Approval with runoff

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Single-winner election

A set of voters $\mathcal{V} = \{v_1, \ldots, v_n\}$

A set of candidates $\mathcal{C} = \{\text{Ann}, \text{Bob}, \text{Carl}, \text{Dan}, \ldots\}$

Single-winner election

A set of voters $\mathcal{V} = \{v_1, \ldots, v_n\}$

A set of candidates $\mathcal{C} = \{$Ann, Bob, Carl, Dan, $\ldots\}$

$\Rightarrow$ Let's use **Plurality with Runoff !**

Plurality with Runoff

First round: Voters vote for their favorite candidate (ideally)

candidates	Ann	Bob	Carl	Dan
scores	28%	30%	20%	22%

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The two candidates with the highest scores advance to the second round

Second round: Majority vote

candidates	Ann	Bob
scores	54%	46%

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Ann

Plurality with Runoff: Is it a good rule?

Monotonicity

If a candidate $a \in \mathcal{C}$ is the winner of an election, and one voter changes their vote in favor of a, then a should remain the winner.

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candidates	Ann	Bob	Carl	Dan
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candidates	Ann	Dan
scores	48%	**52%**

Plurality with Runoff: Is it a good rule?

Monotonicity ⇒ Failed

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Monotonicity violations happen quite often in real life, for instance in 1988 during the French presidential election between Barre, Mitterrand and Chirac.

Plurality with Runoff: Is it a good rule?

Resistance to cloning

Introducing a clone of an existing candidate in the election should not change significantly the result of the election.

More formally:

- a' is a **clone** of a if for all voters v_i and for all candidates $x \neq a, a'$, $x \succ_i a \Leftrightarrow x \succ_i a'$.

Let P' be a a-**clone extension** of a profile P, i.e. we add a clone a' of a. A rule f is resistant to cloning if

- for all $x \neq a, a'$, $x \in f(P) \Leftrightarrow x \in f(P')$,
- if $a \in f(P)$, then $f(P') \cap \{a, a'\} \neq \emptyset$.

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candidates	Ann	Bob	Bobby	Carl	Dan
scores	28%	21%	9%	20%	22%

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- Clone effect occurs very often in real elections, for instance during the French presidential election in 2002. There were 8 candidates from the left, so none of them went to the second round.
- It forces voters to vote ”strategically” and not for their favorite candidate.

Plurality with runoff: Is it a good rule?

But plurality with runoff also fails:

- **Condorcet-consistency**, in a severe way: even if a candidate has a majority $\approx 1 - \frac{1}{m}$ against every other candidates, it might not go to the second round.
- **Participation**: similar reasons as for monotonicity
- **Reinforcement** (because of the runoff)

Plurality with runoff: Is it a good rule?

Pareto-efficient

If every voter prefers a to b, then b should not be a winner.

Plurality with runoff: Is it a good rule?

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Condorcet loser criterion

A candidate who is defeated in a head-to-head competition against every other candidate should not win.

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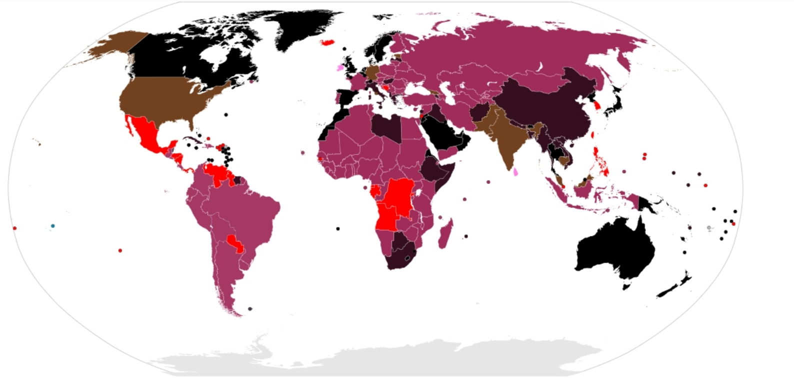
Moreover, having a runoff gives more time to voters to decide, as they only have to focus on the two finalists.

It is also a rule **simple to compute and to implement** as a voting protocol.

A widely used rule



All countries in purple use plurality with runoff for electing the head of state.

Everywhere in France

In France, we like this rule so much that we use it everywhere (or variants of it):

- Presidential election
- Parliament elections (districtwise)
- Party primaries
- A lot of low-stake elections

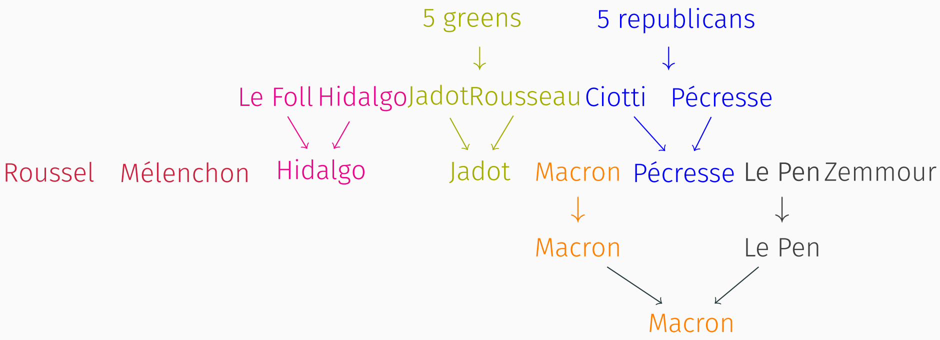
The example of the 2022 presidential election

First round was on *April 10th*, second round was on *April 24th*.

To avoid the 2002 effect, parties (and more generally sets of close candidates) have an incentive to run primaries (and again they chose to use **plurality with runoff**).

- Set of ecologist parties, October 2021
  - First round: Five candidates
  - Second round: Yannick Jadot, Sandrine Rousseau.
- Parti socialiste, October 2021
  - Two candidates: Anne Hidalgo, Stéphane Le Foll.
- Les Républicains, December 2021
  - First round: Five candidates
  - Second round: Valérie Pécresse, Eric Ciotti.

Plurality with runoff with primaries



Iterated plurality with runoff?

Can we keep **the benefits of the two-round protocol** without having to bear all the **drawbacks of plurality** in the first round?

Moreover, we do not want to change the voting system too much such that voters are more likely to **understand it and accept it**.

$\Rightarrow$ What happens if we replace the plurality ballots in the first round by approval ballots?

Approval with Runoff: As a protocol

First round: Voters can approve as many candidates as they like

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$$\Downarrow$$

From these approval ballots, we use an **approval-based committee rule** to select the two finalists

Approval with Runoff: As a protocol

First round: Voters can approve as many candidates as they like

$\Downarrow$

From these approval ballots, we use an **approval-based committee rule** to select the two finalists

$\Downarrow$

Second round: Majority vote between the two finalists

$\Downarrow$

The candidate that **wins the majority vote** is declared winner

Approval with Runoff: The model

$\mathcal{V} = \{v_1, \ldots, v_n\}$ the set of **voters**
$\mathcal{C} = \{c_1, \ldots, c_m\}$ the set of **candidates**

Data structure of preferences:

$P = \langle (A_1, \succ_1), \ldots, (A_n, \succ_n) \rangle$ an **approval-preference profile** (Brams & Sanver 2009) where each voter v_i is associated to an **approval ballot** $A_i \subseteq \mathcal{C}$ and a **ranking** $\succ_i$

We assume *ballot consistency*: if $x \in A_i$ and $y \notin A_i$ then $x \succ_i y$.

$V = \langle A_1, \ldots, A_n \rangle$ is an **approval profile**

$S_V(c) = |\{i | c \in A_i\}|$ is the **approval score** of c

Approval with runoff rules

F an (irresolute) **2-committee approval-based rule** that takes as input an approval profile V and outputs pairs of candidates in $\mathcal{C}$

F^R an (irresolute) **approval with runoff rule** based on F that takes as input an approval-preference profile P and outputs winners in $\mathcal{C}$

- **Step 1**: Use F and V to select pairs of finalists,
- **Step 2**: Run a majority vote between the two finalists of each pair using the rankings.

Multiwinner Approval Voting

Multi-winner Approval Voting: MAV

Select the two candidates with the highest number of approvals

	Approval ballot
10×	Bob
20×	Ann, Bob, Carl
30×	Ann, Bob
20×	Carl, Dan
5×	Dan

$\Rightarrow$

c	$S_V(c)$
Ann	
Bob	
Carl	
Dan	

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	Approval ballot
10×	Bob
20×	Ann, Bob, Carl
30×	Ann, Bob
20×	Carl, Dan
5×	Dan

$\Rightarrow$

c	$S_V(c)$
Ann	50
Bob	
Carl	
Dan	

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30×	Ann, Bob
20×	Carl, Dan
5×	Dan

$\Rightarrow$

c	$S_V(c)$
Ann	50
Bob	60
Carl	
Dan	

Multiwinner Approval Voting

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Select the two candidates with the highest number of approvals

	Approval ballot
10×	Bob
20×	Ann, Bob, Carl
30×	Ann, Bob
20×	Carl, Dan
5×	Dan

$\Rightarrow$

c	$S_V(c)$
Ann	50
Bob	60
Carl	40
Dan	25

Multiwinner Approval Voting

Multi-winner Approval Voting: MAV

Select the two candidates with the highest number of approvals

	Approval ballot
10×	Bob
20×	Ann, Bob, Carl
30×	Ann, Bob
20×	Carl, Dan
5×	Dan

⇒

c	$S_V(c)$
Ann	50
Bob	60
Carl	40
Dan	25

⇒ {Bob, Ann}

Multi-winner Approval Voting: MAV

Select the two candidates with the highest number of approvals

	Approval ballot
10×	Bob, Bobby
20×	Ann, Bob, Bobby, Carl
30×	Ann, Bob, Bobby
20×	Carl, Dan
5×	Dan

⇒

c	$S_V(c)$
Ann	50
Bob	60
Bobby	60
Carl	40
Dan	25

⇒ {Bob, Bobby}

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Monotonicity $\Rightarrow$ Satisfied

If a candidate $a \in \mathcal{C}$ is the winner of an election, and one voter that did not approve a now approves him, then a should remain the winner.

Chamberlin–Courant Approval Voting

Chamberlin–Courant Approval Voting: CCAV

Select the pair of candidates that maximizes the number of voters approving at least one of them

	Approval ballot
10×	Bob
20×	Ann, Bob, Carl
30×	Ann, Bob
20×	Carl, Dan
5×	Dan

⇒

	score
Bob, Ann	60
Bob, Carl	80
Bob, Dan	85
...	...

⇒ {Bob, Dan}

Chamberlin–Courant Approval Voting

Chamberlin–Courant Approval Voting: CCAV

Select the pair of candidates that maximizes the number of voters approving at least one of them

	Approval ballot
10×	Bob, Bobby
20×	Ann, Bob, Bobby, Carl
30×	Ann, Bob, Bobby
20×	Carl, Dan
5×	Dan

$\Rightarrow$

	score
Bob, Ann	60
Bob, Carl	80
Bob, Dan	85
Bob, Bobby	60
...	...

$\Rightarrow$ {Bob, Dan}

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Introducing a clone of an existing candidate in the election should not change significantly the result of the election.

Monotonicity $\Rightarrow$ Failed

If a candidate $a \in \mathcal{C}$ is the winner of an election, and one voter that did not approves a is now approving it, then a should remain the winner.

Impossibility theorem

Theorem

No AVR rule is resistant to cloning, monotonic.

This set of properties is minimal:

- **MAV** satisfies monotonicity but not resistance to cloning,
- **CCAV** satisfies resistance to cloning but not monotonicity

Spectrum of rules

These rules are part of the more general family of rules called αAV-rules

$$\alpha\text{AV}(V) = \text{argmax}_{x,y\in\mathcal{C}}\ S_V(x) + S_V(y) - \alpha S_V(xy)$$

$S_V(x)$ is the number of voters who approve x

$S_V(xy)$ is the number of voters who approve both x and y

	MAV	PAV	CCAV
α	0	$\frac{1}{2}$	1

$$\text{MAV}(V) = \text{argmax}_{x,y\in\mathcal{C}}\ S_V(x) + S_V(y)$$

$$\text{PAV}(V) = \text{argmax}_{x,y\in\mathcal{C}}\ S_V(x) + S_V(y) - \tfrac{1}{2}\, S_V(xy)$$

$$\text{CCAV}(V) = \text{argmax}_{x,y\in\mathcal{C}}\ S_V(x) + S_V(y) - S_V(xy)$$

Proportional Approval Voting

Proportional Approval Voting: PAV

$\text{PAV}(V) = \text{argmax}_{x,y \in \mathcal{C}}\, S_V(x) + S_V(y) - \frac{1}{2} S_V(xy)$

	Approval ballot
10×	Bob,
20×	Ann, Bob, Carl
30×	Ann, Bob
20×	Carl, Dan
5×	Dan

⇒

	score
Bob, Ann	$60 + 50 - \frac{1}{2}50 = 85$
Bob, Carl	$60 + 40 - \frac{1}{2}20 = 90$
Bob, Dan	$60 + 25 - 0 = 85$
...	...

⇒ {Bob, Carl}

Favorite-consistency

Chamberlin–Courant Approval Voting: CCAV

Select the pair of candidates that maximizes the number of voters approving at least one of them.

$\mathrm{CCAV}(V) = \mathrm{argmax}_{x,y \in \mathcal{C}}\, S_V(x) + S_V(y) - S_V(xy)$

	Approval ballot
10×	Bob,
40×	Ann, Bob
40×	Ann, Carl
10×	Carl

⇒

	score
Bob, Carl	100
Ann, Bob	90
Ann, Carl	90

⇒ {Bob, Carl}

But Ann is approved by 80% of voters and the others are approved by 50% of the voters each

Favorite-consistency and sequential rules

Favorite-consistency

At least one finalist is an approval winner

$\Rightarrow$ MAV satisfies it, but not CCAV and PAV,
so we use the sequential versions of these rules:

1. The first finalist x is an approval winner (i.e. it maximizes $S_v(x)$)
2. The second finalist y is the one that maximizes the marginal contribution score of y given that x has already been selected.

$\Rightarrow$ Instead of looking at all possible pairs, we constrain the first finalist of the pair to be x

Sequential rules

$$\text{S-PAV}(V): \quad \text{argmax}_{y \in \mathcal{C}}\, S_V(x) + S_V(y) - \tfrac{1}{2}\, S_V(xy)$$
$$\text{S-CCAV}(V): \quad \text{argmax}_{y \in \mathcal{C}}\, S_V(x) + S_V(y) - S_V(xy)$$

Sequential PAV

Select x_1 maximizing S_V and $x_2 = \text{argmax}_x\, S_V(x) - \frac{1}{2} S_V(x_1 x)$

Sequential CCAV

Select x_1 maximizing S_V and $x_2 = \text{argmax}_x\, S_V(x) - S_V(x_1 x)$

α-seqAV: Select x_1 maximizing S_V and $x_2 = \text{argmax}_x\, S_V(x) - \alpha S_V(x_1 x)$

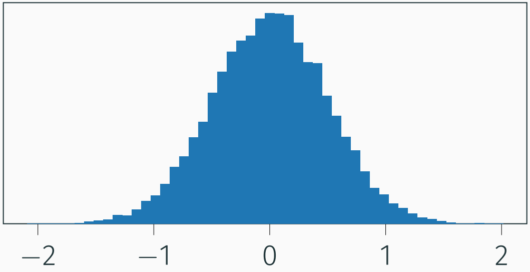
Properties

	MAV^R	$S\text{-}PAV^R$	$S\text{-}CCAV^R$	PAV^R	$CCAV^R$
Pareto-efficiency	✓	✓	✓*	✓	✓*
monotonicity	✓				
resistance to cloning			✓		✓
favorite-consistency	✓	✓	✓		

* Depends on the tie-breaking used

Simulation with 1D Euclidean preferences

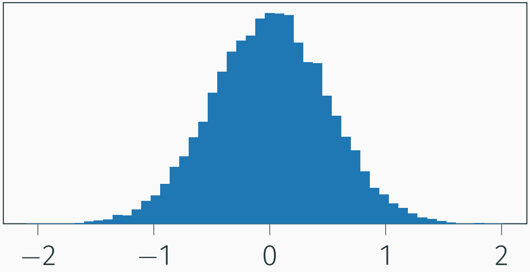
- Gaussian distribution of voters, centered at 0 and with standard deviation 1/2



- Candidates are uniformly distributed in [-1, 1]
- A voter **approves candidates at distance $\leq d$** (approval radius)

Simulation with 1D Euclidean preferences

- Gaussian distribution of voters, centered at 0 and with standard deviation 1/2

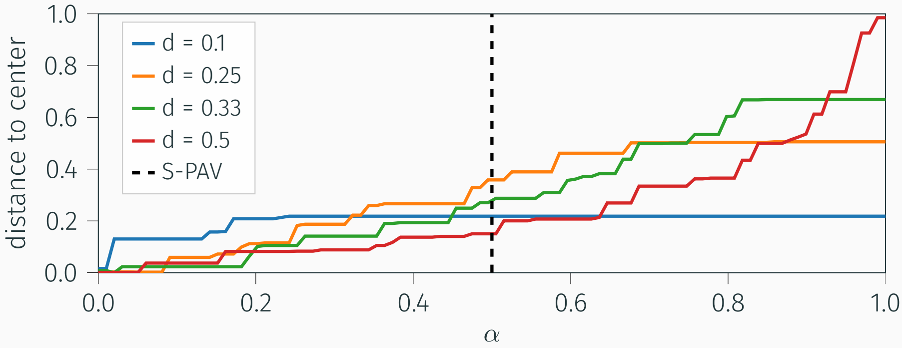


- Candidates are uniformly distributed in [-1, 1]
- A voter **approves candidates at distance $\leq d$** (approval radius)

Question: what are the positions of the finalists depending on the parameter α?

Simulation with 1D Euclidean preferences

- With α-seqAV rules, the first finalist is always the closest to the center (i.e. 0), so the other finalist y maximizes $S_V(y) - \alpha S_V(0y)$
- We depict the position of the second finalist as a function of α and d



Experiments with real data

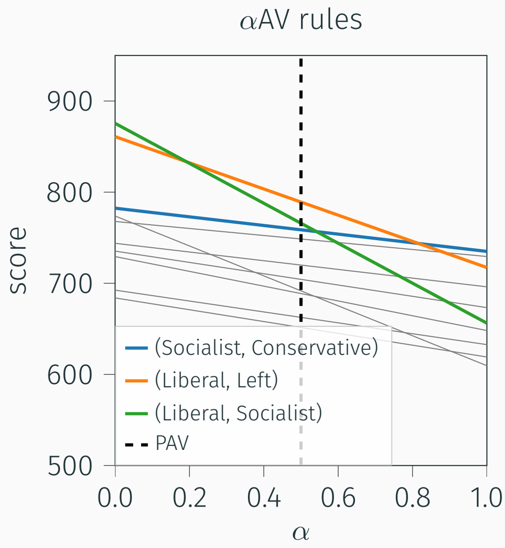
- Datasets collected during the **2017 French presidential election** (**Voter Autrement 2017**, Bouveret et al.) in several cities, each dataset with ~ 1000 voters and 11 candidates (with reweighed voters, so as to unbaised the dataset)
- Dataset from the online experiment *Un autre vote* during the **2022 French presidential election**. ~ 2000 voters and 12 candidates (with reweighed voters).
- Two datasets, **poster competition**, collected at the Summer School on Computational Social Choice. San Sebastian 2016. Available on **PrefLib**, 17 candidates, ~ 60 voters per dataset.

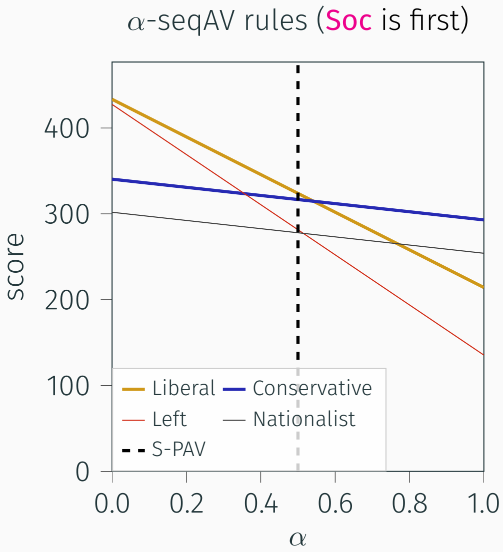
Experiments with real data

	MAV	PAV	S-PAV	CCAV	S-CCAV
2017-Strasbourg	Lib/ Left	Lib/ Left	Lib/ Left	Lib/ Left	Lib/ Left
2017-Grenoble	Soc/ Lib	Lib/ Left	Lib/Soc	Soc/ Cons	Soc/ Cons
2017-Crolles	Lib/ Left	Lib/ Left	Lib/ Left	Lib/ Nat	Lib/ Nat
2022-Online	Grn/ Left	Grn/ Nat	Grn/ Nat	Grn/ Nat	Grn/ Nat
Best-Poster-A	P. 1/P. 2	P. 1/P. 4	P. 1/P. 4	P. 1/P. 6	P. 1/P. 6
Best-Poster-B	P. 1/P. 2	P. 1/P. 2	P. 1/P. 2	P. 1/P. 2	P. 1/P. 2

Left Socialist Grn(Green) Liberal Conservative Nationalist

Experiments with real data: Grenoble dataset





Conclusion

Plurality with runoff:

- Many **unnecessary complications** such as primaries
- Massive **strategic behaviour**
- Hypersensitivity to **cloning**
- Invisibilization of ”small” parties

Approval with runoff:

- Retains the idea of a **two-round protocol** and is very simple
- Is not one rule but a **family of rules**, parameterized by the ABC rule chosen for determining the finalists
- We obtained **axiomatic** and **experimental** results that show that this choice actually makes a big difference

Further work

Questions:

- Will citizens **understand and accept such rules** especially in comparison with plurality with runoff and standard (single-winner) approval voting?
- Will there be **a difference in voting behaviour** under AVR rules between citizens used to runoff voting in their country and those who are not?