Measuring a Priori Voting Power in Liquid Democracy

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A voting game

Nimö (5)   Mich (1)

Stan (2)   **Total weight: 20**   Rach (4)

Esla (3)   Dan (5)

Voters can vote in favor or against.

Example rule:

a vote is successful if there is more weight in favor than against.

A voting game

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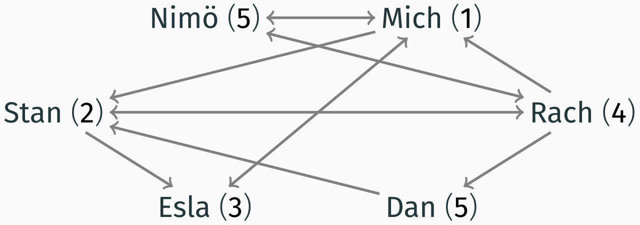

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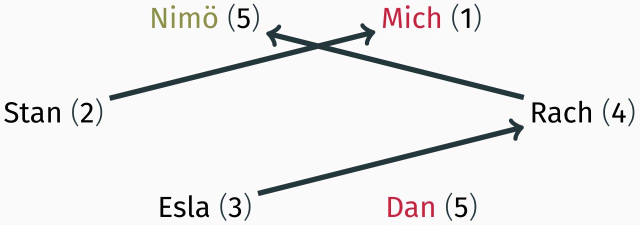

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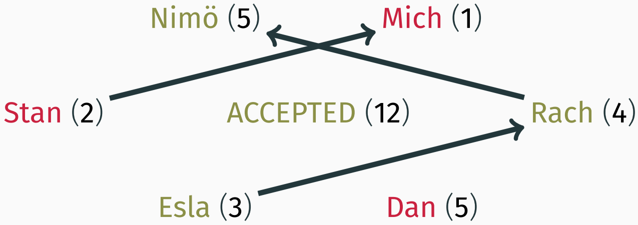

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Question:

Take a voting game and a social network.

Without assuming anything about the bill or the voters,

what is the a priori **voting power** of each voter in the network?

Motivation

Given that voting models with **delegations** are receiving more attention, both theoretically and in practice, how can voting power be measured in these more complex models?

- **Proxy Voting (PV)**: there is a fixed set of representatives to whom voters can delegate their votes.
- **Complete Liquid Democracy (LD)**: every voter can either vote directly or delegate their voting power to someone else.

Formal definitions

We assume the voters are connected by a graph $G = (V, E)$ and each voter $v \in V$ can vote in favour, against or **delegate** to a neighbour.

G-delegation partition

A G-**delegation partition** D is a map on V (voters) s.t. $D(i) \in \{-1, +1\} \cup \mathrm{NB}_{out}(i)$.

$\mathrm{NB}_{out}(i)$: set of out-neighbours of $i \in V$.

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D	+	-	N	-	M	R

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D	+	-	N	-	M	R
T_D	+	-	+	-	-	-

$\Rightarrow$ A G-delegation partition D naturally induces a direct-vote partition T_D.

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Weighted voting rule (= Weighted Voting Game)

A **weighted voting rule** with weights $w : V \to \mathbb{N}$ and a quota $q \in (0.5, 1]$ is such that $W(T) = +1$ if and only if $\sum_{i \in T^+} w(i) > q \cdot \sum_{i \in T^+ \cup T^-} w(i)$.

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The example from the introduction is a WVG with $q = 0.5$.

Penrose-Banzhaf

Liquid Democracy (LD) Penrose-Banzhaf measure

Given a digraph $G = (V, E)$ and a ternary voting rule W, the **LD Penrose-Banzhaf measure** of voter $i \in V$ is defined as:

$$\mathcal{M}_i^{ld}(W, G) = \sum_{D \in \mathcal{D}} \mathbb{P}(D) \frac{W(T_{D_{i+}}) - W(T_{D_{i-}})}{2},$$

where $\mathbb{P}(D)$ is the probability of the G-delegation partition D occurring.

- Probability to **delegate** $p_d^i \in [0, 1]$ and to **vote** $p_v^i = 1 - p_d^i$.
- If **vote**: probability to vote **in favor**/**against**: $p_+ = p_- = 1/2$.
- If **delegate**: probability to delegate to $j \in \mathrm{NB}_{out}(i)$: $1/|\mathrm{NB}_{out}(i)|$.

Computational complexity

Theorem

Computing the LD Penrose-Banzhaf:

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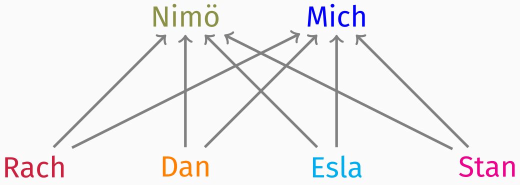
Computing the LD Penrose-Banzhaf:

- #P-hard, even for Weighted Voting Games (WVG).
- For *bipartite* graphs and *complete* graphs, and WVG, it can be computed by a pseudo-polynomial algorithm that runs in polynomial time w.r.t. $|V|$ and $\max_{i \in V} w(i)$.

Proxy voting

In **Proxy voting (PV)**, we have *delegatees* $i \in V_d$ (*proxies*) and *delegators* $i \in V_v$.

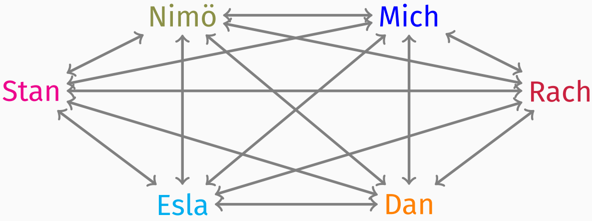


Complexity

The LD Penrose-Banzhaf can be computed by a pseudo-polynomial algorithm that runs in polynomial time w.r.t. $|V|$ and $\max_{i \in V} w(i)$.

Complete liquid democracy

In **complete liquid democracy**, any voter can delegate to any other voter, or vote themselves.



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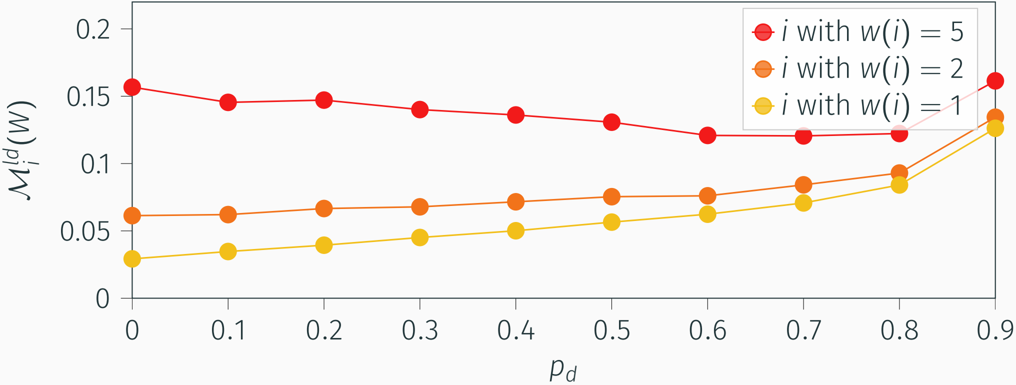
Complete liquid democracy: experiments

Weighted voting game with 100 **voters** of weights $w(i) = 1$ (50%), $w(i) = 2$ (30%) or $w(i) = 5$ (20%), and quota $q = 0.5$.



Criticality distribution and degree distribution

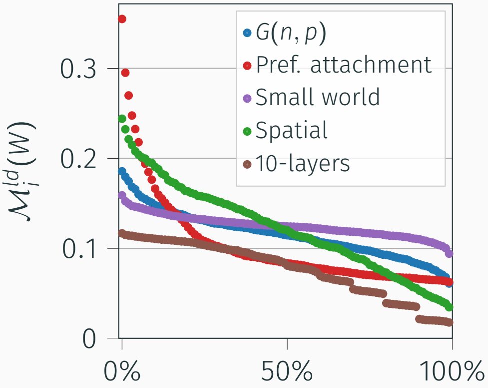


Figure 1: Distribution of the criticality of the voters in the network, from the highest criticality to the smallest criticality.

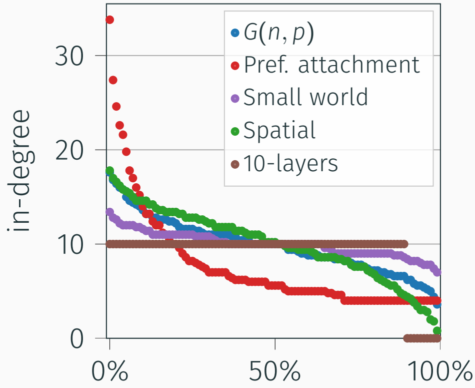


Figure 2: Distribution of the degree of the voters in the network, from the highest degree to the smallest degree.

Conclusion

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- **Experimental analysis** of the criticality in various networks and with varying parameters.

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- This paper continues the tradition of extending the notion of a priori voting power **to new voting models**.
- **Complexity** and hardness results, and pseudo-polynomial algorithms for PV and LD.
- **Experimental analysis** of the criticality in various networks and with varying parameters.
- **Further research directions:** analysis with real data (e.g. with real networks), study other delegation models (e.g. including abstention or ranked delegations).

Thanks for your attention!

Come to our poster!

Paper #4920 (Board R)