Reallocating Wasted Votes in Proportional Parliamentary Elections with Thresholds

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Paris School of Economics

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Université Paris Dauphine

- 0 -
Introduction and motivation

Electing a parliament









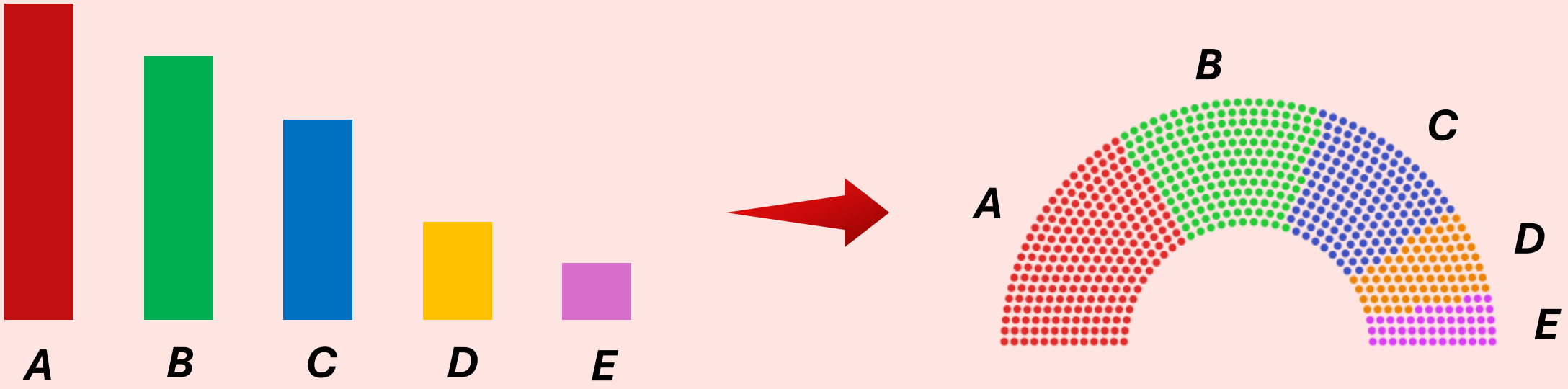




Proportional representation

Voters vote for **one** party.

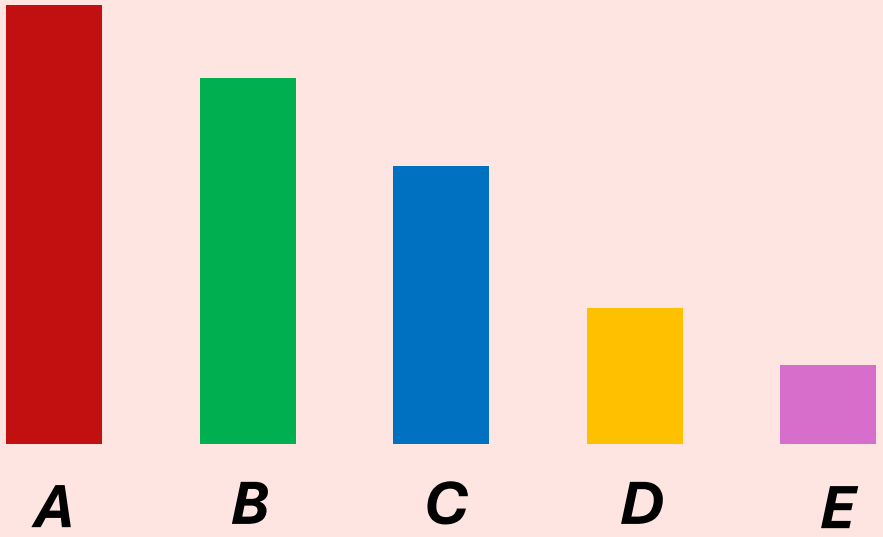
Seats are allocated to parties **proportionally to their scores**.

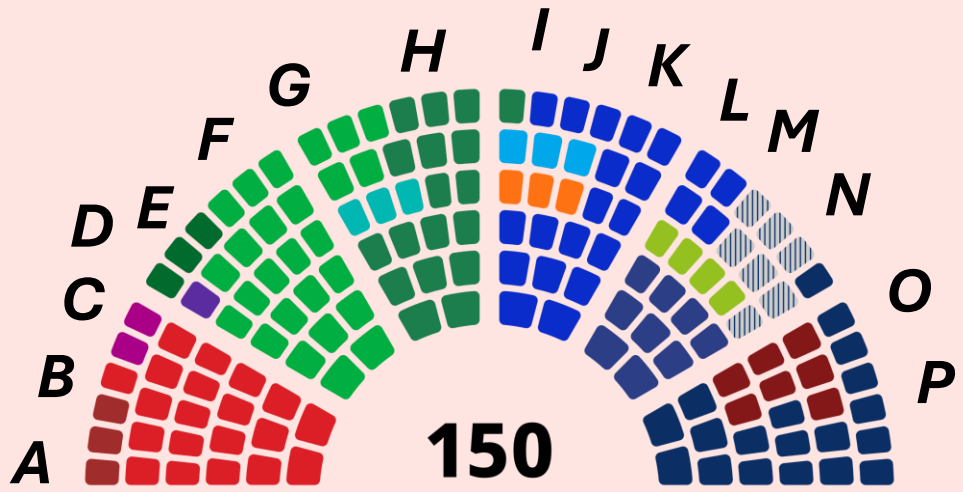


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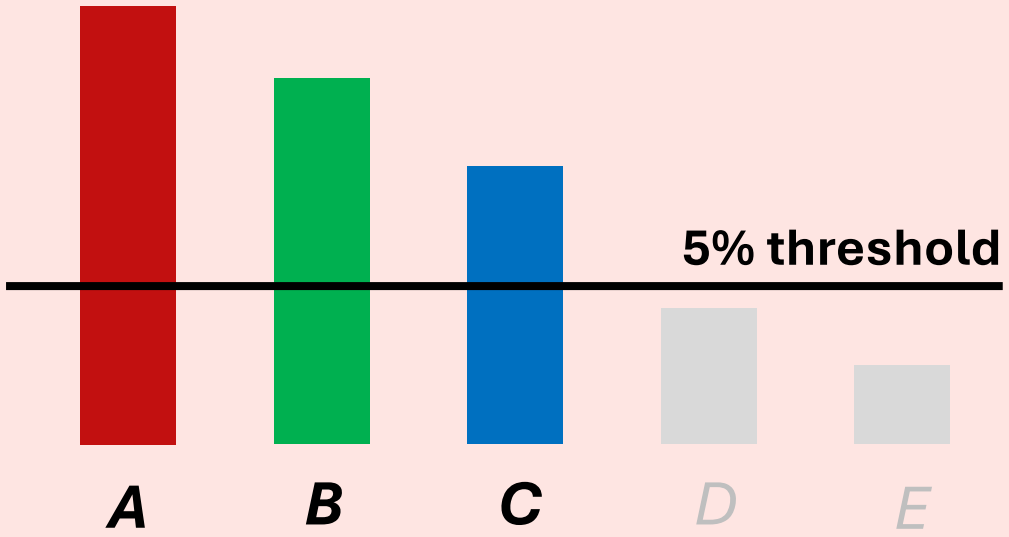


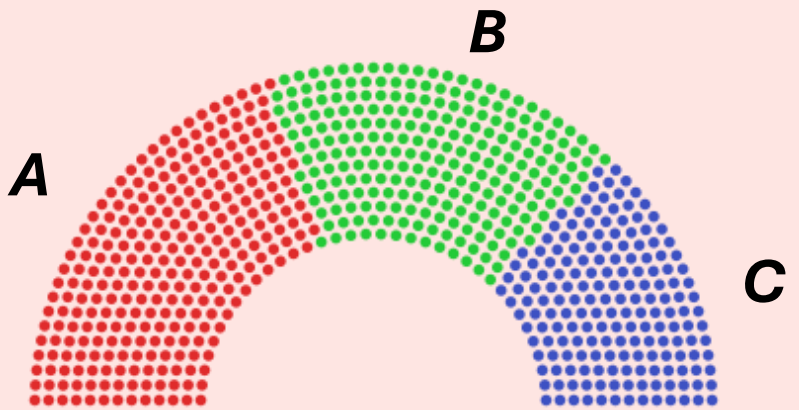


Problem: possible political fragmentation (many parties get a seat).

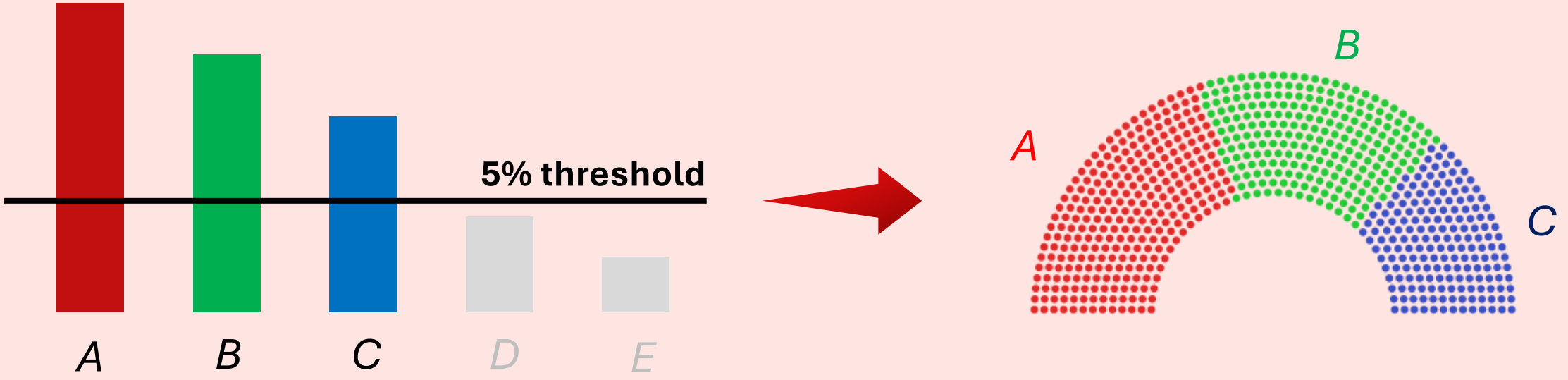
Proportional representation with *threshold*

Many countries impose an **electoral threshold** to reduce political fragmentation.





Proportional representation with *threshold*



- **Some votes are "lost":** *D* and *E* supporters have no influence on the seat distribution.
- This incentivizes forms of **tactical voting.**

The “lost” votes



2019 election of the French *representatives to the EU Parliament*.



2025 election of the *Bundestag* members.



2002 election of the *Turkish Parliament* members.

Threshold	*“Lost” votes*
5%	20%
5%	14%
10%	46%

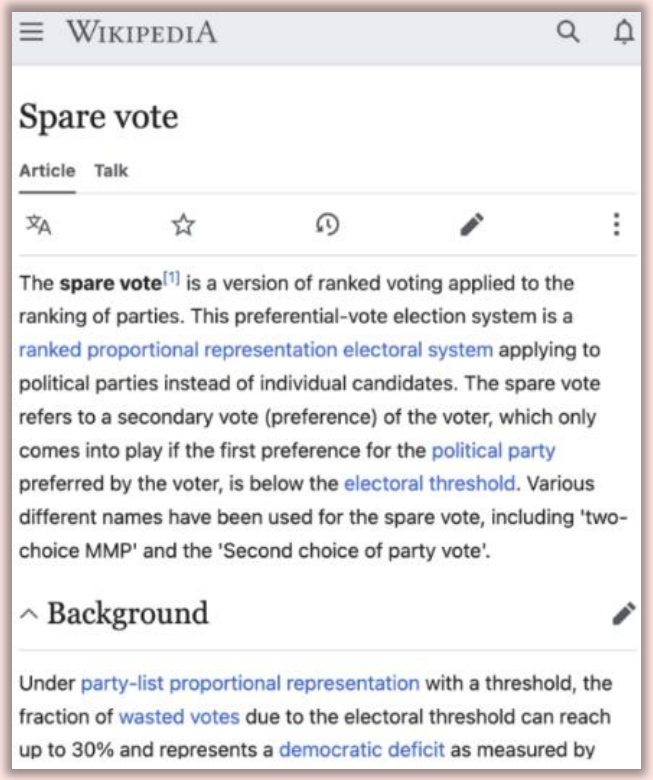
increasing in recent decades

Idea: a *replacement* vote

We could let voters **indicate a second choice** to be used in case their first choice does not reach the threshold.

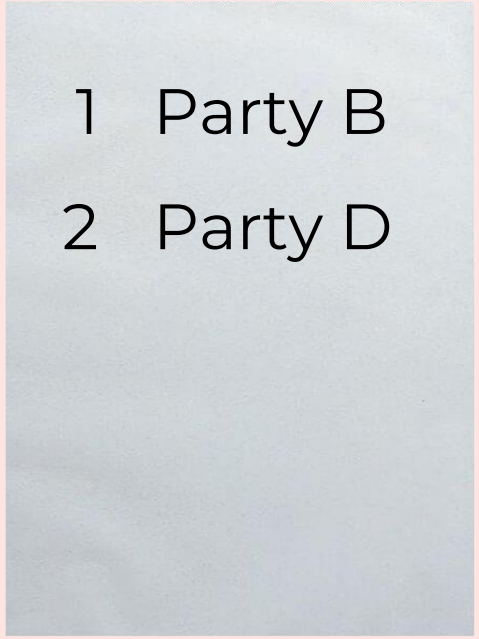




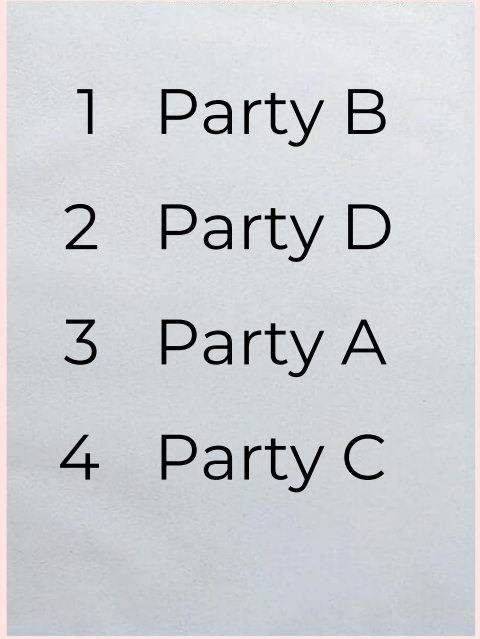


Idea: a *replacement* vote

We could ask voters to rank **two parties**

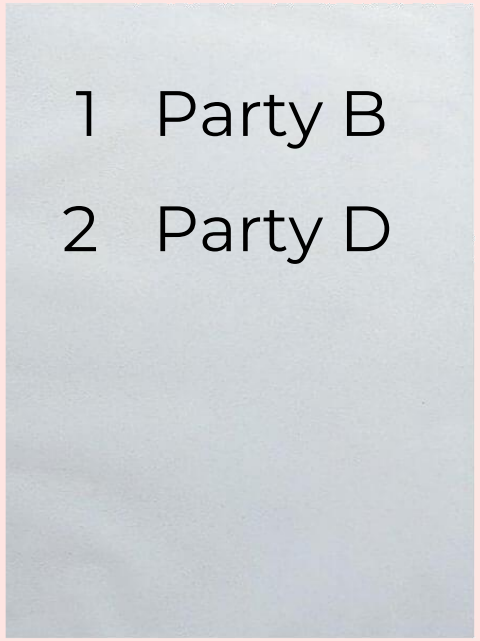


We could even ask for a **truncated ranking**

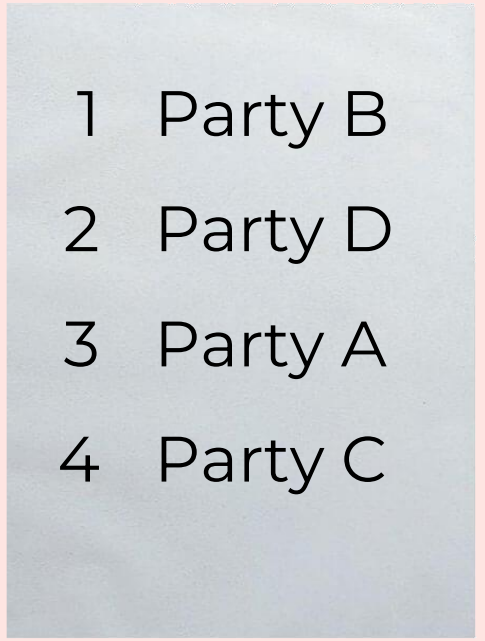


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Question: how to decide which parties are “above the threshold”?

A not-so-easy problem

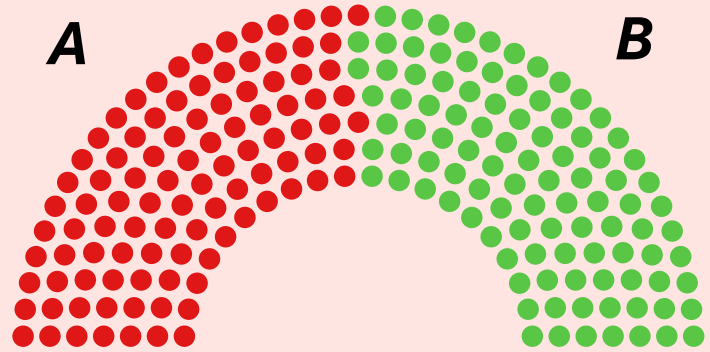
47% A

47% B

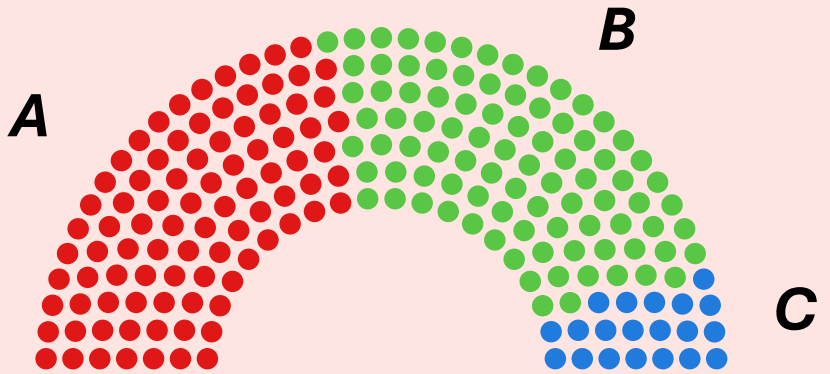
4% $C \succ D$

2% $D \succ C$

5% threshold



or



The formal model

We have:

- Set of **voters** $\boldsymbol{V = \{1, 2, \dots, n\}}$ and **parties** $\boldsymbol{C = \{p_1, \dots, p_m\}}$.
- A preference **profile** $\boldsymbol{P = (\succ_1, \dots, \succ_n)}$ of truncated rankings of voters over parties.
- A given **threshold** τ (absolute number of voters).

We want:

- A set of selected partis $S \subseteq C$, called the **outcome**.
- Voters are **represented** by their most-preferred party in S (if any).
- An outcome S is **feasible** if every party represents $\geq \tau$ voters.
- We assume that parties in S get **a number of seats proportional** to the **share** of voters they represent.

Feasible outcomes

- **Profile** *P*:

$6 \times \quad \boldsymbol{A} \succ B \succ D \succ C$

$4 \times \quad \boldsymbol{B} \succ C \succ E \succ A \succ D$

$3 \times \quad \boldsymbol{C} \succ B$

$3 \times \quad D \succ E \succ \boldsymbol{B} \succ A \succ C$

$2 \times \quad \boldsymbol{C} \succ A \succ E$

- **Threshold** $\tau = 5$.

Outcome $\{A, B, C\}$ is feasible.

Feasible outcomes

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$2 \times \quad C \succ A \succ E$

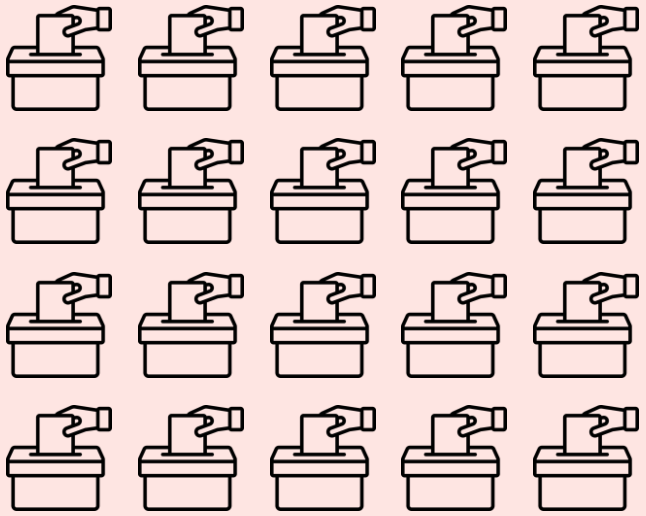
- **Threshold** $\tau = 5$.

✓ Outcome $\{A, B, C\}$ is feasible.

✗ Outcome $\{B, D\}$ is not feasible.

Summary of the problem

Profile P



Threshold τ

Party Selection Rule f

Feasible Outcome

$$S \subseteq C$$

- 1 -
Party Selection Rules

Rule 1: Direct Winners Only (DO)

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The selected parties are all those which receive more first-place votes than required by the threshold.

35% $A \succ C$

30% B

20% $C \succ E \succ A$

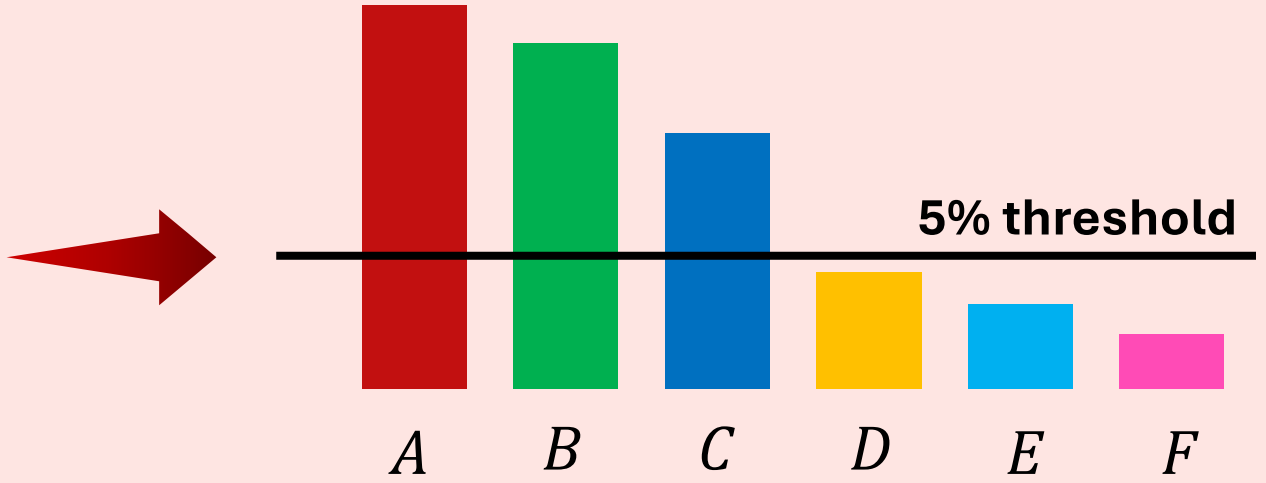
6% $C \succ A \succ B$

3% $D \succ E \succ F \succ C$

3% $E \succ F \succ A$

2% $F \succ E \succ D$

1% $D \succ F \succ B$



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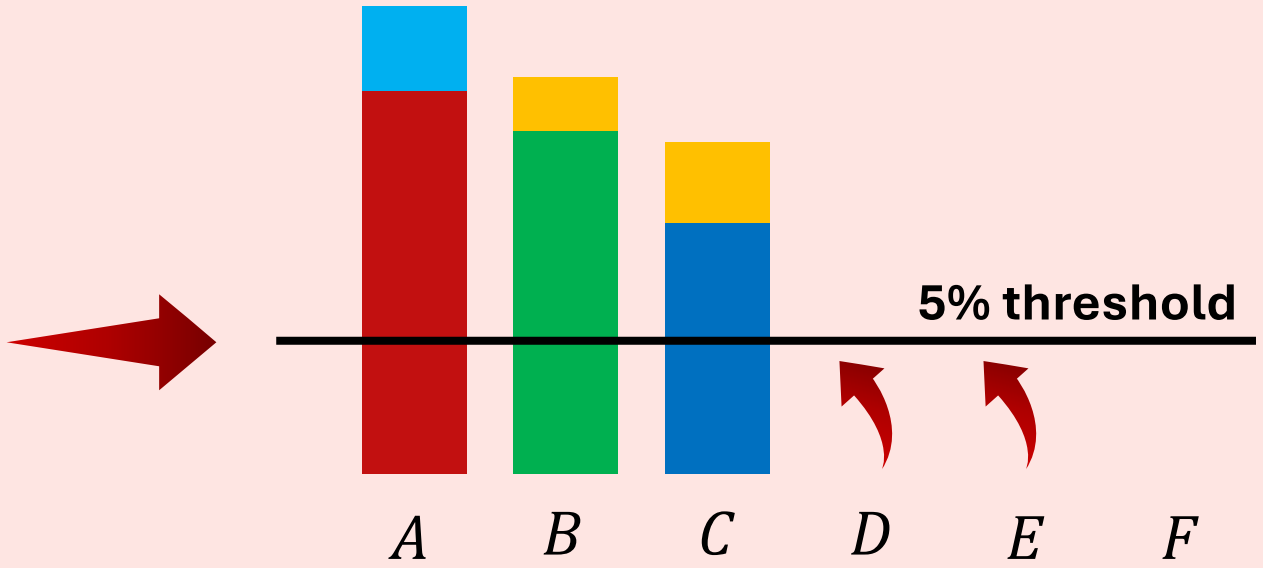
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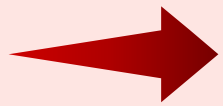
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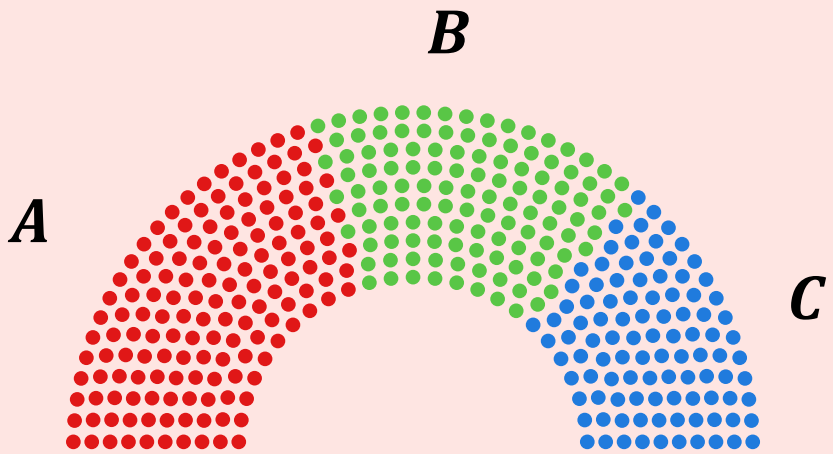
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Rule 2: Single Transferable Vote (STV)

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Parties that receive the fewest votes are successively eliminated until all parties receive more votes than required by the threshold.

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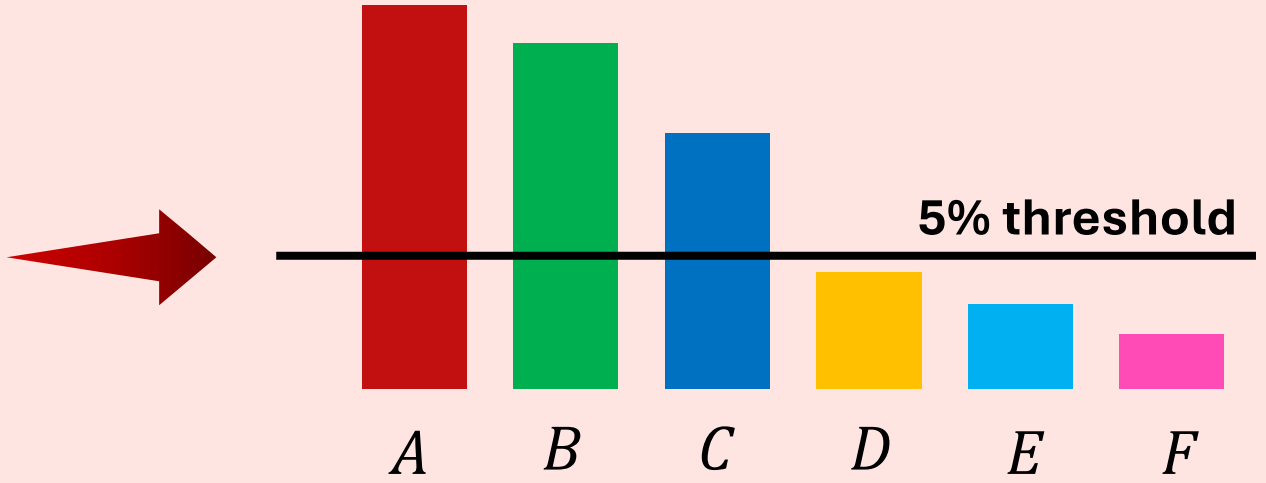
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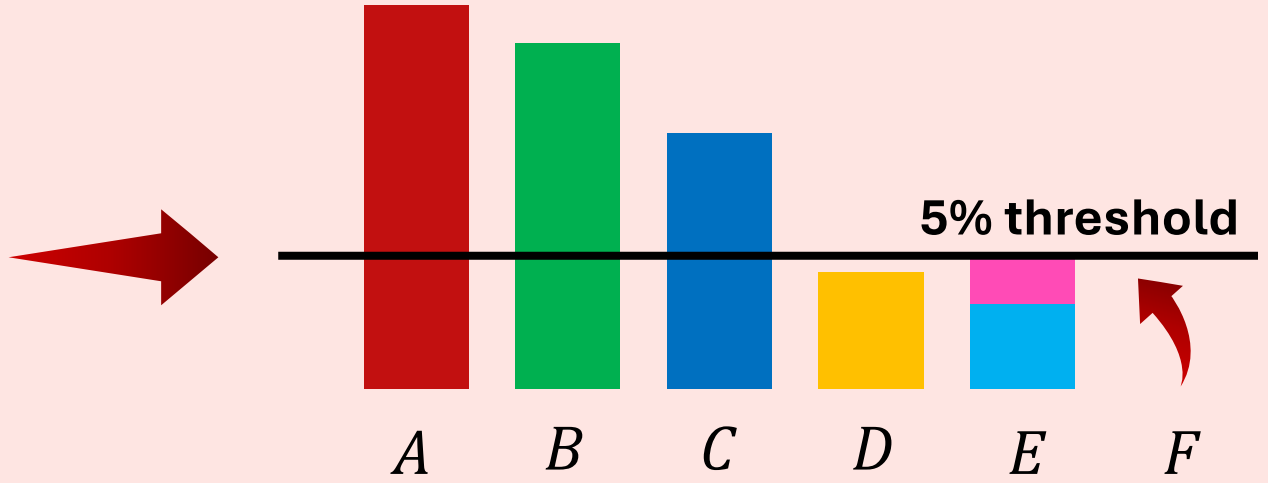
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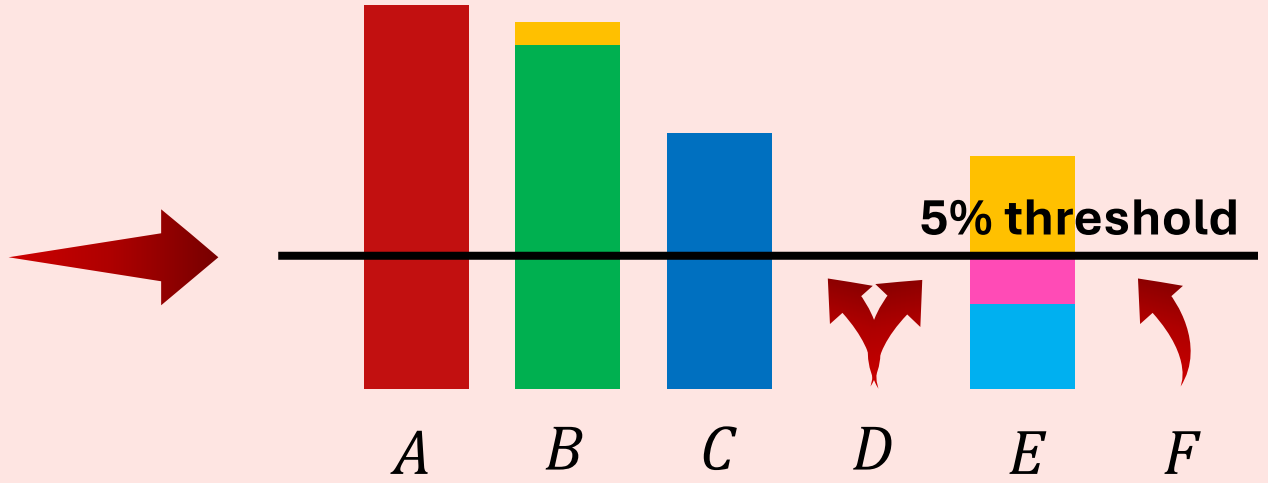
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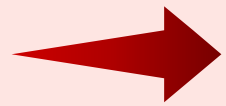
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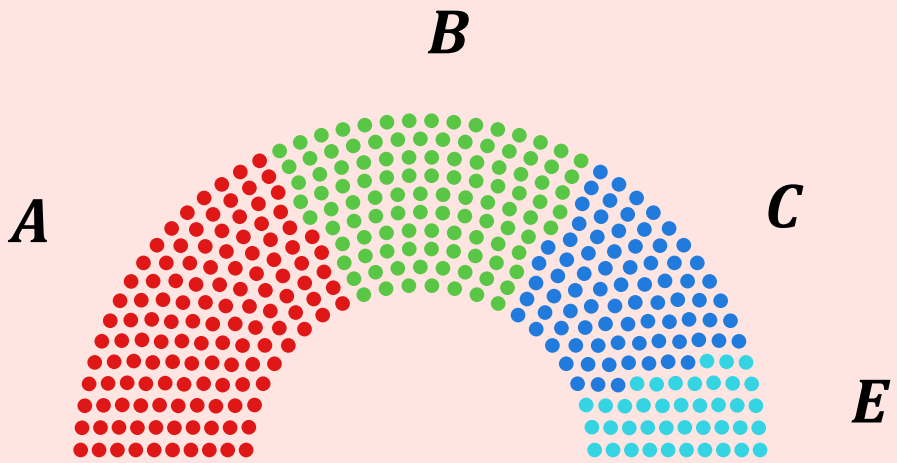
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Rule 3: Greedy Plurality (GP)

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Parties are considered in decreasing order of plurality score, and added to the outcome if it remains feasible.

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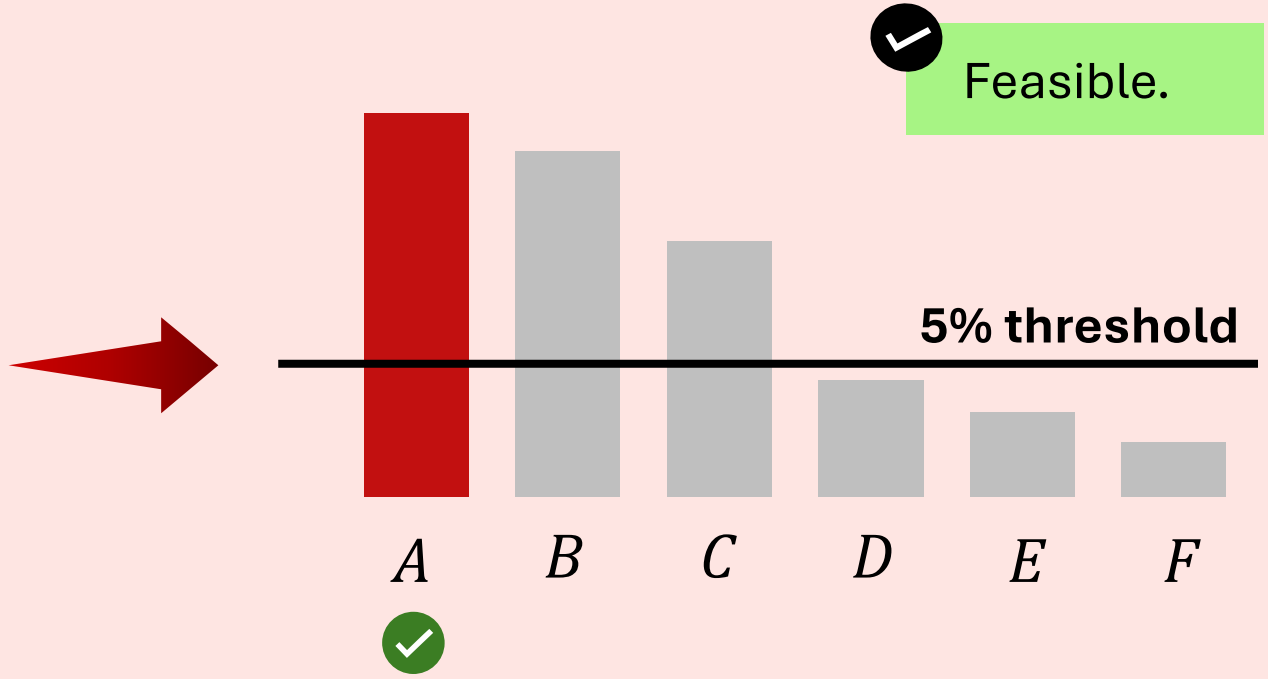
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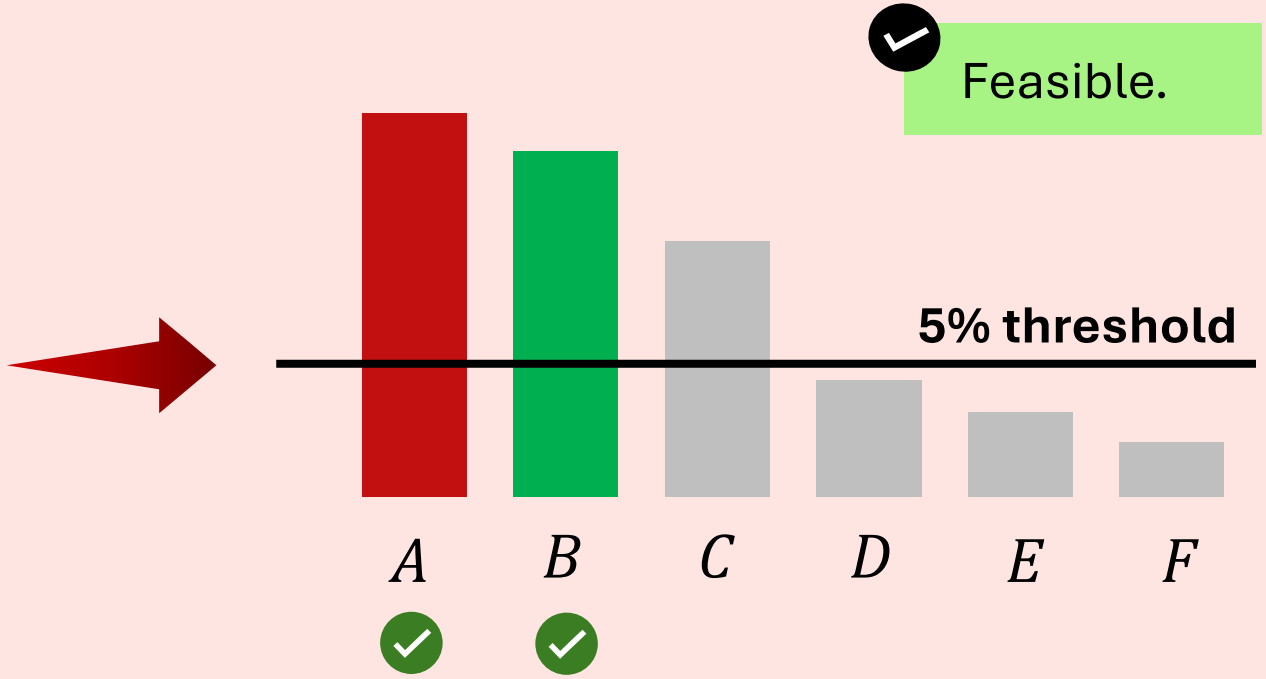
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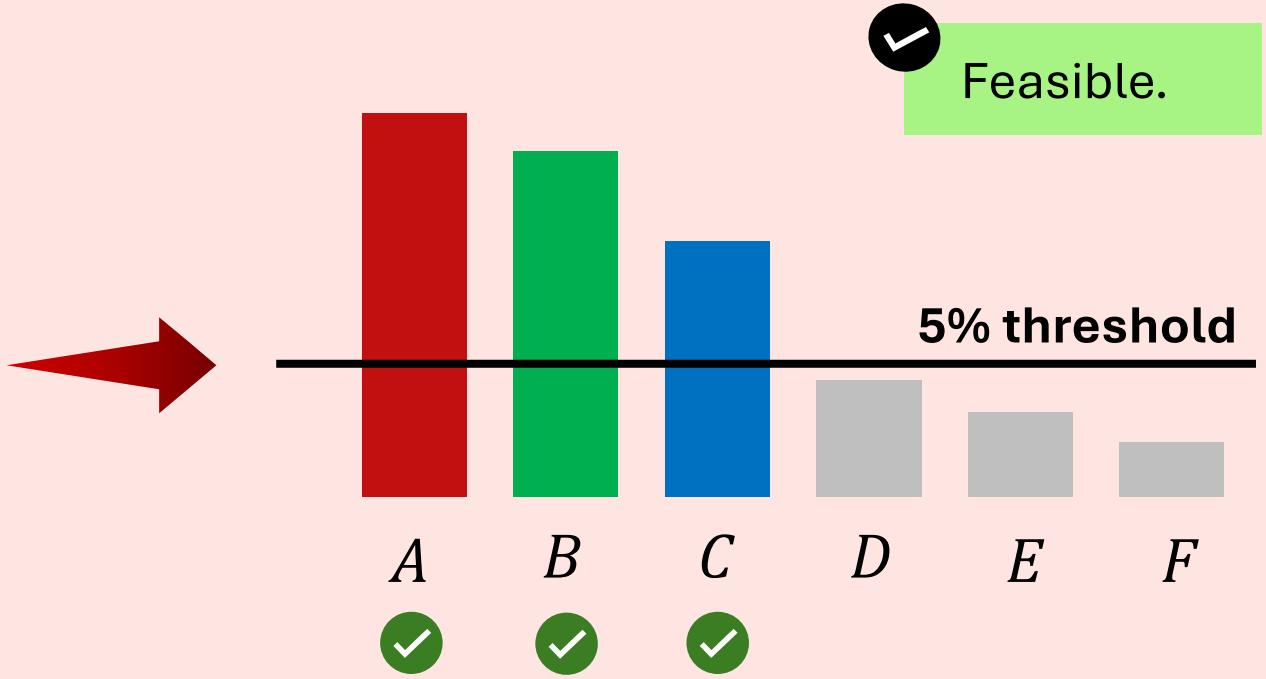
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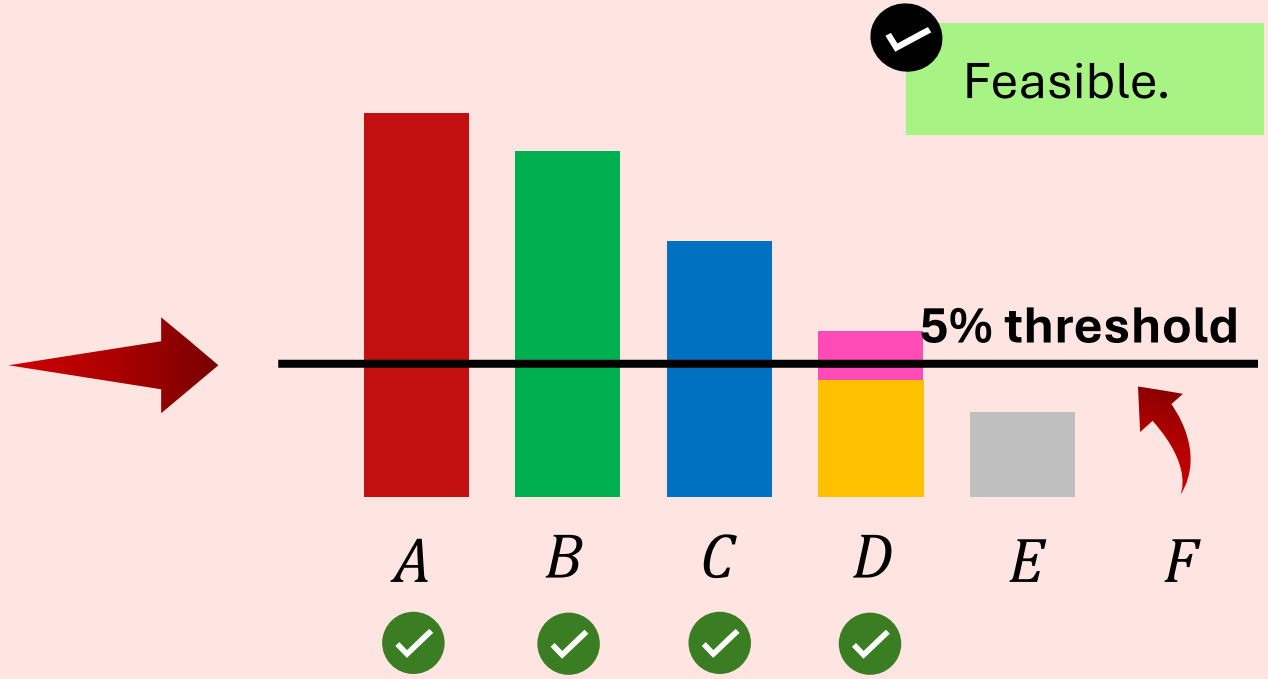
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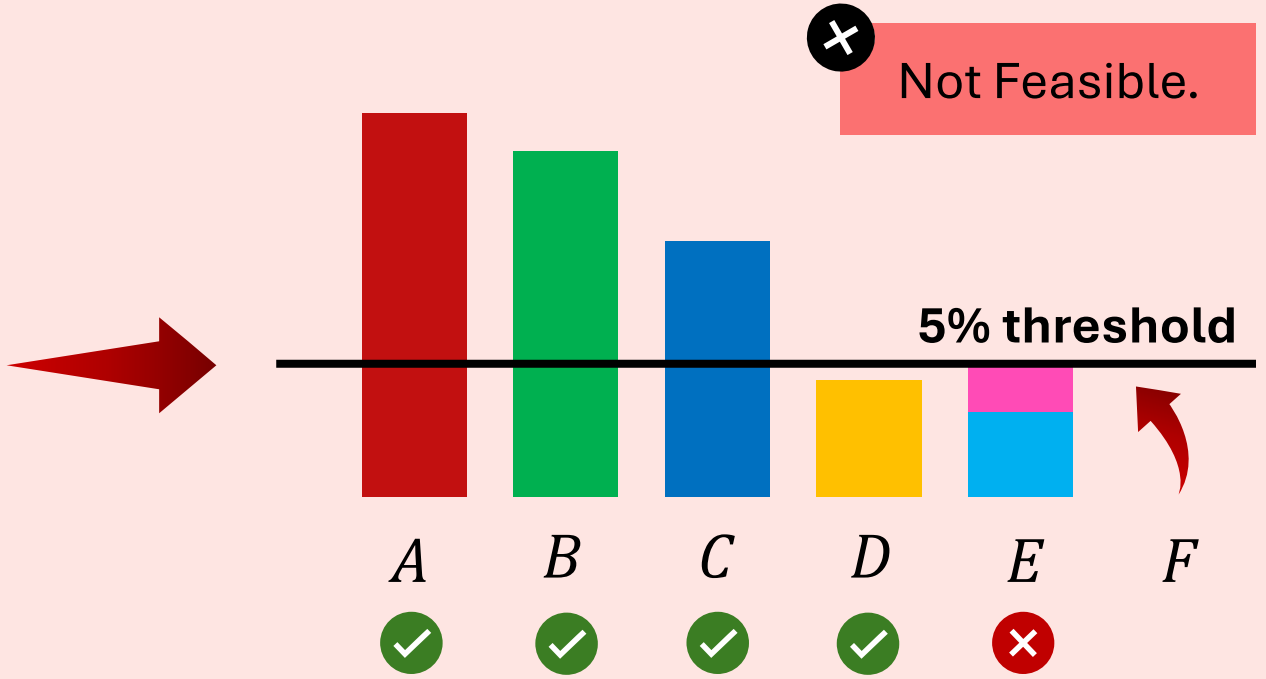
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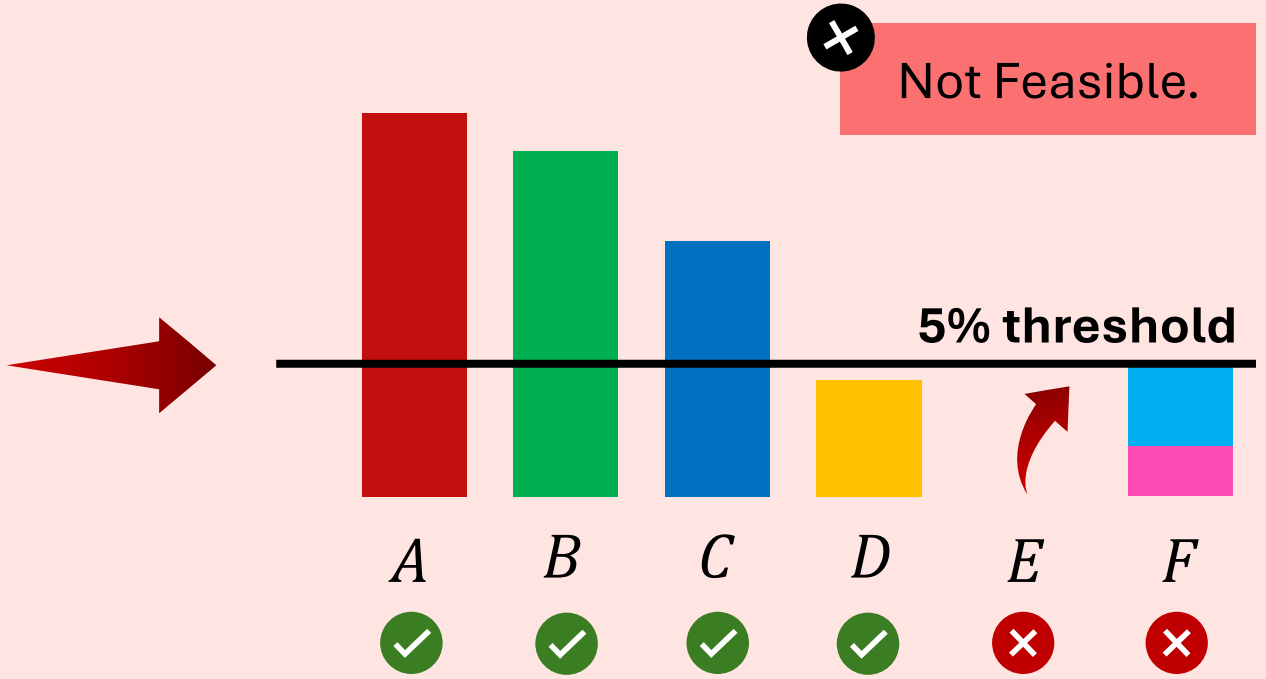
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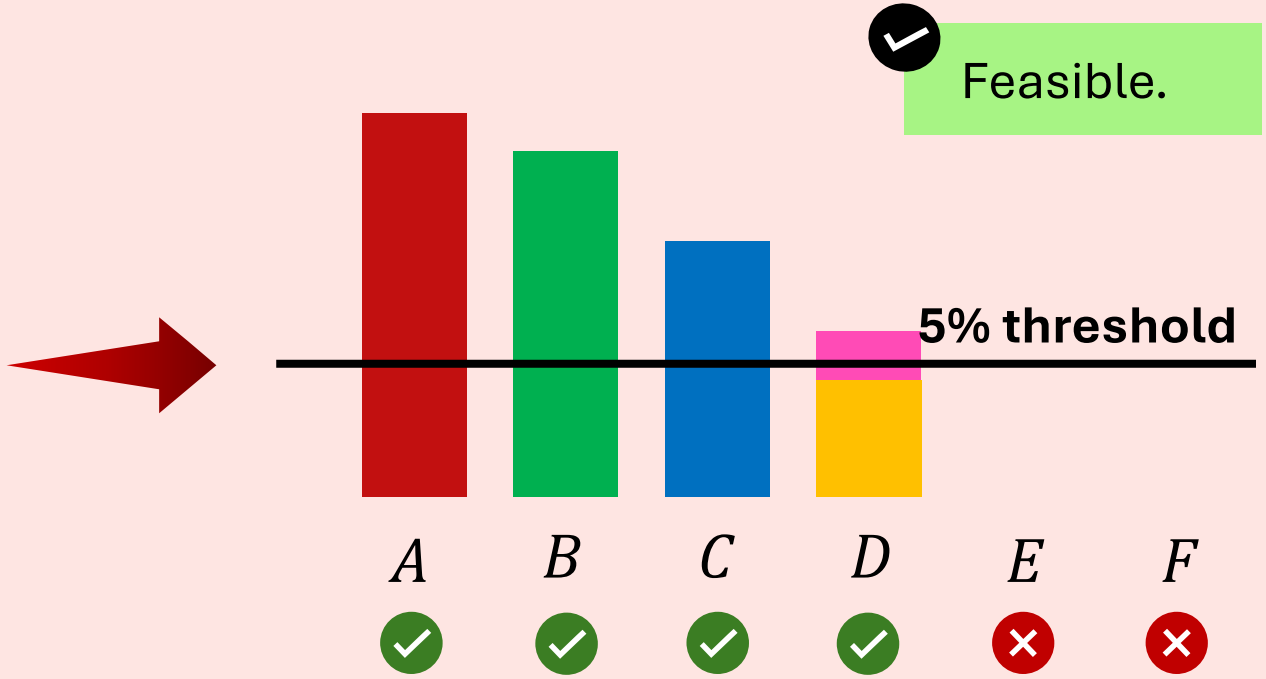
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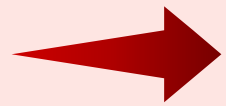
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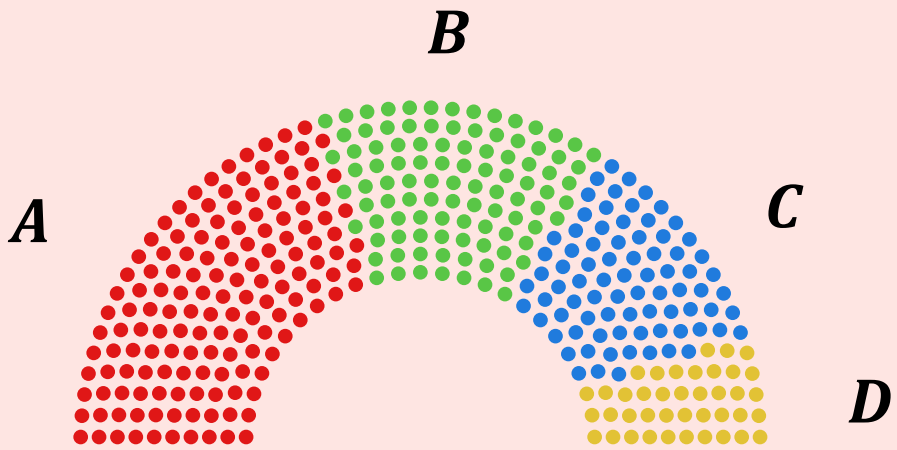
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- 3 -
Axiomatic Analysis

Inclusion of Direct Winners

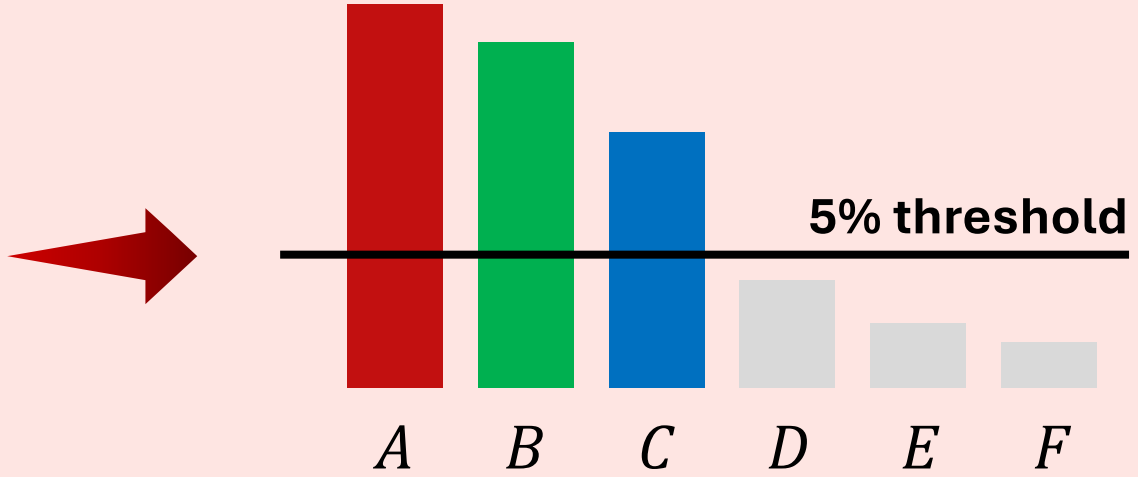
Axiom: Inclusion of Direct Winners

If τ voters or more rank a party x on top of their rankings, this party should be selected.

Satisfy

DO
STV
GP

35%	$A \succ C$	**4%**	$D \succ B \succ A$
30%	B	**3%**	$E \succ F \succ A$
26%	$C \succ E \succ A$	**2%**	$F \succ E \succ D$



Representation of Solid Coalitions

Axiom: Representation of solid coalitions

If τ voters or more rank **a set** of parties T on top of their rankings, at least one of these parties should be selected.

Satisfy	Break
STV	**DO** **GP**

Inspired by *Proportionality for Solid Coalitions* **[Dummet 94]**

35% $A \succ C$

30% B

26% $C \succ E \succ A$

4% $D \succ B \succ A$

3% $E \succ F \succ A$

2% $F \succ E \succ D$

= 5%

Either
E* or *F should be selected

Varying the Threshold

Axiom: Threshold Monotonicity

If a party is selected for threshold τ, then it is also selected for threshold $\tau' < \tau$.

Satisfy	Break
DO STV	GP

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If a party is selected for threshold τ, then it is also selected for threshold $\tau' < \tau$.

Satisfy	Break
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Axiom: Independence of Definitely Losing Parties

Once some parties are losing at some threshold τ, then for all larger thresholds $\tau' > \tau$, the rule should behave **as if none of the losing parties had been available**.

Satisfy	Break
STV	DO GP

Varying the Threshold

Characterization Theorem: **STV** is the only party selection rule that satisfies *inclusion of direct winners* and *independence of definitely losing parties*.

Reinforcement

Axiom: Reinforcement for Winning Parties

If a party is selected for profile P_1 with threshold τ_1 and for profile P_2 with threshold τ_2, then it should be selected for profile $P_1 + P_2$ with threshold $\tau_1 + \tau_2$.

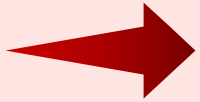
✓ Satisfy	✗ Break
DO	**STV** **GP**

Characterization Theorem: **DO** is the only party selection rule that satisfies *inclusion of direct winners* and *reinforcement for winning parties*.

Incentive Issues

Axiom: Representative-strategyproofness

Voters cannot cause a party to be selected that they prefer to all currently selected parties by misreporting their preferences.



No hope for strategyproofness in general (Gibbard-Satterthwaite applies).

We thus consider scenarios in which **we require additional conditions** on the parties or on the possible manipulations.

Incentive Issues



Proposition: **GP** satisfies representative-strategyproofness when there is *at most one risky party* from the perspective of each voter. (DO and STV do not.)

Summary of the Axiomatic Analysis

	DO	STV	GP
Set-Maximality			✓
Inclusion of Direct Winners	✓	✓	✓
Representation of Solid Coalitions		✓	
Threshold Monotonicity	✓	✓	
Ind. of Definitely Losing Parties		✓*	
Ind. of Clones		✓	
Reinforcement for Winning Parties	✓*		
Monotonicity	✓		
Rep-SP (one risky party)			✓
Share-SP (safe party first or second)	✓		
Share-SP (rep. ranked first)	✓		✓

- 4 -
Empirical Analysis

Context of the Experiment

To collect appropriate preference data, **we ran a voting experiment** during the 2024 election of the French representative to the EU parliament.

Candidate parties: 38
Threshold: 5%
Parties above the threshold: 7
Lost votes: 12.1%



Experimental Setup

1. Explanation of the issues caused by the threshold.
2. Presentation of the candidate lists.
3. **Vote with alternative voting methods.**
4. Questionnaire.

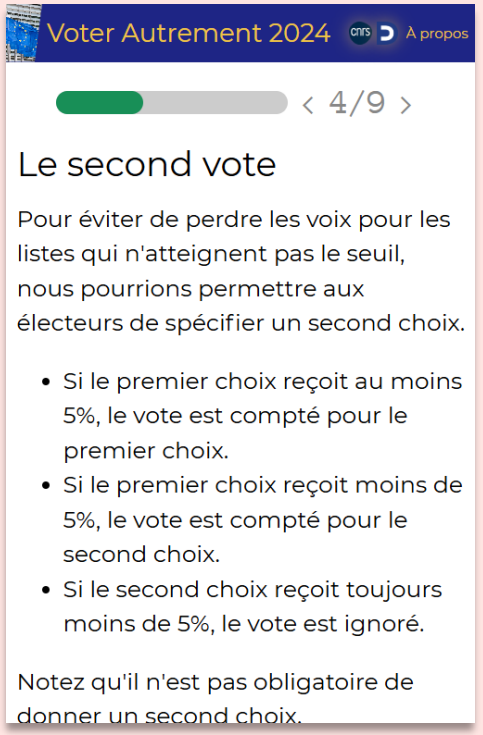


Fig. Screenshot of the website of the experiment conducted during the 2024 election of the French representative to the EU Parliament.

Two Samples of Participants

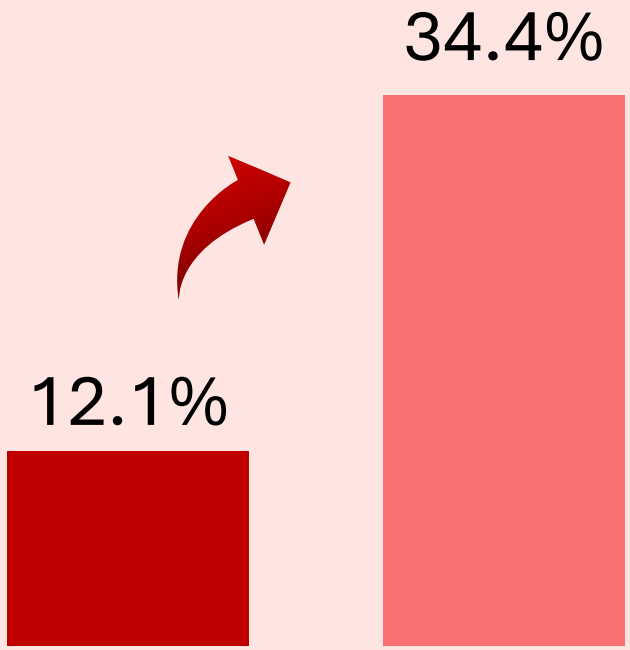
1 Self-selected sample

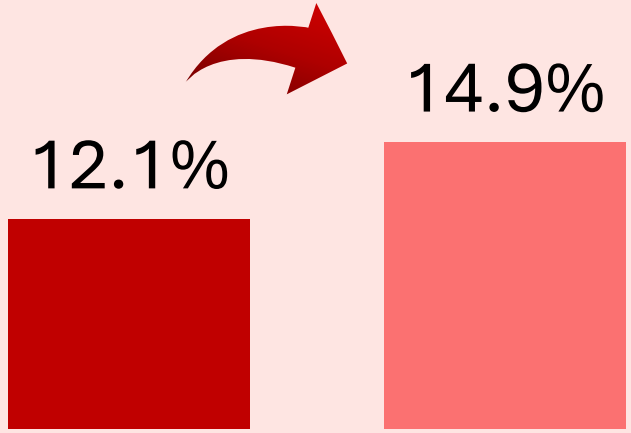
- 3046 participants in a week.
- Recruited through social media, unpaid.
- Overrepresentation of left-wing, young and educated people.

2 Representative sample

- 1000 participants.
- Recruited via a polling institute and paid a fixed amount to participate.
- Representative of the French population.

Observation 1: More votes for “small parties”





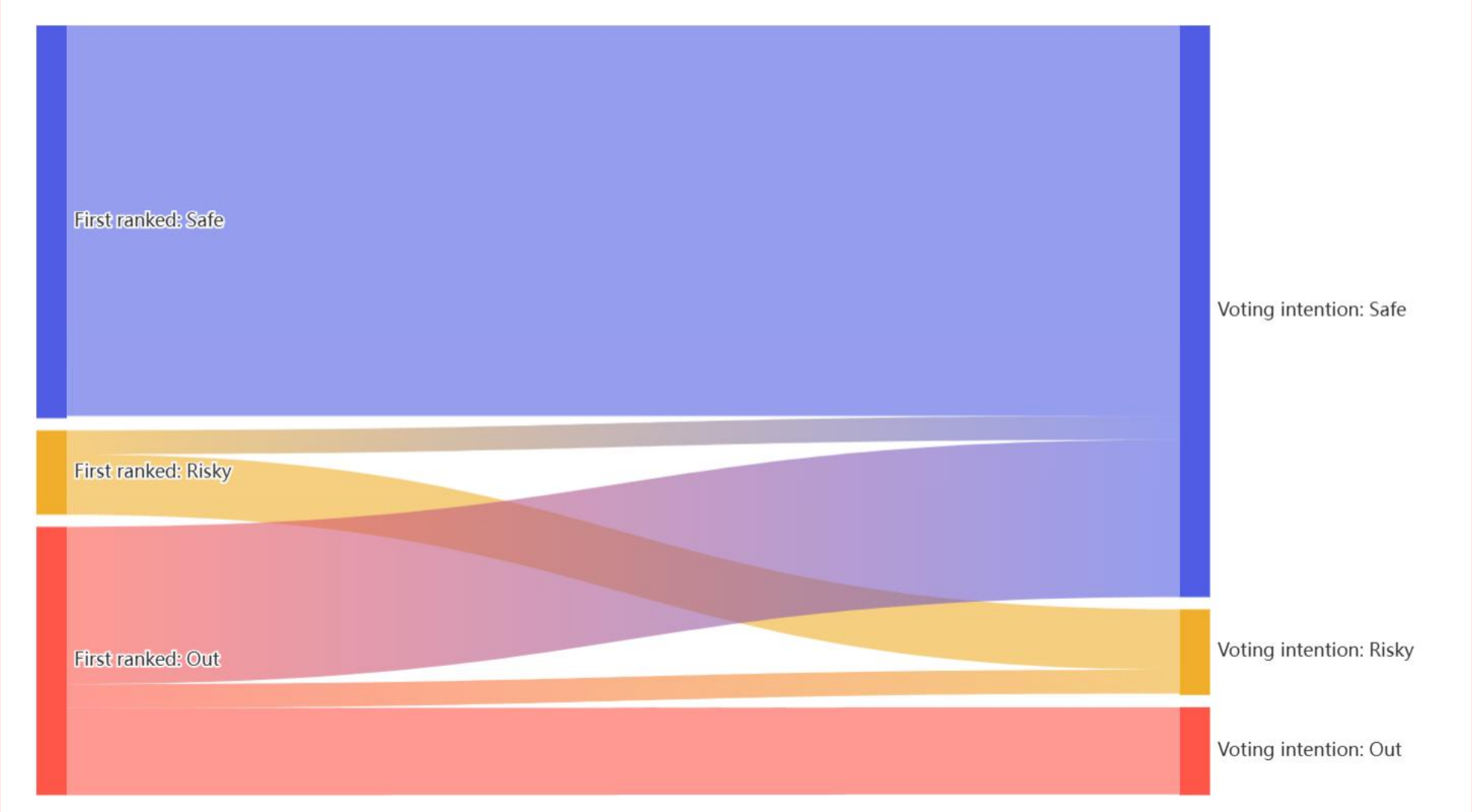
1 *Self-selected* sample

2 *Representative* sample

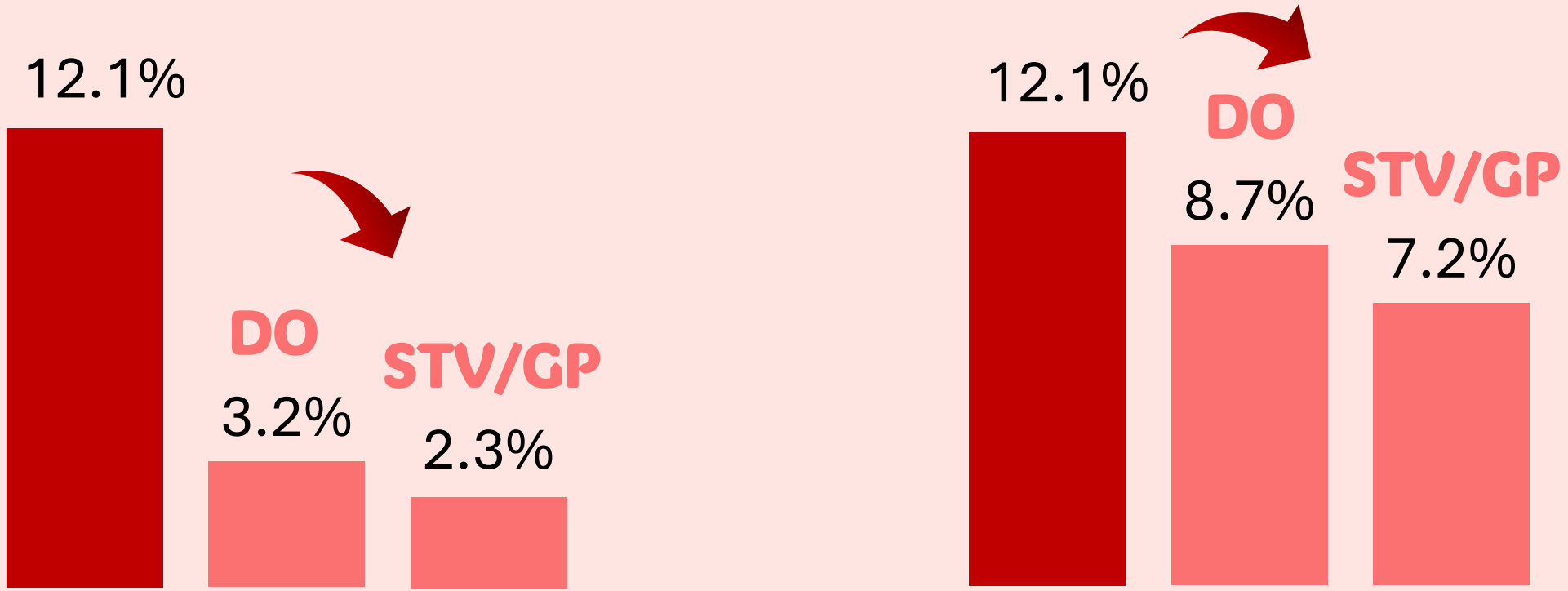
Share of voters that vote for a “small” party in the actual election.

Share of voters that put a “small” party first in their ranking.

Observation 1: More votes for “small parties”



Observation 2: Less “lost votes”



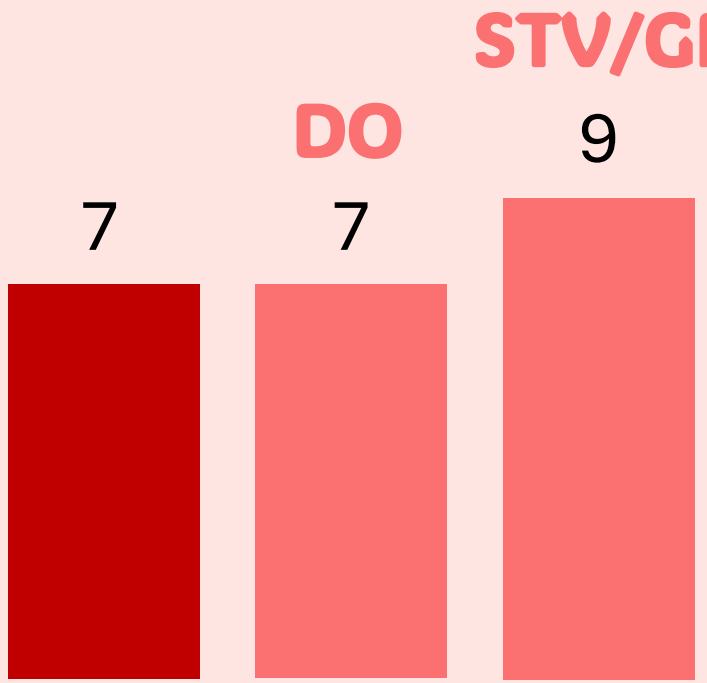
1 *Self-selected* sample

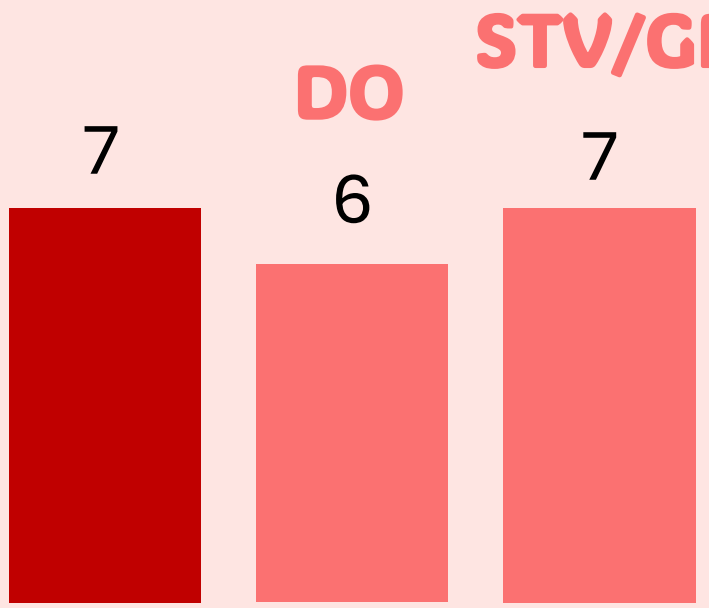
2 *Representative* sample

Share of unrepresented voters in the actual election.

Share of unrepresented voters with ranking-based rules.

Observation 3: No political fragmentation





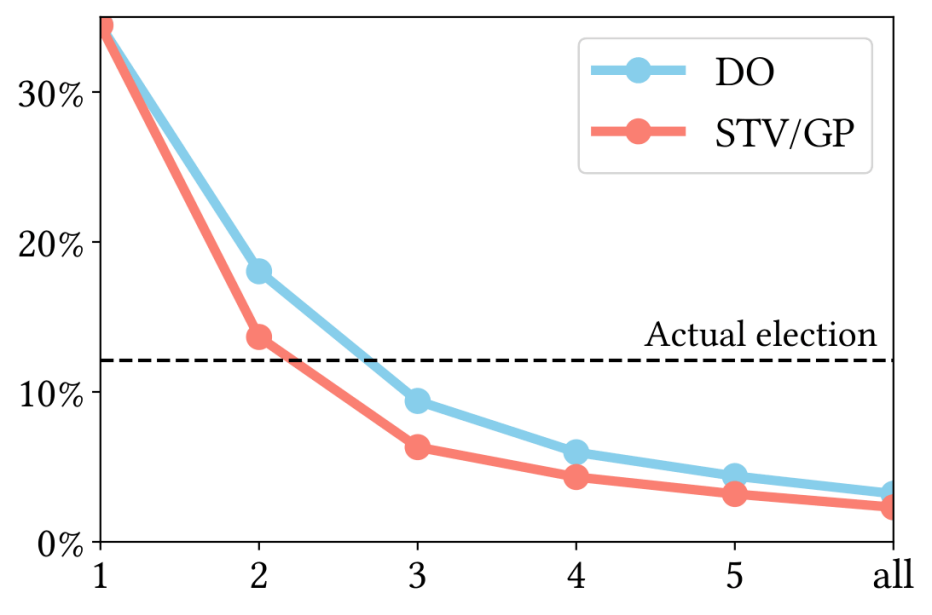
1 *Self-selected* sample

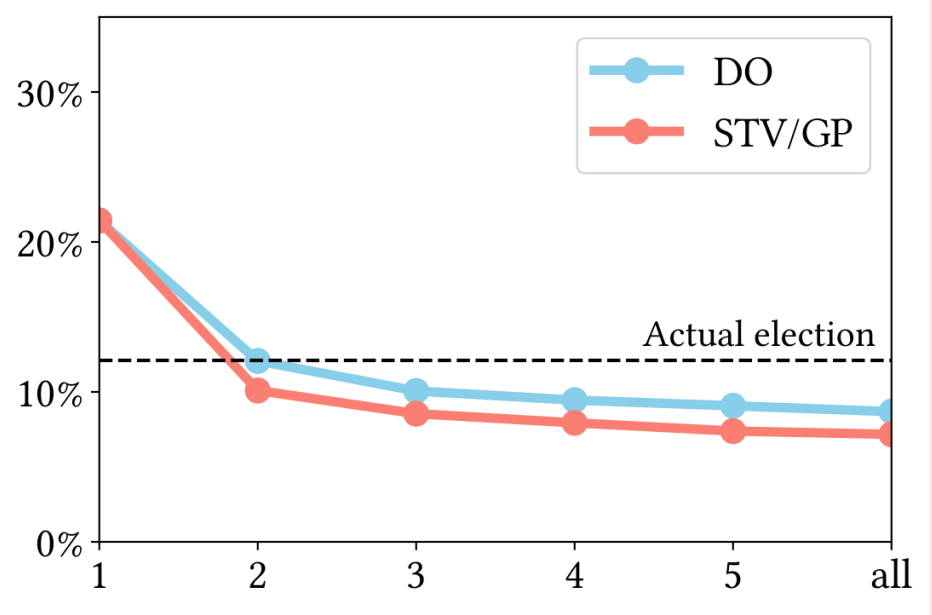
2 *Representative* sample

Number of parties receiving a seat in the actual election.

Number of parties receiving a sear with ranking-based rules.

Observation 4: We can ask for short rankings





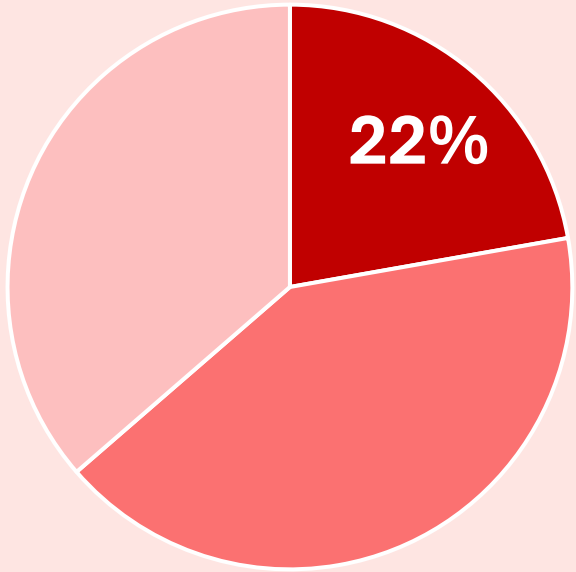
1 *Self-selected* sample

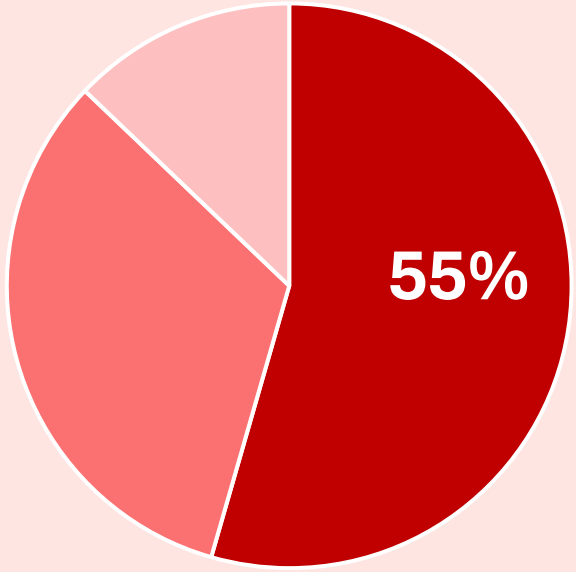
2 *Representative* sample

Share of unrepresented voters if all rankings are **truncated** to rank k (x-axis).

Opinions of the participants

“Which system do you think is better suited for the election of your representatives to the EU parliament?”





1 *Self-selected* sample

2 *Representative* sample

Current system

Two choices

Truncated ranking

- / -
Conclusion ?

Conclusion

We axiomatically and empirically studied rules for electing parliaments with electoral thresholds.

Main takeaway: We can significantly increase representativeness by allowing voters to **rank parties**.

- STV and GP leave **fewer voters unrepresented** than DO.
- DO and GP have **stronger strategyproofness guarantees** than STV.
- STV satisfies **independence of clones and represents solid coalitions**.

Conclusion

We axiomatically and empirically studied rules for electing parliaments with electoral thresholds.

Main takeaway: We can significantly increase representativeness by allowing voters to **rank parties**.

Questions?

- / -
Appendix

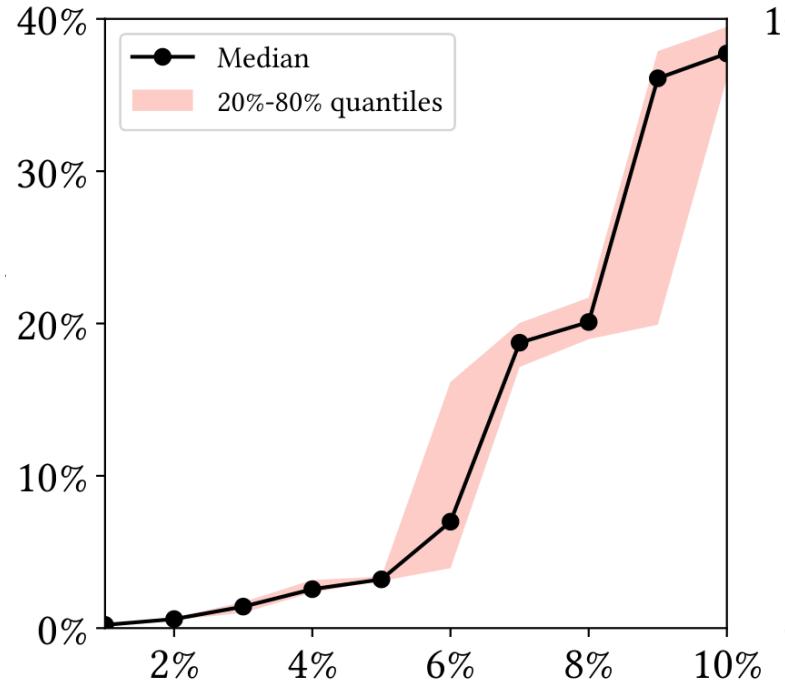
Two Samples of Participants

We assign weights to the voters to reduce the biases: weights are selected based on the vote of the participant at the actual election, to match the share of votes received by each party.

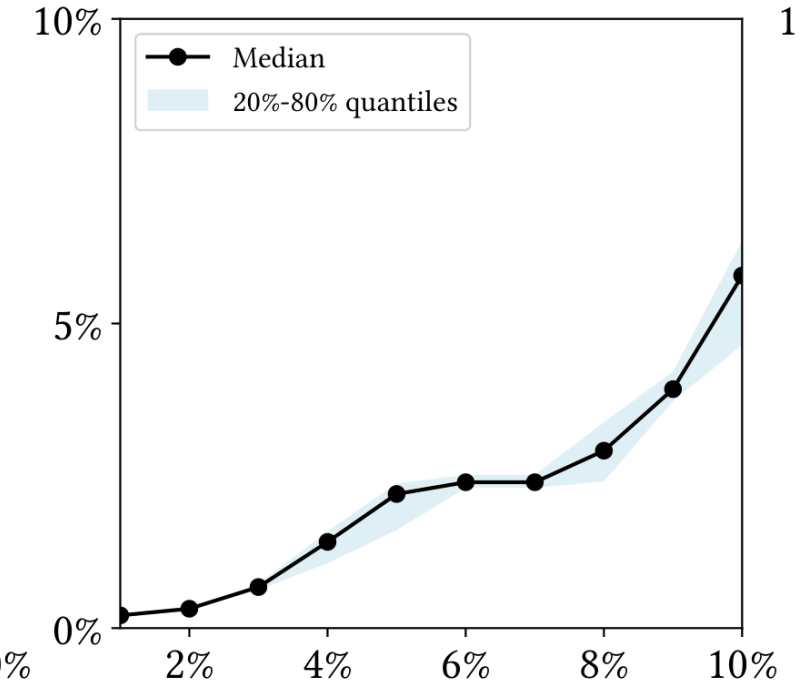
Example: if 5% of participants said they voted RN in the experiment, but the RN got 20% of the votes, each of these participants **is assigned a weight of 4.**

Robustness tests

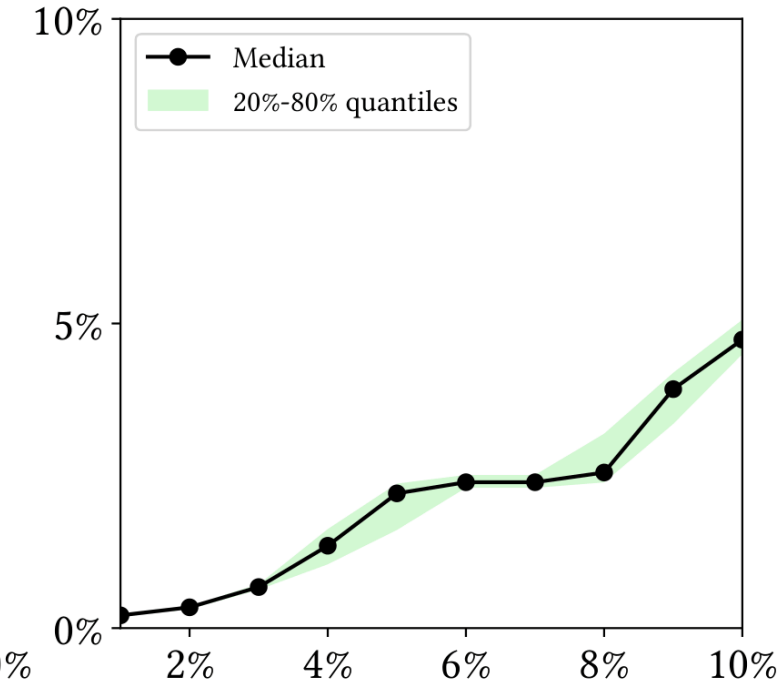
DO



STV



GP



Share of unrepresented voters with **different threshold values** and with **random noise** added to the preferences (*self-selected sample*).