Approval with Runoff

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Single-winner election

A set of voters $\mathcal{V} = \{v_1, \ldots, v_n\}$

A set of candidates $\mathcal{C} = \{\text{Ann}, \text{Bob}, \text{Carl}, \text{Dan}, \ldots\}$

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$\Rightarrow$ Let's use **Plurality with Runoff !**

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First round: Voters vote for their favorite candidate

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scores	28%	30%	20%	22%

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Second round: Majority vote

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Ann

Plurality with Runoff: Is it a good rule?

Monotonicity

If a candidate $a \in \mathcal{C}$ is the winner of an election, and one voter changes his vote in favor of a, then a should remain the winner.

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Resistance to cloning

Introducing a clone of an existing candidate in the election should not change significantly the result of the election.

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candidates	Ann	Bob	Bobby	Carl	Dan
scores	28%	21%	9%	20%	22%

$\Rightarrow$

candidates	Ann	Dan
scores	48%	52%

- Happens quite often, e.g. French presidential election in 2002.

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But also **participation**, **Condorcet-consistency** and **reinforcement**

Plurality with runoff: Is it a good rule?

Condorcet loser criterion $\Rightarrow$ Satisfied

A candidate who can be defeated in a head-to-head competition against every other candidate should not win.

Moreover, having a runoff give more time to voters to decide, as they only have to focus on the two finalists.

It is also a rule **simple to compute and to implement** as a voting protocol.

Can we keep **the benefits of the two-round protocol** without having to bear all the **drawbacks of plurality** in the first round?

⇒ What happens if we replace the plurality ballots in the first round by approval ballots?

Approval with Runoff: As a protocol

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Second round: Majority vote between the two finalists

Approval with Runoff: The model

$P = \langle (A_1, \succ_1), \ldots, (A_n, \succ_n) \rangle$ an **approval-preference profile** where each voter v_i is associated to an **approval ballot** $A_i \subseteq \mathcal{C}$ and a **ranking** $\succ_i$

$V = \langle A_1, \ldots, A_n \rangle$ an **approval profile**

$S_V(c) = |\{i | c \in A_i\}|$ is the **approval score** of c

F an (irresolute) **2-committee approval-based rule** that takes as input an approval profile V and outputs a pair of candidates in $\mathcal{C}$

F^R an (irresolute) **approval with runoff rule** based on F that takes as input an approval-preference profile P and outputs a winner in $\mathcal{C}$

Multiwinner Approval Voting

Multi-winner Approval Voting: MAV

Select the two candidates with the highest number of approvals

	Approval ballot
10	Bob
20	Ann, Bob, Carl
30	Ann, Bob
20	Carl, Dan
5	Dan

$\Rightarrow$

c	$S_V(c)$
Ann	50
Bob	60
Carl	40
Dan	25

$\Rightarrow \{\text{Bob}, \text{Ann}\}$

Multiwinner Approval Voting

Multi-winner Approval Voting: MAV

Select the two candidates with the highest number of approvals

	Approval ballot
10	Bob, Bobby
20	Ann, Bob, Bobby, Carl
30	Ann, Bob, Bobby
20	Carl, Dan
5	Dan

$\Rightarrow$

c	$S_V(c)$
Ann	50
Bob	60
Bobby	60
Carl	40
Dan	25

$\Rightarrow \{\text{Bob}, \text{Bobby}\}$

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Monotonicity $\Rightarrow$ Satisfied

If a candidate $a \in \mathcal{C}$ is the winner of an election, and one voter that did not approve a now approves him, then a should remain the winner.

Chamberlin-Courant Approval Voting

Chamberlin-Courant Approval Voting: CCAV

Select the pair of candidates that maximizes the number of voters approving at least one of them

	Approval ballot
10	Bob
20	Ann, Bob, Carl
30	Ann, Bob
20	Carl, Dan
5	Dan

$\Rightarrow$

	score
Bob, Ann	60
Bob, Carl	80
Bob, Dan	85
...	...

$\Rightarrow \{\text{Bob}, \text{Dan}\}$

Chamberlin-Courant Approval Voting

Chamberlin-Courant Approval Voting: CCAV

Select the pair of candidates that maximizes the number of voters approving at least one of them

	Approval ballot
10	Bob, Bobby
20	Ann, Bob, Bobby, Carl
30	Ann, Bob, Bobby
20	Carl, Dan
5	Dan

$\Rightarrow$

	score
Bob, Ann	60
Bob, Carl	80
Bob, Dan	85
Bob, Bobby	60
...	...

$\Rightarrow \{\text{Bob}, \text{Dan}\}$

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Theorem

No neutral AVR rule is resistant to cloning and monotonic

Spectrum of rules

These rules are part of the more general family of αAV-rules

$$\alpha\text{AV}(V) = \textbf{argmax}_{x,y\in\mathcal{C}}\ S_V(x) + S_V(y) - \alpha S_V(xy)$$

$S_V(x)$ is the number of voters who approve x

$S_V(xy)$ is the number of voters who approve both x and y

	MAV	PAV	CCAV
α	0	$\frac{1}{2}$	1

Chamberlin-Courant Approval Voting: CCAV

Select the pair of candidates that maximizes the number of voters approving at least one of them

	Approval ballot
10	Bob,
40	Ann, Bob
40	Ann, Carl
10	Carl

$\Rightarrow$

	score
Bob, Carl	100
Ann, Bob	90
Ann, Carl	90

$\Rightarrow \{\text{Bob}, \text{Carl}\}$

But Ann is approved by 80% of voters and the others are approved by 50% of the voters each

Favorite-consistency and sequential rules

Favorite-consistency

At least one finalist is an approval winner

$\Rightarrow$ MAV satisfies it, but not CCAV and PAV,
so we defined sequential versions of these rules:

1. The first finalist x is an approval winner (i.e. it maximizes $S_V(x)$)
2. The second finalist y is the one that maximizes the score when paired with x:

$\Rightarrow$ Instead of looking at all possible pairs, we constrain the first finalist of the pair to be x

Properties

	MAV^R	S-PAV^R	S-CCAV^R	PAV^R	CCAV^R
Pareto-efficiency	✓	✓	✓*	✓	✓*
monotonic	✓				
resistant to cloning			✓		✓
favorite-consistency	✓	✓	✓		

* Depends on the tie-breaking used

Experiments with real data

- Datasets collected during the **2017 French presidential election** in several cities, each dataset with $\sim$ 1000 voters and 11 candidates
- Two datasets, **poster competition**, collected at the Summer School on Computational Social Choice. 17 candidates, $\sim$ 60 voters per dataset.

	MAV	PAV	S-PAV	CCAV	S-CCAV
2017-Strasbourg	Lib/ Left	Lib/ Left	Lib/ Left	Lib/ Left	Lib/ Left
2017-Grenoble	Soc/ Lib	Lib/ Left	Lib/Soc	Soc/ Cons	Soc/ Cons
2017-Crolles	Lib/ Left	Lib/ Left	Lib/ Left	Lib/ Nat	Lib/ Nat
Best-Poster-A	P. 1/P. 2	P. 1/P. 4	P. 1/P. 4	P. 1/P. 6	P. 1/P. 6
Best-Poster-B	P. 1/P. 2	P. 1/P. 2	P. 1/P. 2	P. 1/P. 2	P. 1/P. 2

Conclusion

- Approval with runoff is not one rule but a **family of rules**, parameterized by the ABC rule chosen for determining the finalists
- We obtained **axiomatic** and **experimental** results that show that this choice actually makes a big difference
- I had to omit a lot of things, so come to the poster session if you have questions or if you want to know more!