k (x-axis).

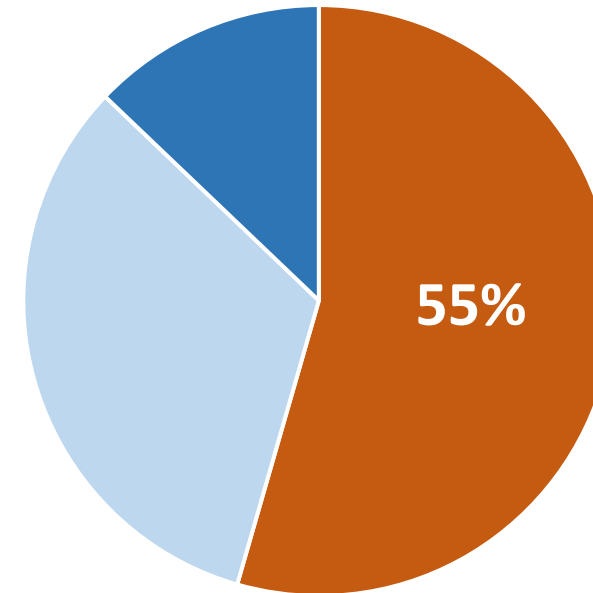


Share of unrepresented voters with different threshold values and with random noise added to the preferences (*self-selected sample*).

Which system do you think is better suited for the election of your representatives to the EU parliament?



1 *Self-selected sample*



2 *Representative sample*



Uninominal (current system)



Two choices



Truncated rankings

Conclusion

We axiomatically and empirically studied rules for electing parliaments with electoral thresholds.

Main takeaway: We can significantly increase representativeness by allowing voters to **rank** parties.

- ➡ STV and GP leave *fewer voters unrepresented* than DO.
- ➡ DO and GP have *stronger strategyproofness guarantees* than STV.
- ➡ STV satisfies *independence of clones* and *represents solid coalitions*.

Thanks for your attention!
Questions?