Aggregating Correlated Estimations with (Almost) no Training

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ECAI 2023

A choice problem.

A set of m **candidates** $\mathcal{C} = \{c_1, \dots, c_m\}$, each having **unknown utility** $\boldsymbol{U(c_j)}$.

A set of n **agents** $\mathcal{A} = \{1, \dots, n\}$.

Agents give **scores = noisy estimates** of candidates' utilities.

$$\boldsymbol{s_i(c_j)} = \boldsymbol{U(c_j)} + \mathcal{E}_i(\boldsymbol{c_j})$$

Our goal: select a candidate with the highest possible utility, based on the agents' estimates.

Hypothesis 1.

We assume agents are pre-selected to have similar (good) accuracies.

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Solutions.

Range Voting (RV)

Select the candidate that **maximizes the sum/average** of the estimates $w_{RV}(c_j) = \sum_{i=1}^{n} s_i(c_j)$.

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Select the candidate that **maximizes the sum/average** of the estimates $w_{RV}(c_j) = \sum_{i=1}^{n} s_i(c_j)$.

Approval Voting (AV)

Select the candidate that maximizes the **number of agents** who estimate its utility greater than the average $w_{AV}(c_j) = \sum_{i=1}^{n} 1_{s_i(c_j) \geq \tilde{s}_i}$.

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Nash Product (NP)

Select the candidate that **maximizes the product** of the estimates $w_{NP}(c_j) = \prod_{i=1}^{n} s_i(c_j)$.

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Solutions.

Range Voting, Approval Voting, Nash Product.

												avg
	17	17	17	17	17	17	17	17	12	10	14	15.6
	15	15	15	15	15	15	15	15	17	16	17	15.4

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~~Solutions.~~

~~Range Voting, Approval Voting, Nash Product~~.

												avg	
(drink icon)	17	17	17	17	17	17	17	17	12	10	14	15.6	👑
(flask icon)	15	15	15	15	15	15	15	15	17	16	17	15.4	

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We assume some diversity among the agents' noises.

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~~Solutions.~~

~~Range Voting, Approval Voting, Nash Product.~~

~~Hypothesis 2.~~

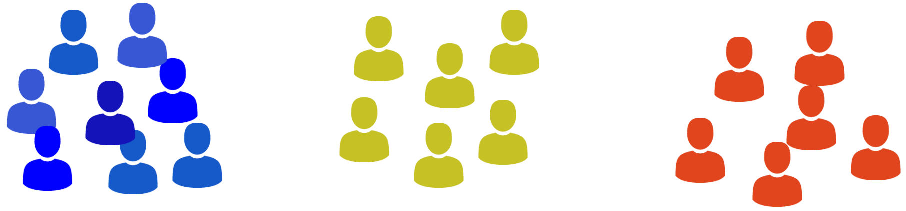
~~We assume some diversity among the agents’ noises.~~

But what if **we don’t?**

Our proposal:
Embedded Voting (EV)

Embedded Voting: Intuition

Let's say agents are divided **in groups G_k.**



The idea: each group should have the **same weight**, whatever its size.

$$w_{EV}(c_j) \propto \prod_{G_k} \sum_{i \in G_k} s_i(c_j)$$

Informally: we do the product of groups scores. Note that this formula is invariant with the sizes of the groups.

Embedded Voting: General case.

Embedded Voting (EV)

Using their estimates of candidates' utilities, **we embed the agents,** such that correlated agents have correlated embeddings.
Using the **Singular Value Decomposition (SVD)** on agents' estimates, we can associate singular values to groups scores. Then, the EV score for one candidate is **the product of the most important singular values**.

Trained Embedded Voting (EV+)

Same as EV, but the features for the embeddings are based on **1,000 estimates**.

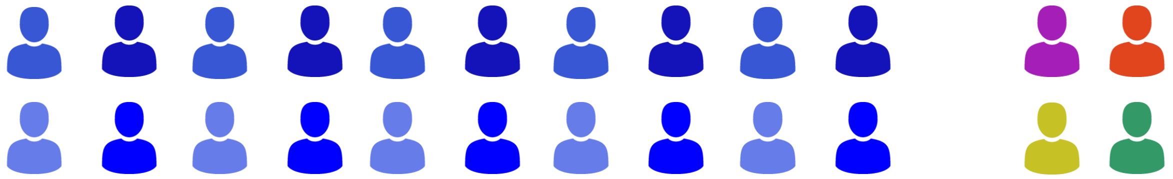
Experimental Validation

Experimental Model

We conducted various experiments **on synthetic data**, with a particular model that is designed to create **a lot of correlations**.

In our default experiment, we consider a group of **20 correlated agents** and **4 totally independent** agents.

Moreover, the **Group noise** is set to be greater than the **Independent noise**.



$$S_i(c_j) = \textbf{True utility} + \textbf{Group noise} + \textbf{Independent noise}$$

Maximum-likelihood approaches

Model Aware (MA)

Maximum Likelihood Estimator, given the noise model and the parameters of the model **(Upper Bound).**

Pseudo likelihood (PL)

Maximum Likelihood Estimator, given the noise model but **approximating the parameters** using agents' estimates.

Trained Pseudo likelihood (PL+)

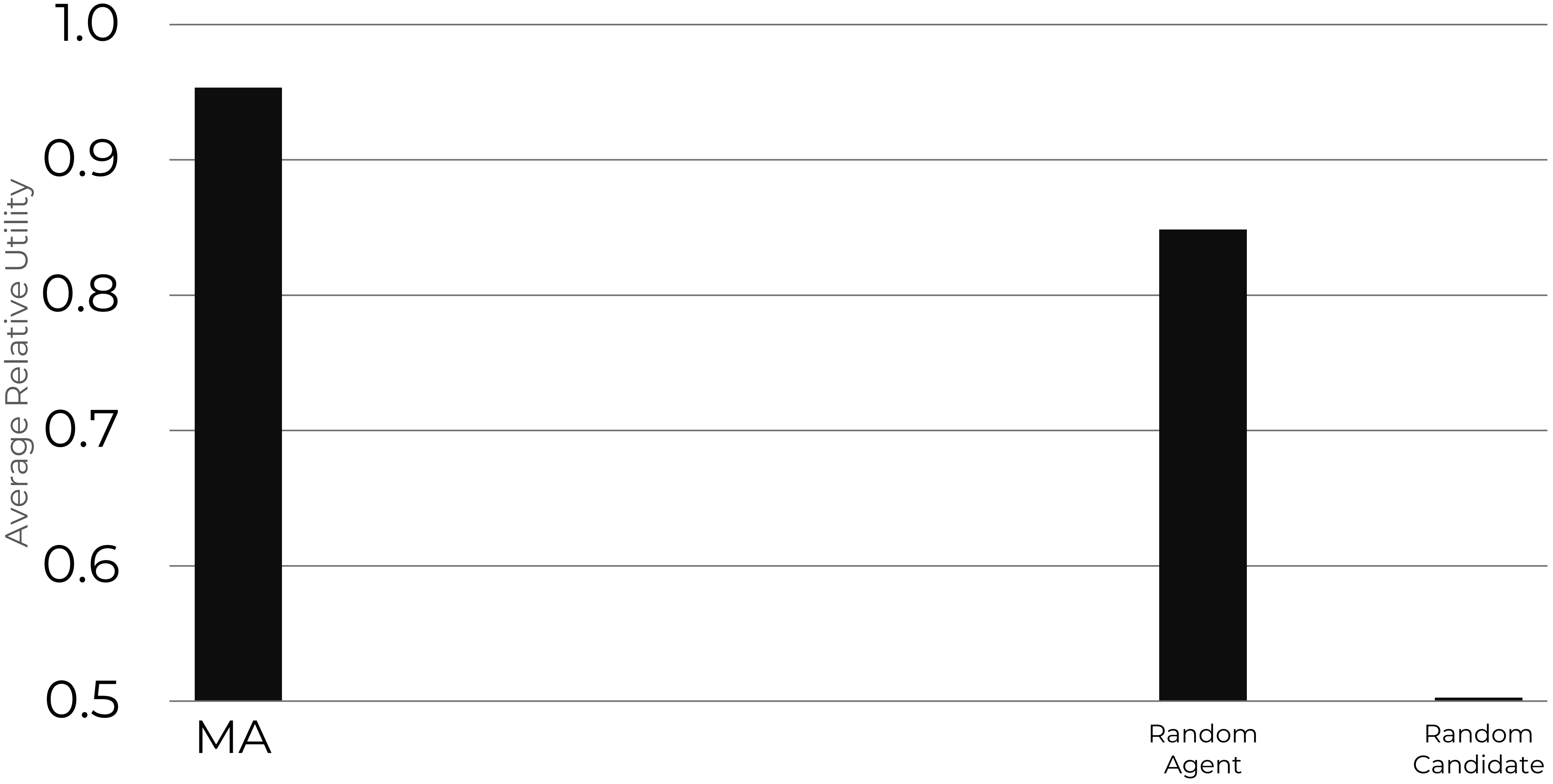
Using **1,000 estimates** for a better approximation of the correlations.

Our metric : Relative utility.

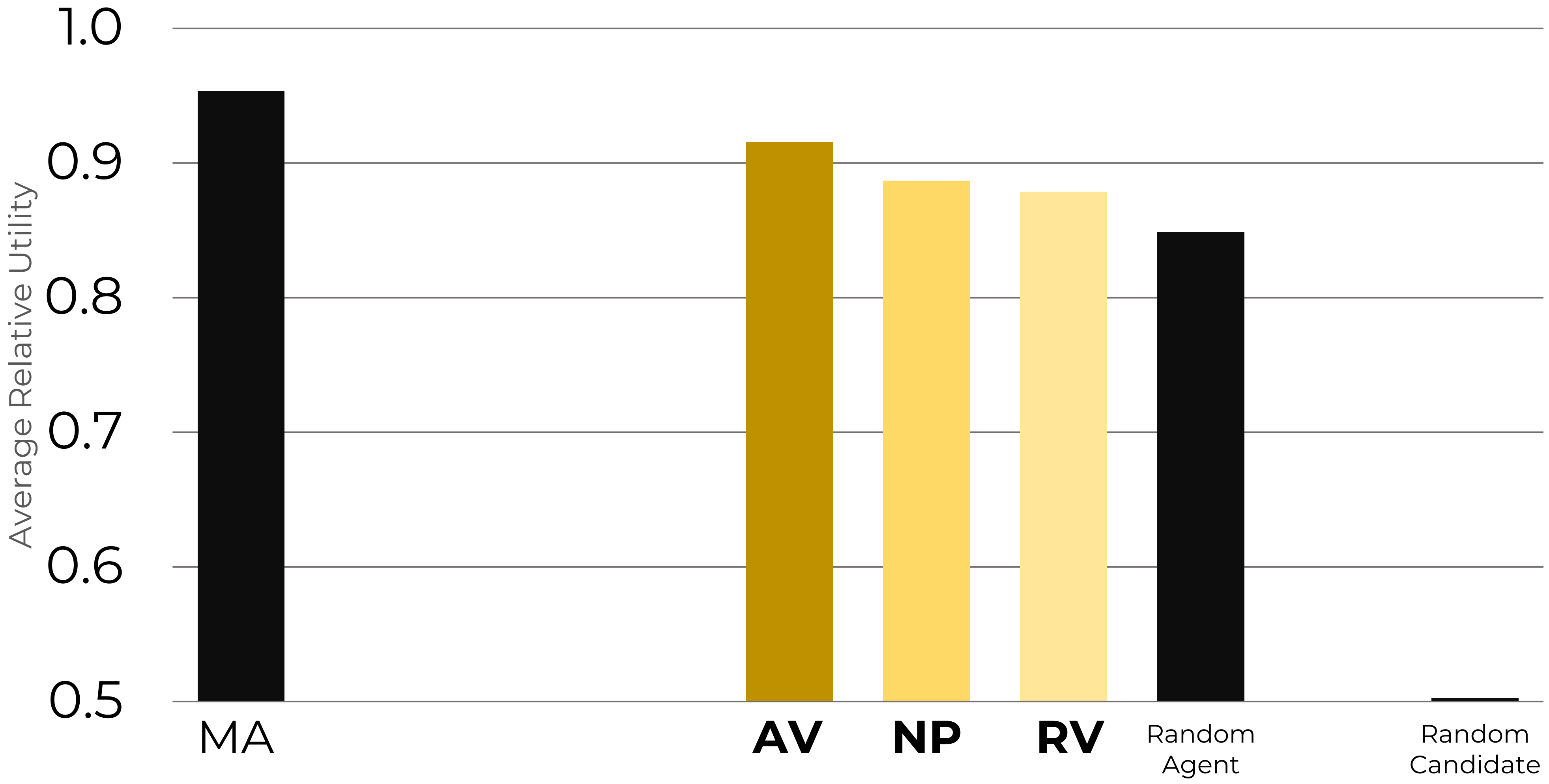
(averaged over *10,000* choices)

$$\frac{U(c_j) - U_{min}}{U_{max} - U_{min}} \in [0,1]$$

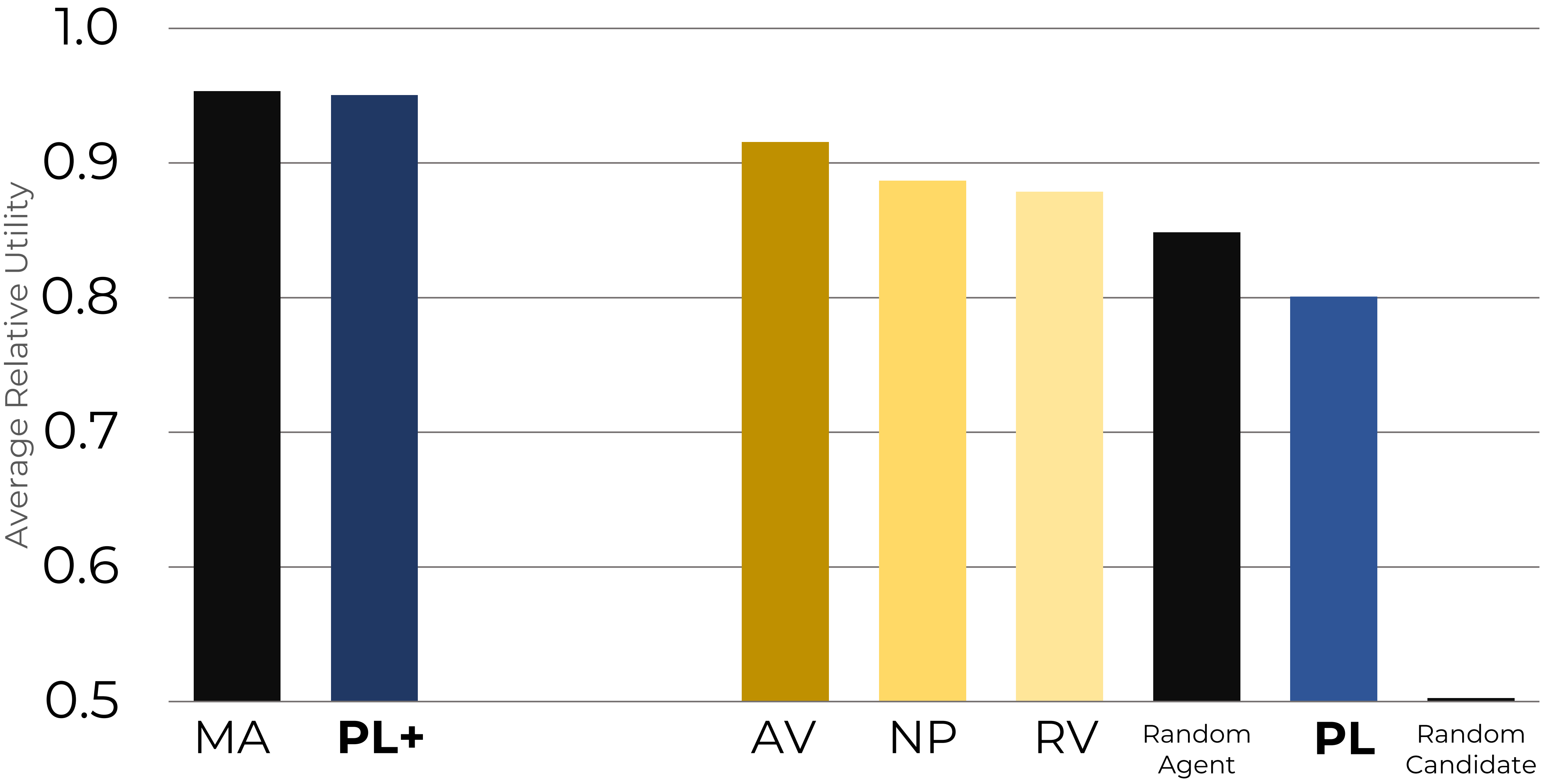
Upper and lower bounds



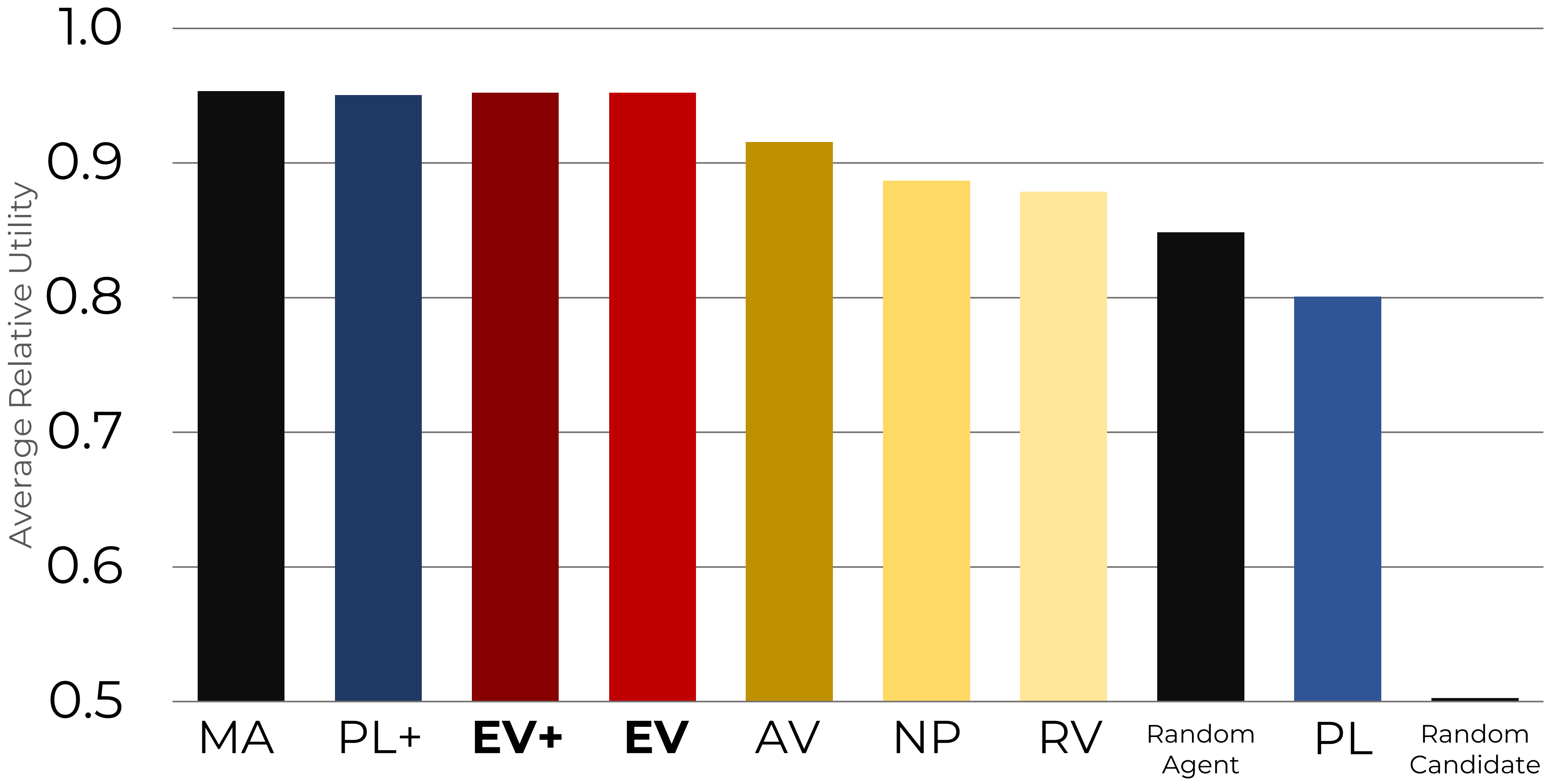
Welfare-based approaches



Pseudo-Likelihood approach

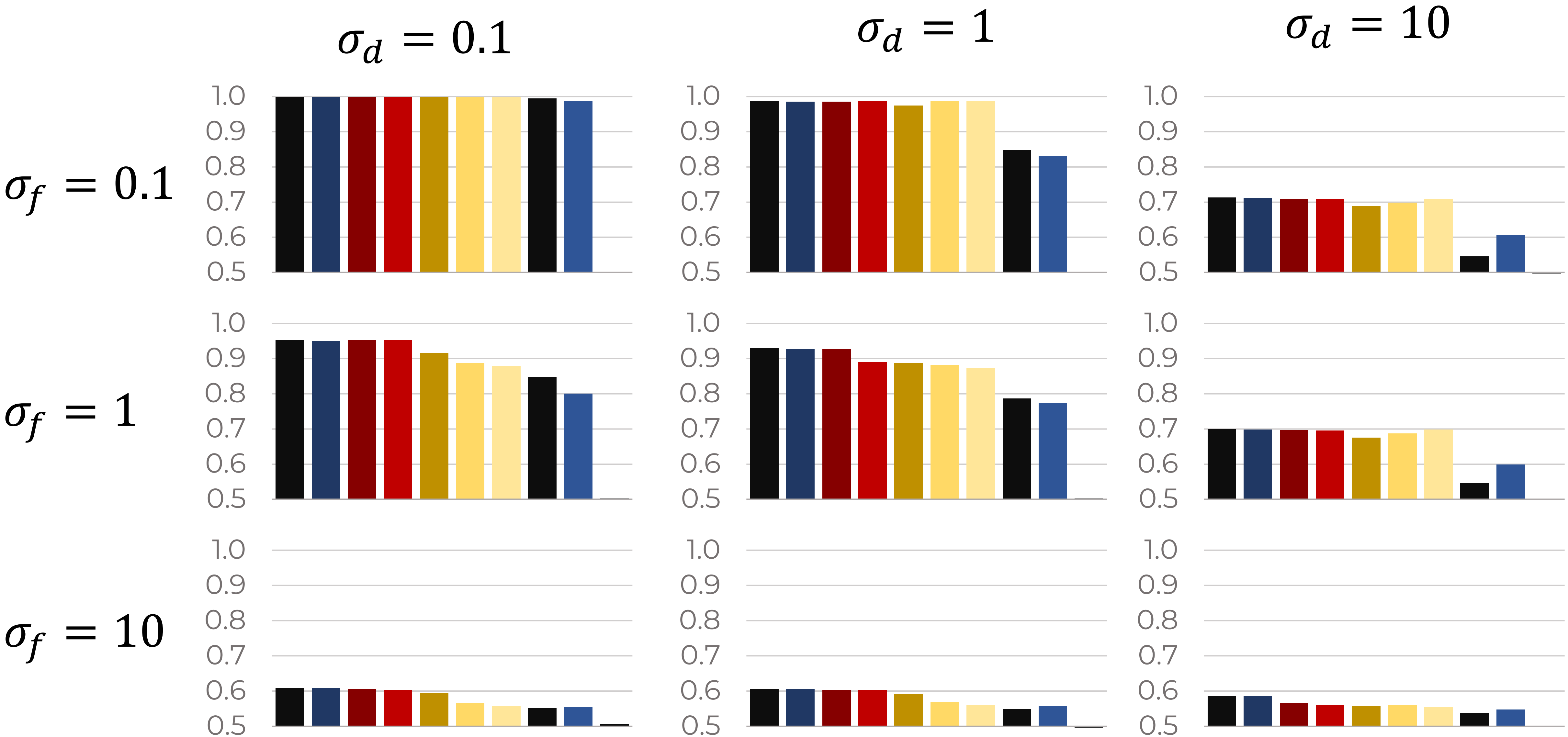


Embedded Voting

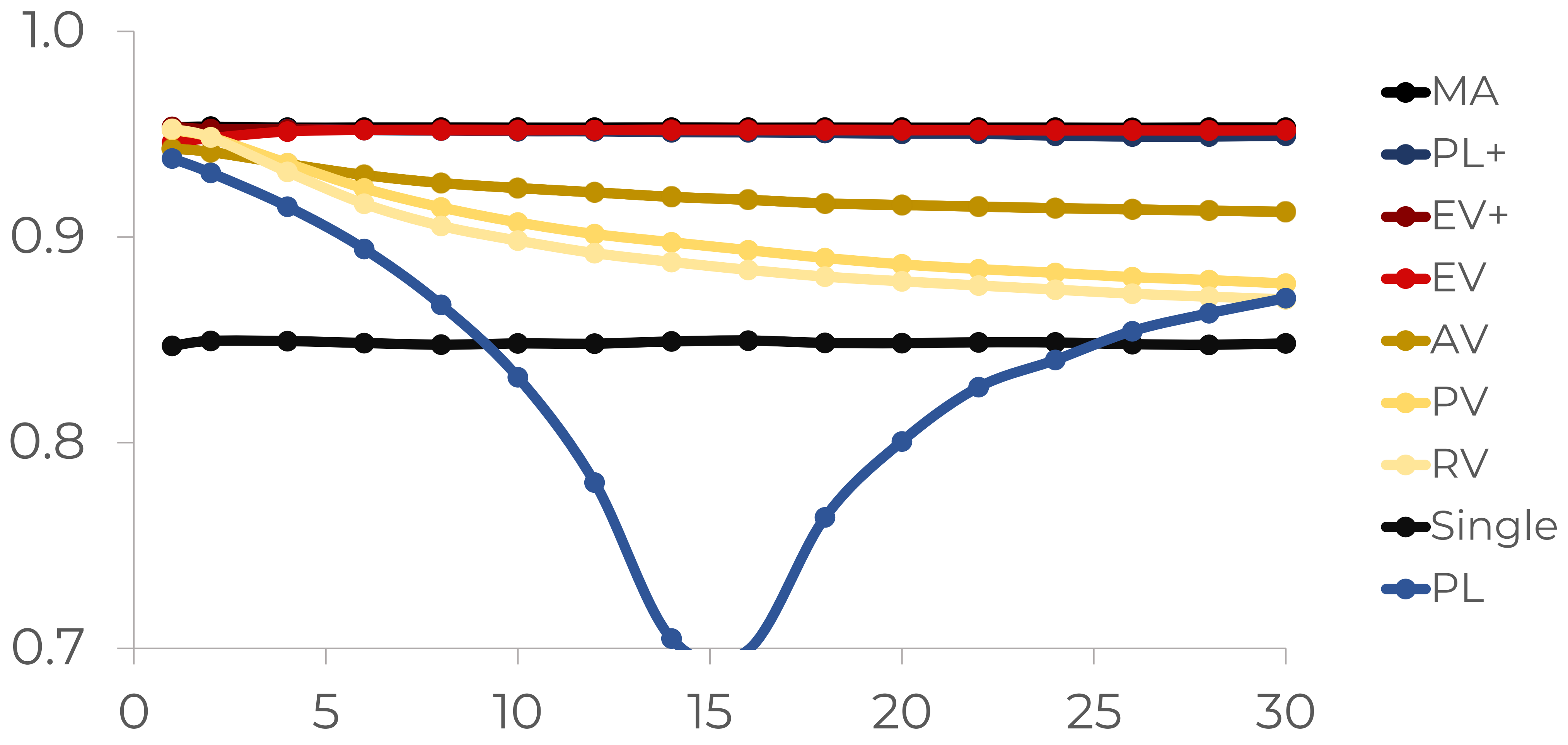


What if we vary…

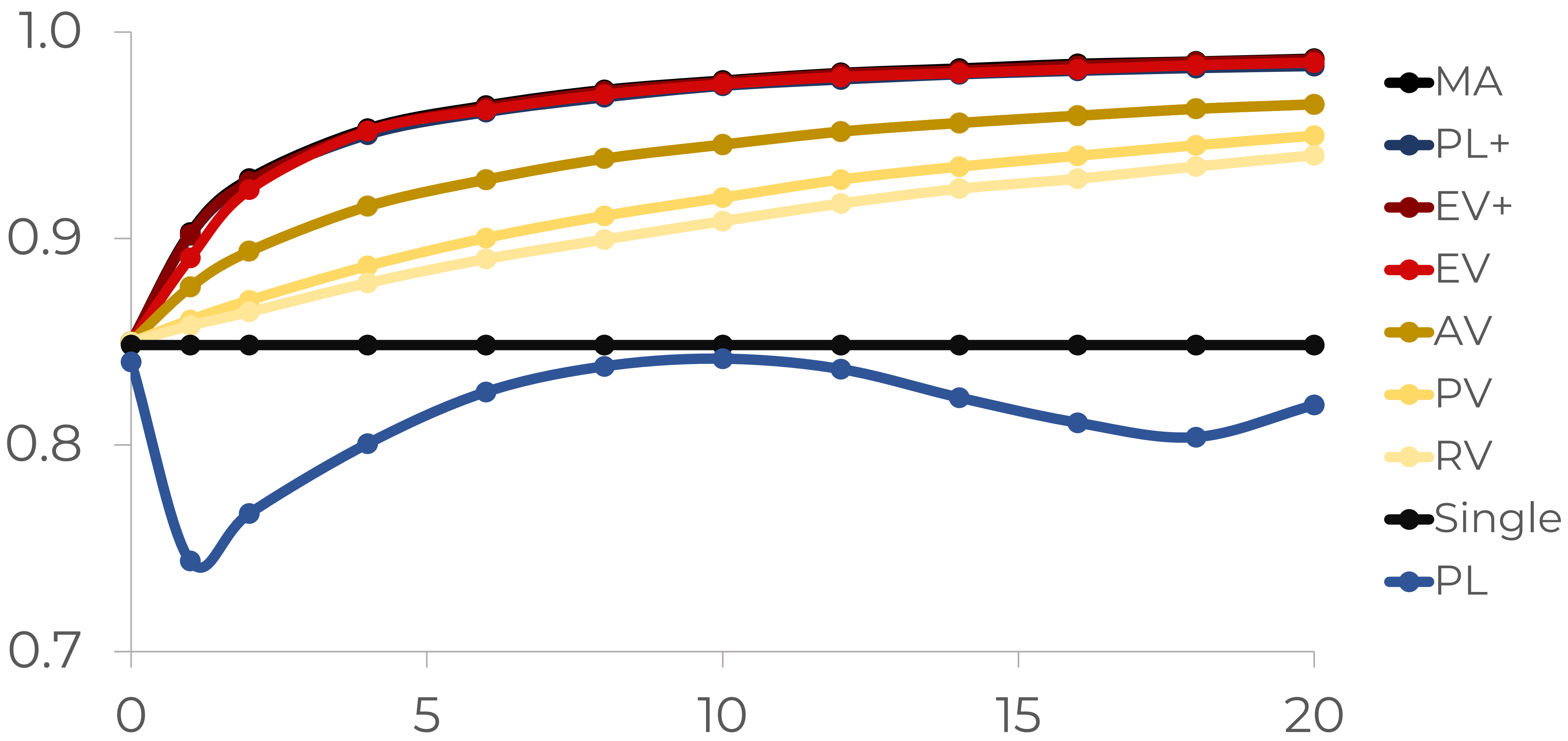
...the **noise intensities?**



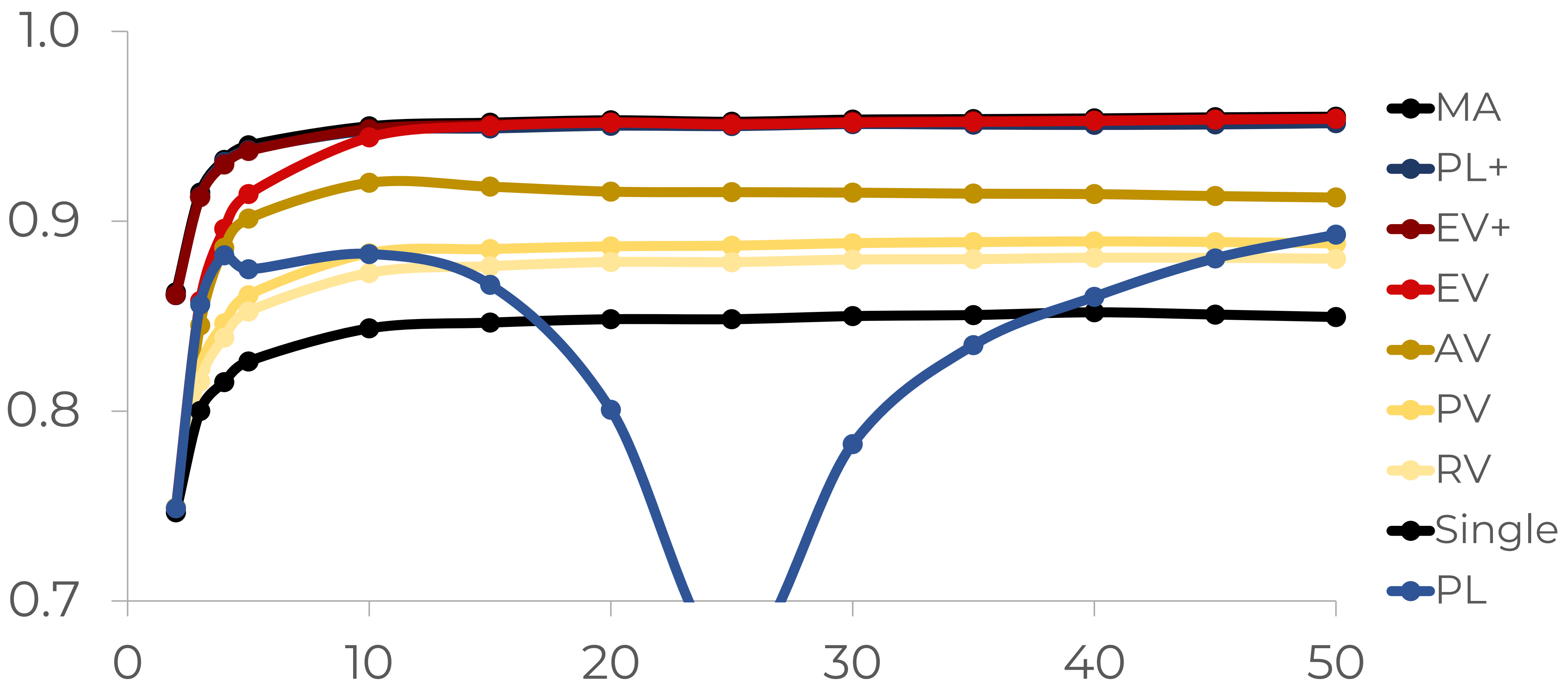
...the **number of agents in the correlated group?**



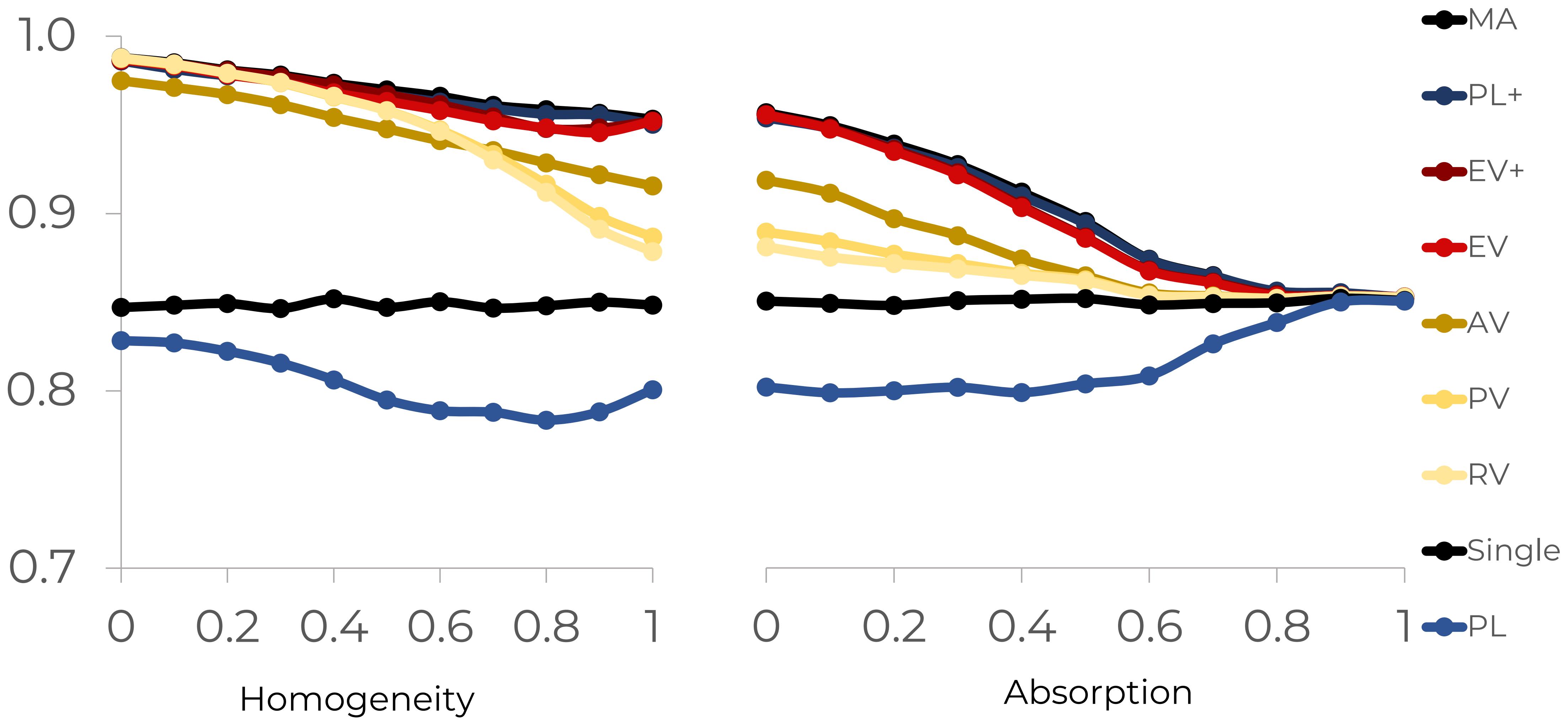
...the **number of independent agents?**



...the **number of candidates?**



And with **more fluid correlations…**



Conclusion

Context

Aggregating correlated agents in a **choice problem**.

Our proposal

Embedded Voting (EV), that uses SVD to embed the agents according to their estimations.

Our results

1. Our method **outperforms** classical ones, particularly when agents are correlated.
2. When a training set is available, a **maximum likelihood** approach is the best option.
3. If there is no such training, **Embedded Voting** should be preferred.

Thanks for your attention!

Our **paper**

Our **python package**