Liquid Democracy with Ranked Delegations

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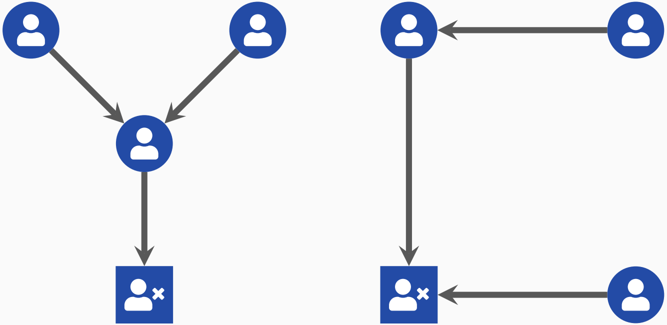
Liquid Democracy Workshop @ Zurich

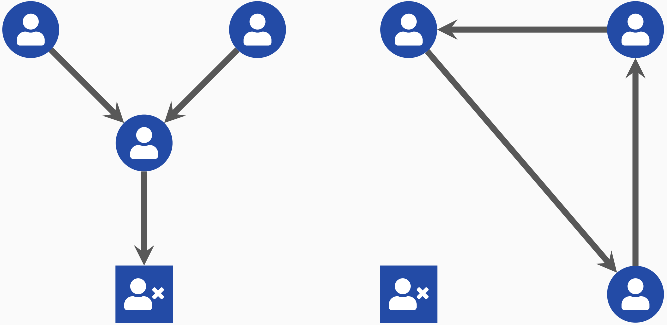
Liquid Democracy with Ranked Delegations



Voters can delegate their vote to **one** other voter.

Implementations: LiquidFeedback, Sovereign, GoogleVotes

Liquid Democracy with Ranked Delegations



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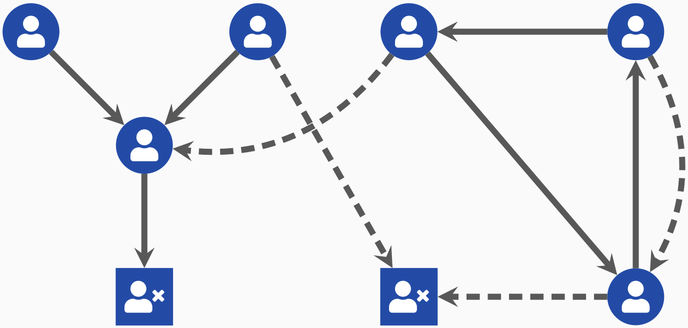
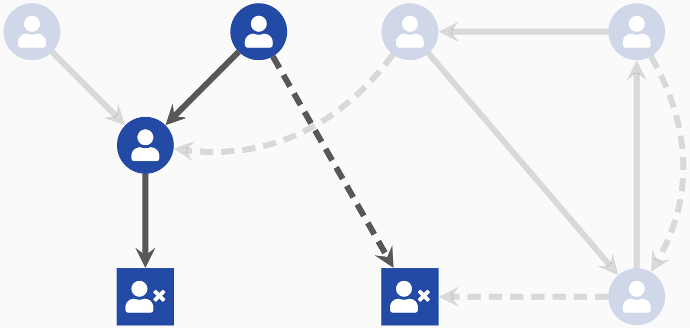


Voters can state **a set of approved delegatees** together with a **ranking** among them.

Implementations: LiquidFeedback, Sovereign, GoogleVotes



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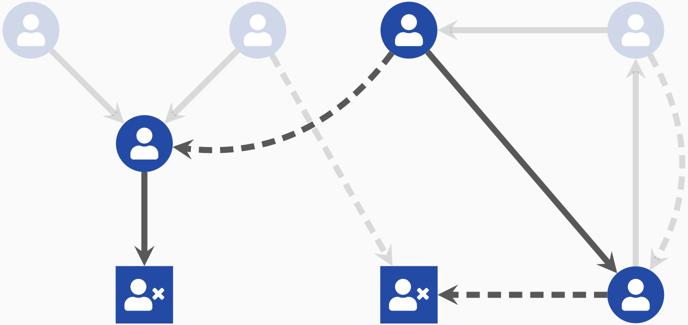

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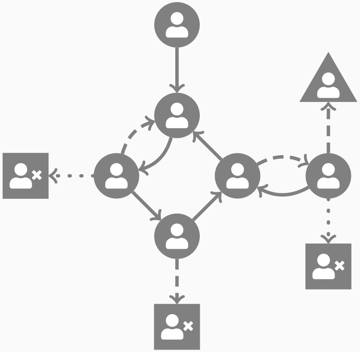
Delegation Rules

Input: A directed delegation graph with a rank for every edge, and a partition of V into:

- casting voters : no outgoing edges
- delegating voters : reach at least one
- isolated voters : do not reach any



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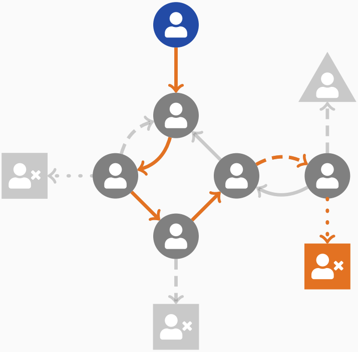
Output: for each **delegating voter**:

- a path to a **casting voter**



A delegation rule indirectly outputs a **weight distribution** over casting voters.

We introduce a simple **graph-theoretical model** that can capture **rules** and **axioms** studied in the literature.

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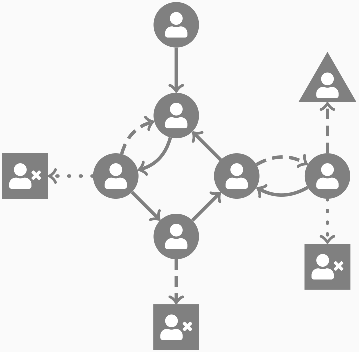
We identify a natural **subclass** of delegation rules, perform an extensive **axiomatic analysis**, and compare all studied rules **empirically**.

Sequence Rules

Sequence Rules

let $\mathcal{S}_v$ be the set of **rank sequences** of paths leading to casting voters for a delegating voter v



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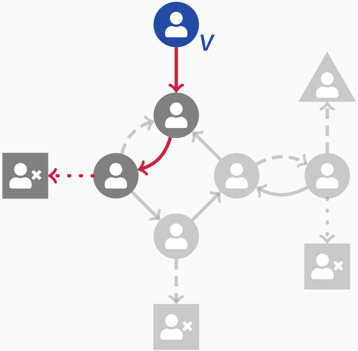

$\mathcal{S}_v = \{(1,1,3), \quad , \quad \}$

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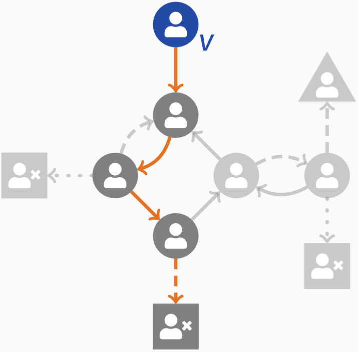

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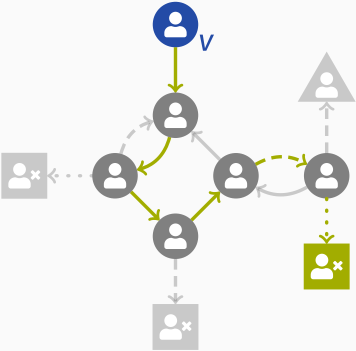

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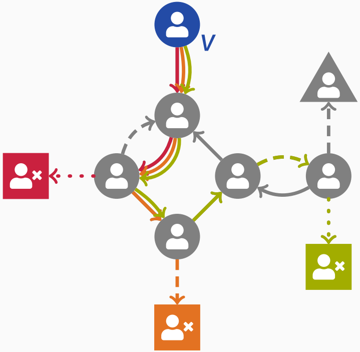
sequence rule: outputs $\max_{\triangleright}\{\mathcal{S}_v\}$ for each delegating voter v, where $\triangleright$ is an order over rank sequences



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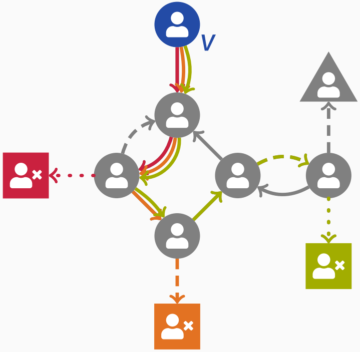
Sequence Rules

Let $\triangleright_{lex}$ be the lexicographical order.



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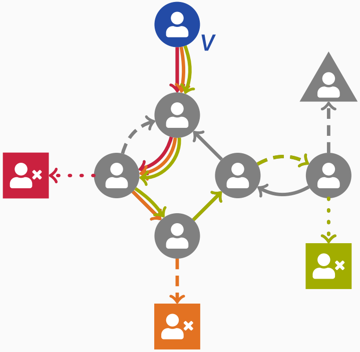
Let $\rhd_{lex}$ be the lexicographical order.

- depth-first delegation: rule induced by $\rhd_{lex}$
- breadth-first delegation: orders sequences by length, tie-breaking according to $\rhd_{lex}$

[Kotsialou and Riley (AAMAS 2020)]



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Sequence Rules

$1 \succ 2 \succ 3$

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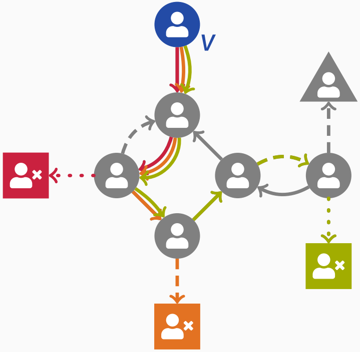
- min-sum: orders sequences by the sum of ranks, breaks ties according to $\rhd_{lex}$



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Sequence Rules

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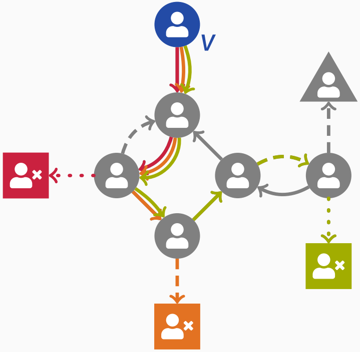
  [Kotsialou and Riley (AAMAS 2020)]
- min-sum: orders sequences by the sum of ranks, breaks ties according to $\triangleright_{\text{lex}}$
- leximax: $s \triangleright s'$ iff $\sigma(s) \triangleright_{\text{lex}} \sigma(s')$, where σ sorts s by non-increasing ranks, e.g.,

  $\sigma(1,1,1,2) = (2,1,1,1) \triangleright_{\text{lex}} (3,1,1) = \sigma(1,1,3)$



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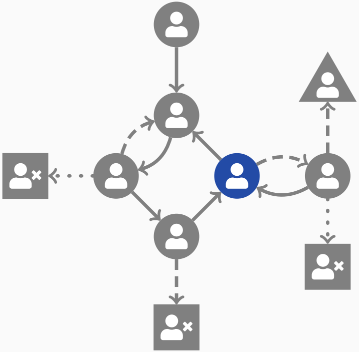
Axiomatic Analysis

Confluence

Confluence: for all : all paths intersecting with use the same outgoing edge of .



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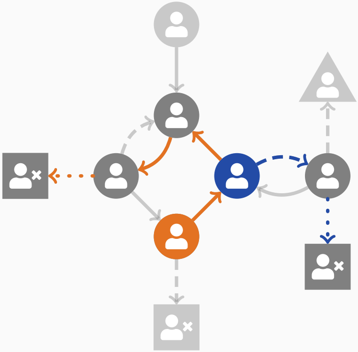
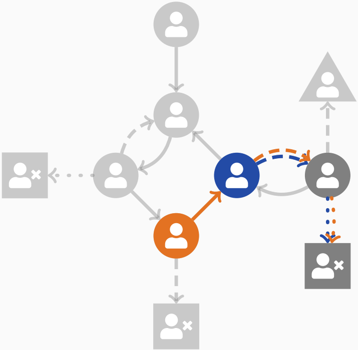


Confluence: for all 👤: all paths intersecting with 👤 use the same outgoing edge of 👤.

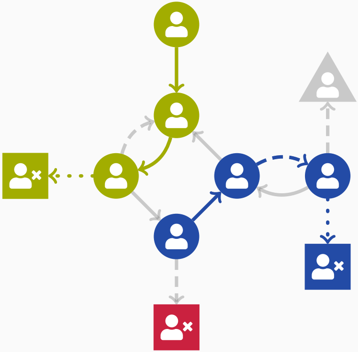


Confluence

Confluence: for all : all paths intersecting with use the same outgoing edge of .

- output of the delegation rule can be **communicated** more easily
- a single representative helps "to preserve the high level of **accountability** guaranteed by classical liquid democracy."

[Gölz et al., WINE 2018]



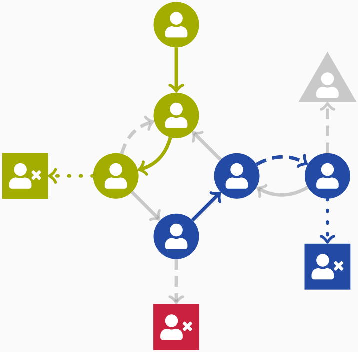
Confluence

Confluence: for all : all paths intersecting with use the same outgoing edge of .

Theorem

*Building upon a **characterization** of orders ▷ that induce **confluent** sequence rules, we show:*

- *breadth-first delegation, min-sum, diffusion, and leximax are confluent*
- *depth-first delegation is not confluent*



Copy-robustness

Copy-robustness: A delegating voter has a direct path to its casting voter. If it becomes a casting voter, the joint voting power of both remains equal.

[Behrens & Swierczek (LDJ, 2015)]

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Impossibility Theorem

No sequence rule is both confluent and copy-robust. Hence, breadth-first delegation, min-sum, diffusion, and leximax are not copy-robust.

Copy-robustness: A delegating voter has a direct path to its casting voter. If [delegating voter] becomes a casting voter, the joint voting power of [delegating voter] & [casting voter] remains equal.
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Impossibility Theorem

No sequence rule is both **confluent** and **copy-robust**. Hence, **breadth-first delegation**, **min-sum**, **diffusion**, and **leximax** are not copy-robust.

Characterization of DFS

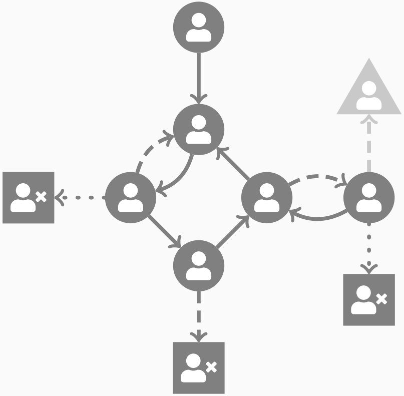
Depth-first delegation is the only **sequence rule** that is **copy-robust** and satisfies **weak lexicographical tie-breaking**.

Can we obtain **confluence** and **copy-robustness** by going **beyond** sequence rules?

Branching Rules

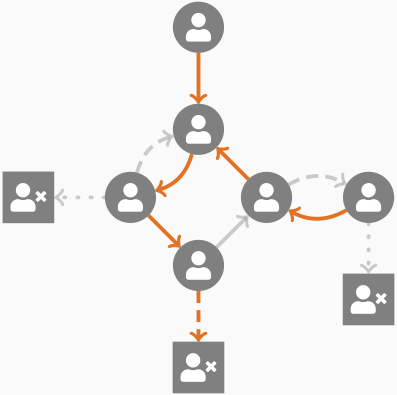
Branching rules

C-branching: Acyclic subgraph such that all delegating voters 👤 have **exactly one** outgoing edge.



C-branching: Acyclic subgraph such that all delegating voters have **exactly one** outgoing edge.

Branching rules select delegations on a global level while **Sequence rules** select delegations for each voter



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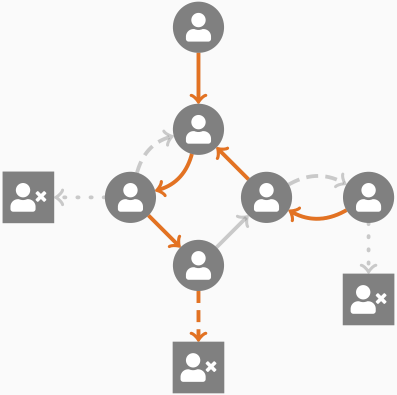
Borda branching: Select a *C*-branching *B* that minimizes the total sum of ranks



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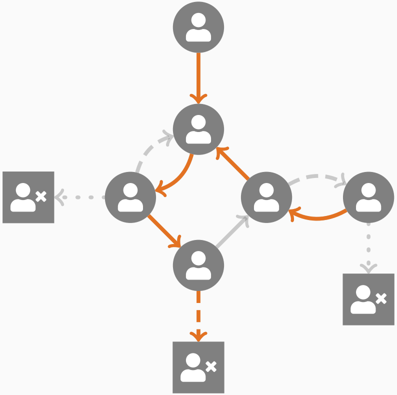


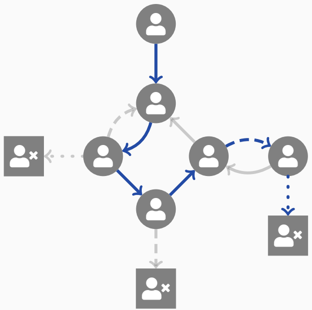
Theorem

*Borda branching (with an appropriate tie-breaking rule) satisfies **confluence** and **copy-robustness**.*

Popular Branchings [Kavitha et al. (Math. Prog., 2021)] $\xrightarrow{1} \succ \overset{2}{\dashrightarrow} \succ \overset{3}{\cdots\rightarrow}$

Pairwise majority comparisons:

$$\Delta(B_1, B_2) := \quad \#\text{ nodes in favor of } B_1$$
$$- \#\text{ nodes in favor of } B_2$$



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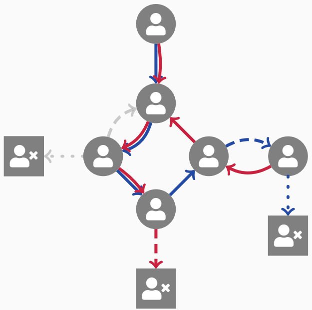

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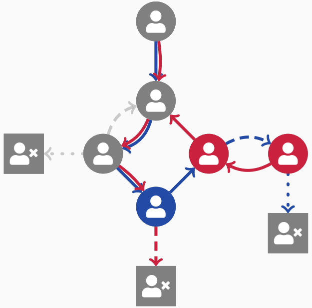

Pairwise majority comparisons:

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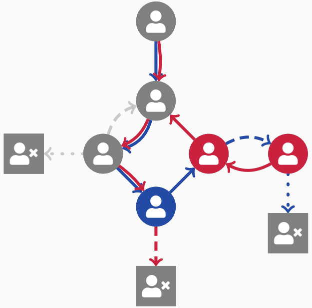


Unpopularity margin:

$$\text{unpopularity}(B) := \max_{B'}(\Delta(B', B))$$

Theorem

*A **popular branching**, i.e., a branching with unpopularity $= 0$ does not always exist.*

Empirical Results

Data generation

- **Prominence-based method** (following the *preferential attachment* principle): the highest your in-degree in the network, the more likely you are to receive delegations.
- **Friendship-based method** (following the *small world* principle): the more you have common *friends* with someone, the most likely you are to receive its delegation.

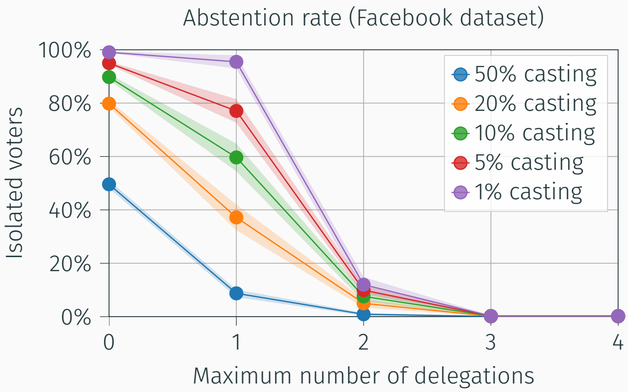
For each method, we used both generated data and **real data**. Here, I will only show the results for experiments on two real datasets:

dataset	method	nodes	edges	avg degree
Twitter	Prominance-based	456K	14,8M	65
Facebook	Friendship-based	63K	817K	26

Impact of backup delegation on abstention rate

On the classic liquid democracy setting, each voter can delegates to **at most one voter**. This cause the issue of **delegation cycles** and **lost ballots**.

With ranked delegation, we achieve **far better participation rate**, even when only 1% of all voters are actually voting.



Results

Twitter dataset ($n = 456626$)	Unpop.	AvgRank	AvgLen	MaxWeight
Breadth-first	223746	3.4	1.16	27397
MinSum	105023	1.37	1.89	31963
Leximax	13699	1.04	5.59	118722
BordaBranching	16	1.0	6.0	132421
Depth-first			6.05	127855

Facebook dataset ($n = 63731$)	Unpop.	AvgRank	AvgLen	MaxWeight
Breadth-first	28678	3.29	1.27	162
MinSum	12746	1.35	2.04	224
Leximax	2567	1.08	3.97	539
BordaBranching	12	1.03	4.79	748
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MaxWeight: Maximum accumulated voting weight of a casting voter. Mechanism avoiding **super voters** were studied by Gölz et al. (WINE, 2018).

Unpopularity: Worst-case **majority comparison** [Kavitha et al. (Math. Prog. 2021)]

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Observations

- **trade-off** between minimizing unpopularity and maximum weight
- delegation rules can be aligned on a **spectrum**

Summary

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In this talk:

- introduction of a simple **graph-theoretical** model
- formalization of the class of **sequence rules**
- **impossibility** result for copy-robust and confluent sequence rules
- Borda branching satisfies copy-robustness and confluence
- **characterization** of depth-first delegation via copy-robustness

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- **characterization** of depth-first delegation via copy-robustness

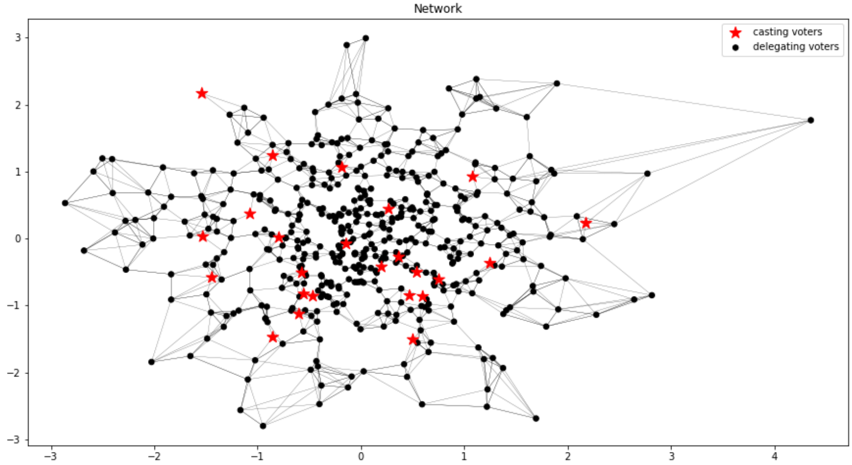
Not mentioned in this talk:

- **characterization** of breadth-first delegation via confluence
- a **generalization** of a result by Kotsialou and Riley (AAMAS 2020) implying that almost all studied sequence rules satisfy **guru participation**
- Borda branching satisfies **guru participation**
- a proof that diffusion is a **sequence rule** by uncovering its respective order
- more experiments !

Thanks

Thanks for your attention !

Bonus : The distance-based method



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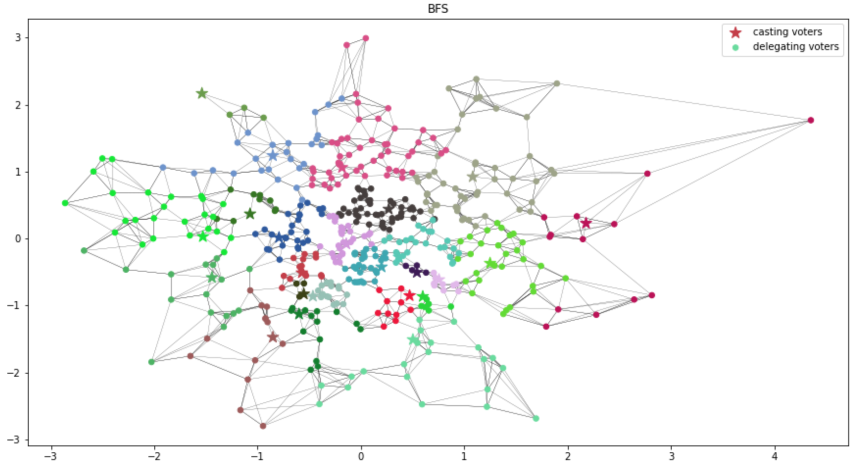

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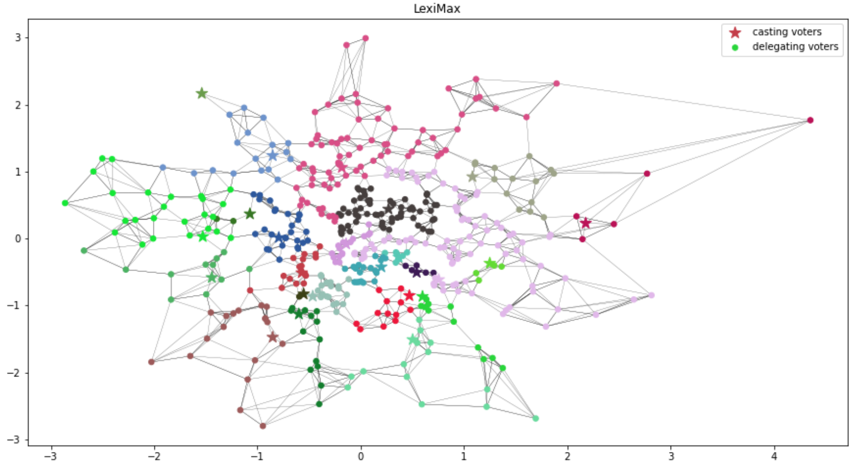

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