Reallocating Wasted Votes in Proportional Parliamentary Elections with Thresholds

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Voting systems for parliamentary elections







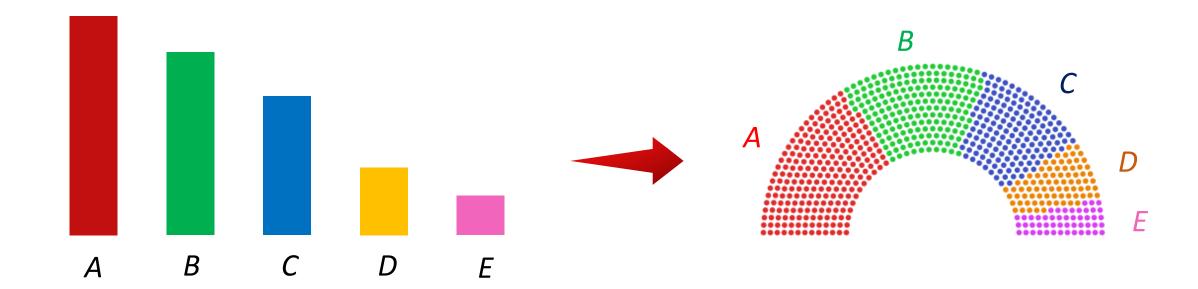






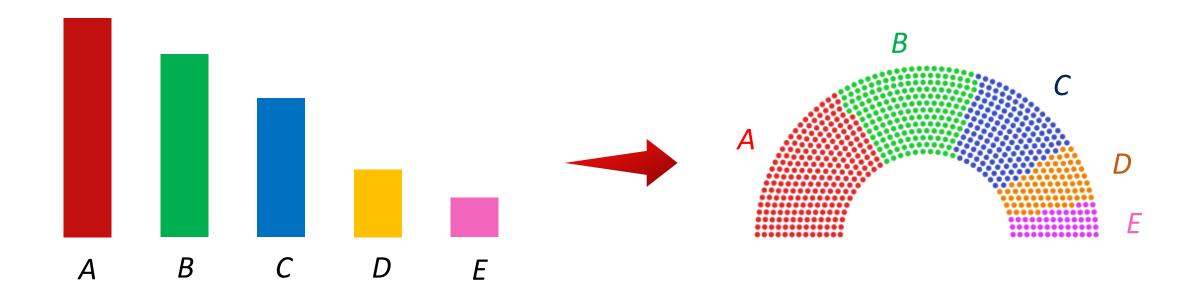
Voters vote for one party.

Seats are allocated to parties proportionally to their scores.



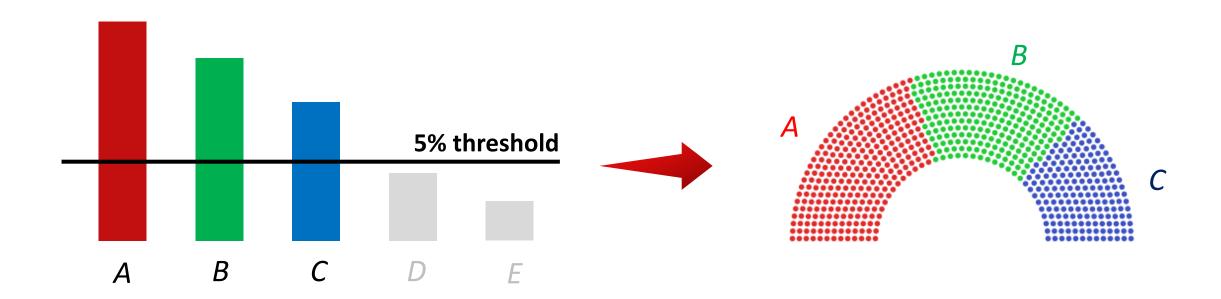
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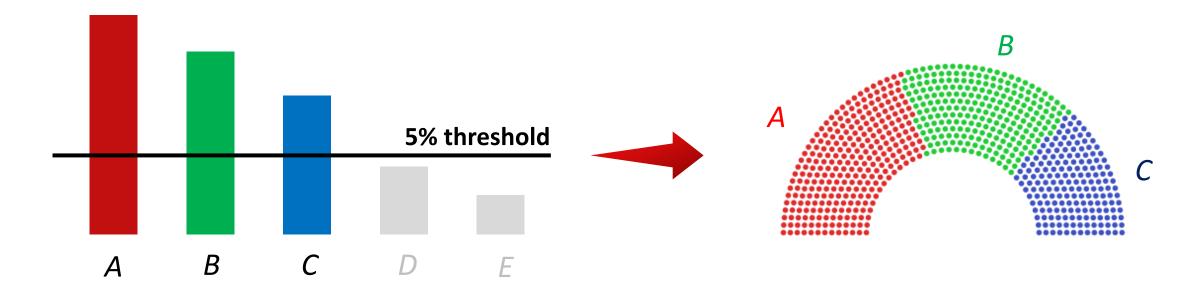
Seats are allocated to parties proportionally to their scores.



Problem: possible political fragmentation (many parties get a seat).

Many countries impose and **electoral threshold** to reduce political fragmentation.





- → Some votes are "lost": D and E supporters have no influence on the seat distribution.
- This incentivizes forms of tactical voting.

The "lost" votes

		Threshold	"Lost" votes
* * * * * * * *	2019 election of the French representative to the EU Parliament.	5%	20%
	2025 election of the <i>Bundestag</i> members.	5%	14% increasing in recent decades
C*	2002 election of the <i>Turkish</i> Parliament members.	10%	46%

Idea: a replacement vote

We could let voters **indicate a second choice** to be used in case their first choice does not reach the threshold.







We could ask voters to rank **two parties**

- 1 Party B
- 2 Party D

We could even ask for a truncated ranking

- 1 Party B
- 2 Party D
- 3 Party A
- 4 Party C

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- 2 Party D

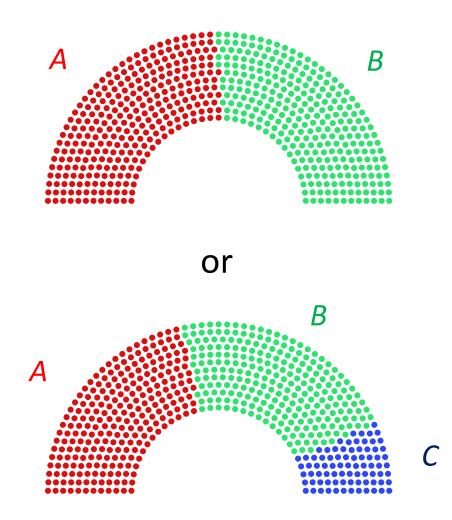
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Question: how to decide which parties are "above the threshold"?

47% A 47% B 4% C > D 2% D > C

5% threshold



We have:

- Sets of voters $V = \{1, 2, ..., n\}$ and parties $C = \{p_1, ..., p_m\}$.

The formal model

- A preference profile $P = (\succ_1, ..., \succ_n)$ of truncated rankings of voters over parties.
- A given threshold τ (absolute number of voters).

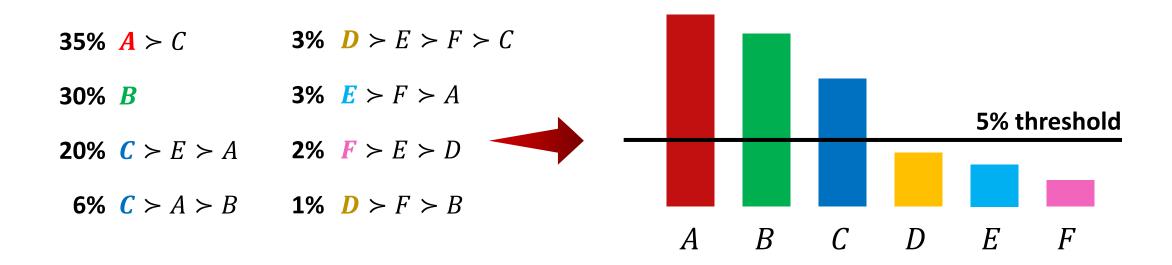
We want:

- A set of selected parties $S \subseteq C$, called the **outcome**.
- Voters are **represented** by their most-preferred party in S (if any).
- An outcome S is **feasible** if every party represents at least τ voters.

Party Selection Rules

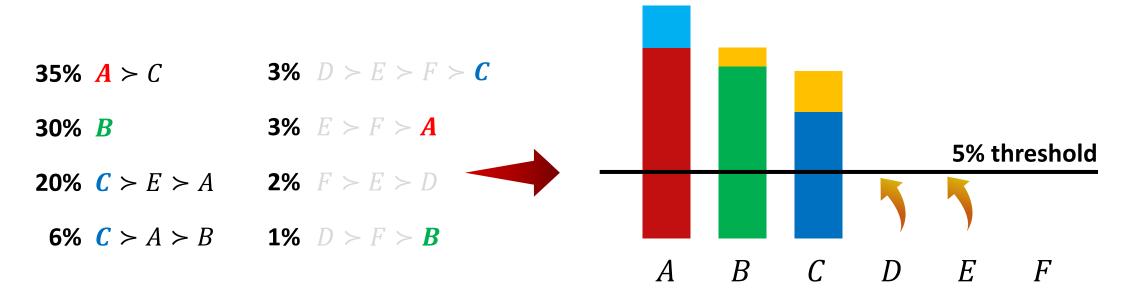
Rule: Direct Winners Only (DO)

The selected parties are all those which receive more first-place votes than required by the threshold.



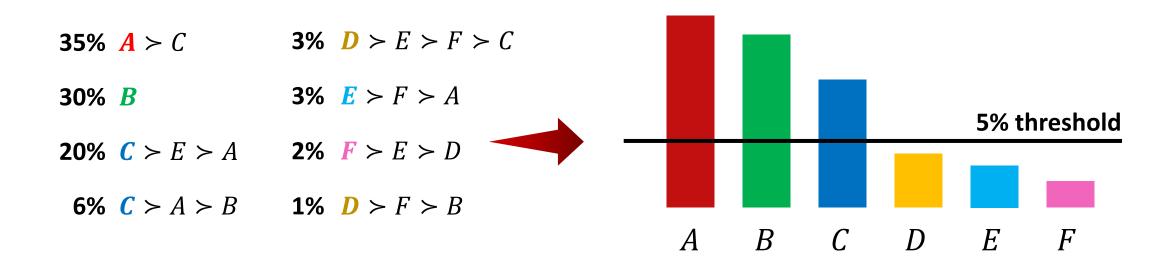
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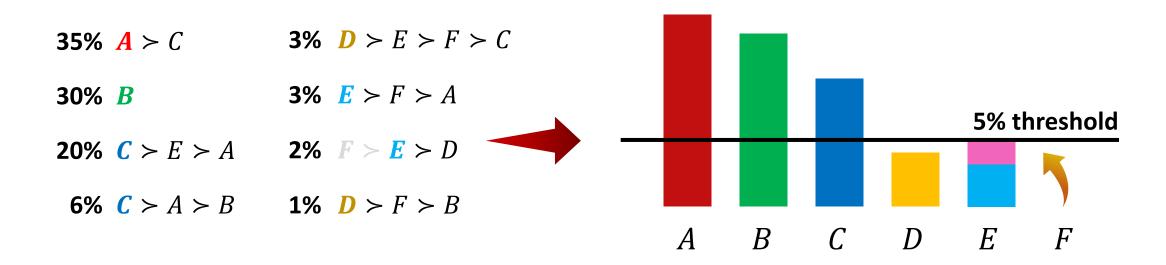
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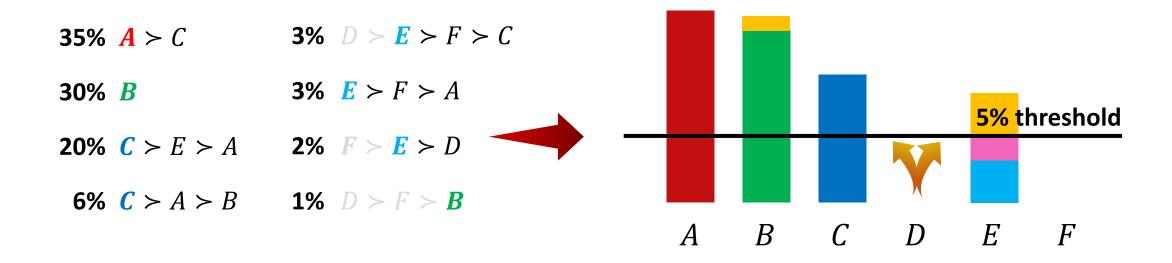


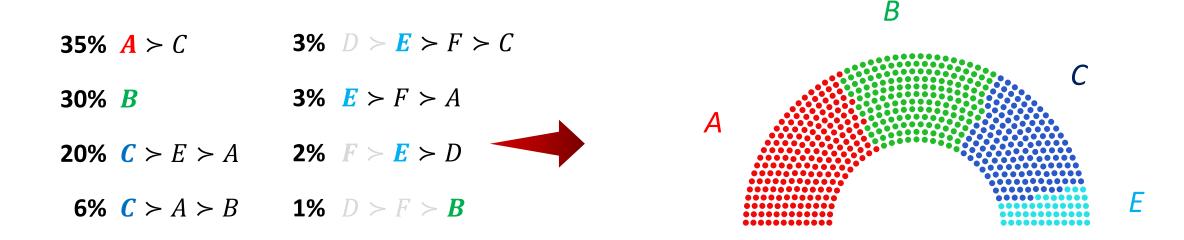
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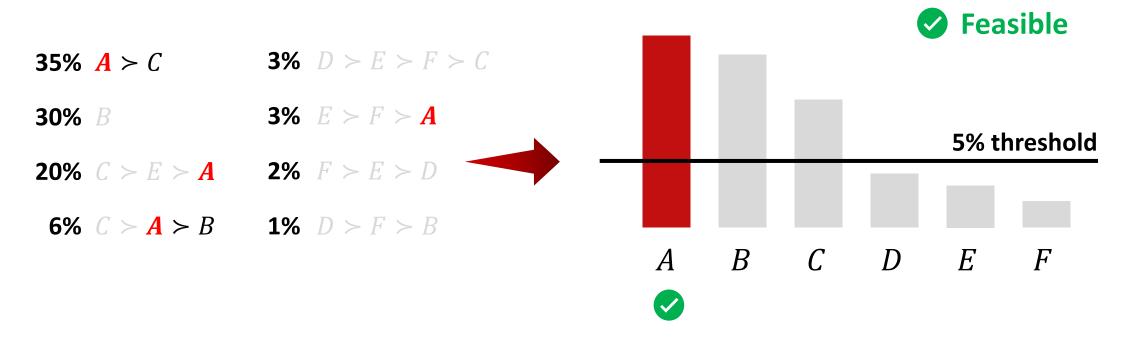
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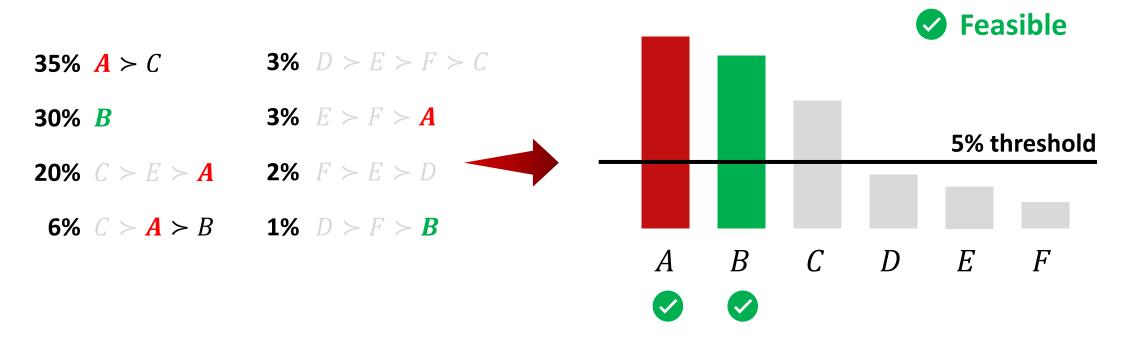


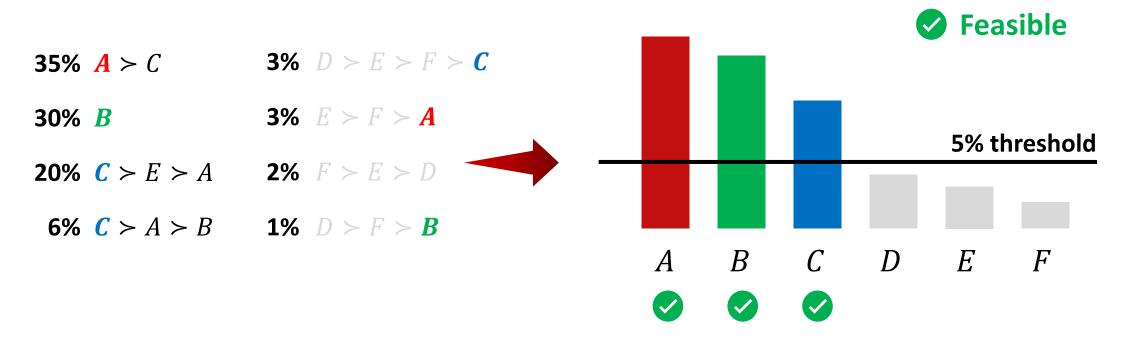


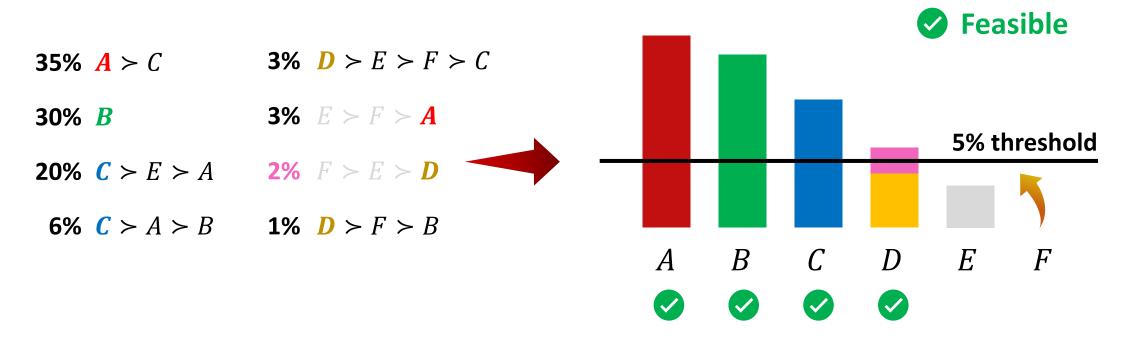




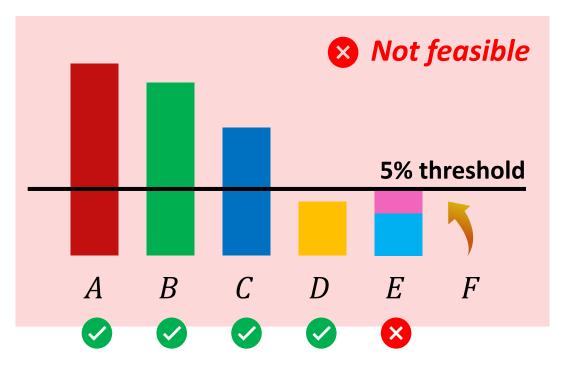




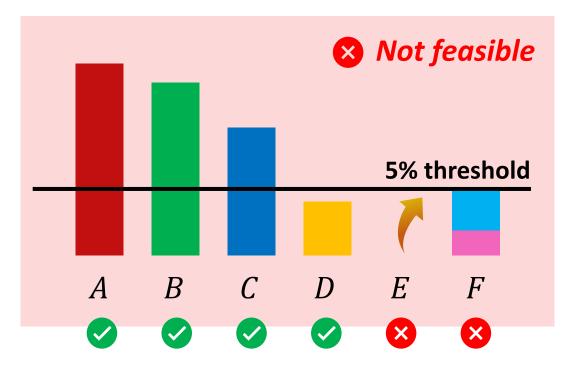


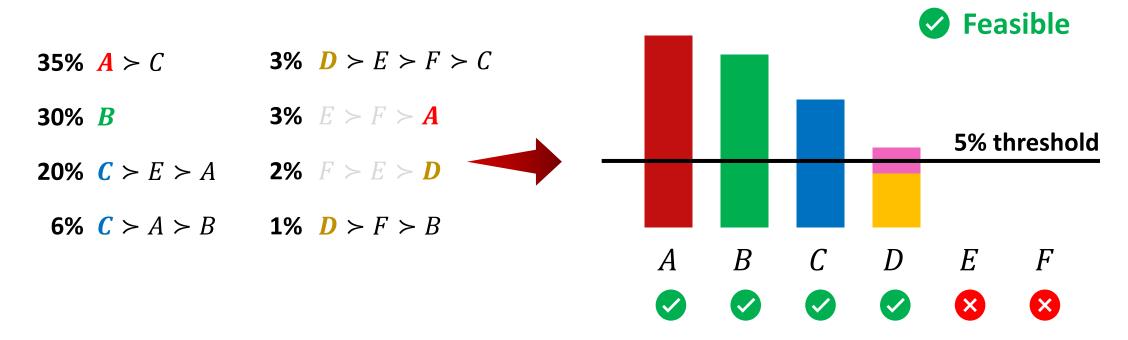


35%
$$A > C$$
 3% $D > E > F > C$
30% B 3% $E > F > A$
20% $C > E > A$ 2% $F > E > D$
6% $C > A > B$ 1% $D > F > B$

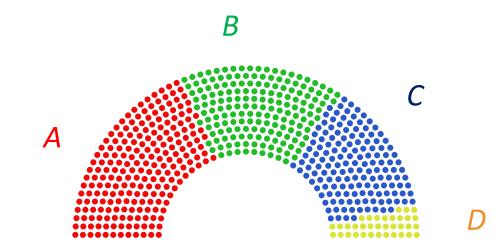


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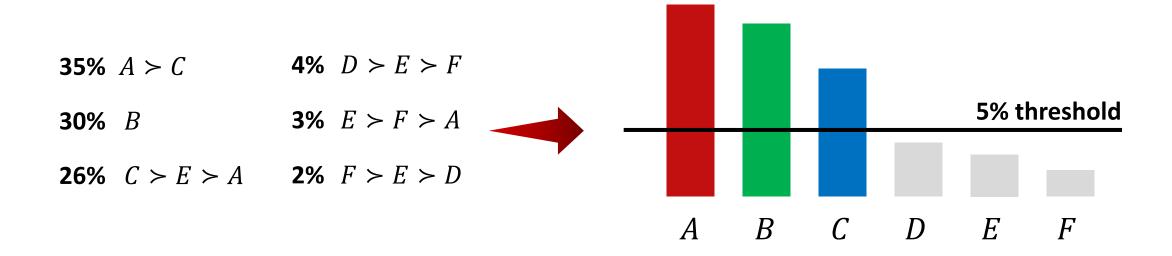


Axiomatic Analysis

Inclusion of Direct Winners

Axiom: Inclusion of Direct Winners

If τ voters or more rank a party x on top of their rankings, this party should be selected.



Representation of Solid Coalitions

Axiom: Representation of solid coalitions

If τ voters or more rank a set of parties T on top of their rankings, at least one of these parties should be selected.





⊗ GP

Inspired by Proportionality for Solid Coalitions [Dummet 94]

35%
$$A > C$$
 4% $D > E > F$

30% B 3% $E > F > A$

26% $C > E > A$ 2% $F > E > D$

= 5%

Party E or F should be part of the outcome. **Axiom:** Threshold Monotonicity



DO





If a party is selected for threshold τ , then it is also selected for threshold $\tau' < \tau$.

Axiom: Independence of Definitely Losing Parties







Once some parties are losing at some threshold τ , then for all larger thresholds $\tau' > \tau$, the rule should behave as if none of the losing parties had been available.

Characterization Theorem : STV is the only party selection rule that satisfies inclusion of direct winners and independence of definitely losing parties.

Axiom: Reinforcement for Winning Parties



DO





If a party is selected for profile P_1 with threshold τ_1 and for profile P_2 with threshold τ_2 , then it should be selected for profile $P_1 + P_2$ with threshold $\tau_1 + \tau_2$.

Characterization Theorem : DO is the only party selection rule that satisfies inclusion of direct winners and reinforcement for winning parties.

Axiom: Representative-strategyproofness

Voters cannot cause a party to be selected that they prefer to all currently selected parties by misreporting their preferences.



No hope for strategyproofness in general (Gibbard-Satterthwaite applies).

Say that a party is (all with respect to a voter i)...

- ...**Safe** if it is always selected no matter how *i* votes.
- ...Risky if it might or might not be selected depending on how i votes.
- ...Out if it is always not selected no matter how *i* votes.

 $\ll \tau$ votes

Out

 $\sim \tau$ votes

Risky

 $\gg \tau$ votes

Safe

Say that a party is (all with respect to a voter i)...

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$\ll au$ votes $\sim au$ votes $\gg au$ votes $\sim au$ votes

1 maximum

Proposition: GP satisfies representative-strategyproofness when there is *at most one risky* party from the perspective of each voter. (DO and STV do not.)

Axiomatic Analysis

Summary

	DO	STV	GP
Set-maximality			
Inclusion of direct winners			
Representation of solid coalitions			
Threshold monotonicity			
Ind. of definitely losing parties			
Ind. of clones			
Reinforcement for winning parties			
Monotonicity			
Rep-SP (one risky party)			
Share-SP (safe 1 st or 2 nd)			
Share-SP (rep. ranked 1st)			

Empirical Analysis

Context of the experiment

To collect appropriate preference data, we **ran a voting experiment** during the 2024 election of the French representative to the EU parliament.

Candidate parties: 38

Threshold: 5%

Parties above the threshold: 7

Lost votes: 12.1%



The experimental setup

- Explanation of the issues caused by the threshold.
- Presentation of the candidate lists.
- Vote with alternative voting methods.
- 4 Questionnaire.

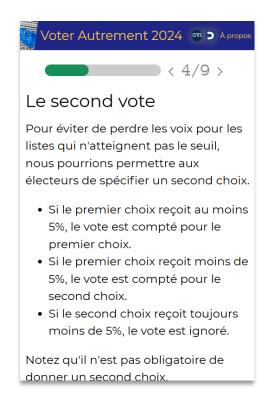


Fig. Screenshot of the website of the experiment conducted during the 2024 election of the French representative to the EU Parliament.

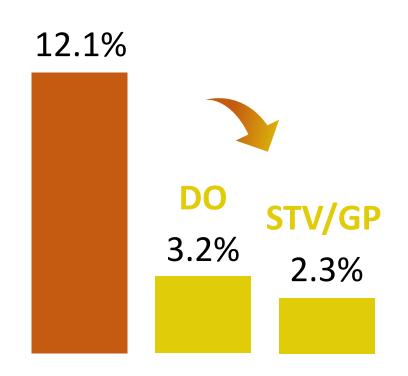
Two samples of participants

1 Self-selected sample

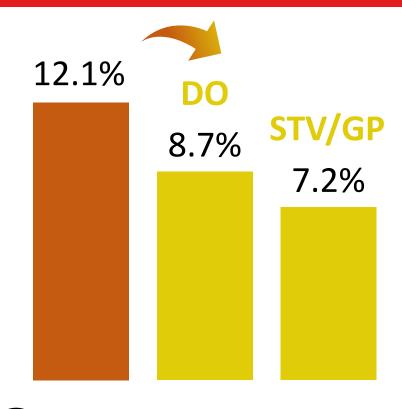
- 3 046 participants in a week.
- Recruited through social media, unpaid.
- Overrepresentation of leftwing, young and educated people.

2 Representative sample

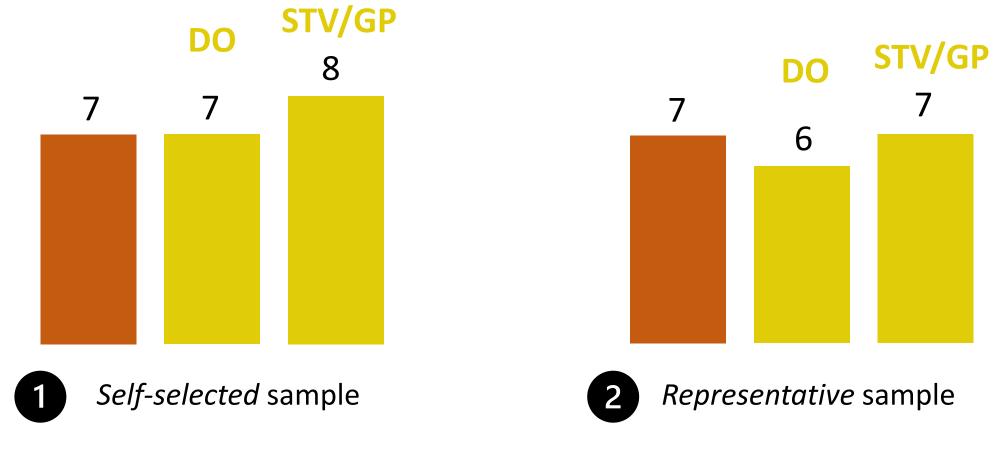
- 1 000 participants.
- Recruited via a polling institute and paid a fixed amount to participate.
- Representative of the French population.



1 Self-selected sample

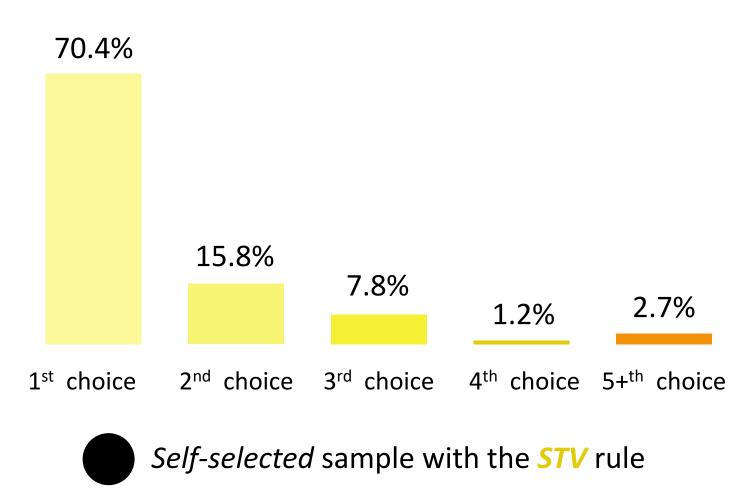


- 2 Representative sample
- Share of unrepresented voters in the actual election
- Share of unrepresented voters with ranking-based rules



- Number of parties receiving a seat in the actual election
- Number of parties receiving a sear with ranking-based rules

We still reduce the number of lost votes if we impose short rankings



Conclusion

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We axiomatically and empirically studied rules for electing parliaments with electoral thresholds.

Main takeaway: We can significantly increase representativeness by allowing voters to **rank** parties.

- STV and GP leave fewer voters unrepresented than DO.
- DO and GP have stronger strategyproofness guarantees than STV.
- STV satisfies independence of clones and represents solid coalitions.

Thanks for your attention! Questions?