Expressive Ballots for Voting Systems and Political Analysis

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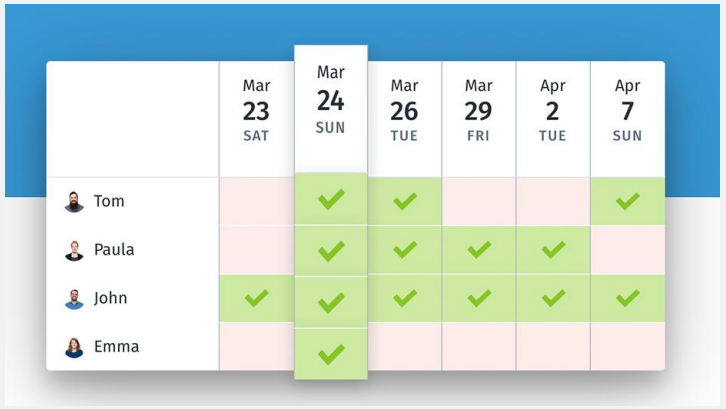
ILLC – University of Amsterdam

PhD Thesis supervised by *Jérôme Lang* and *Dominik Peters*

and conducted at the **Lamsade, Université Paris Dauphine - PSL**

Context: Voting and preferences

Political election



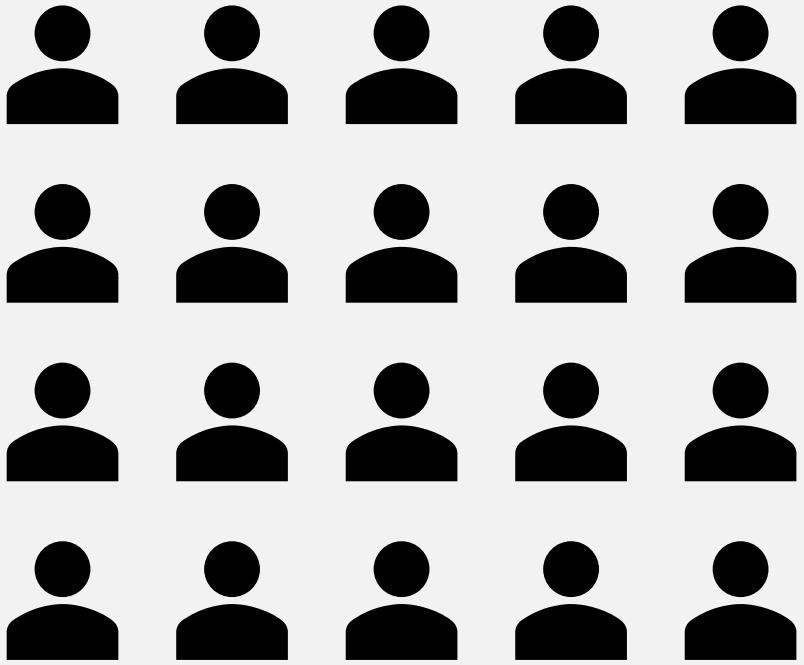
Decide on a date

Jury decision

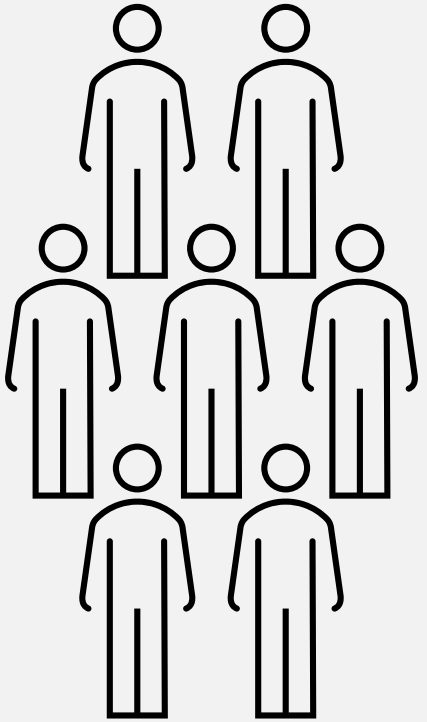
Choosing the pope

Voting systems

Voters

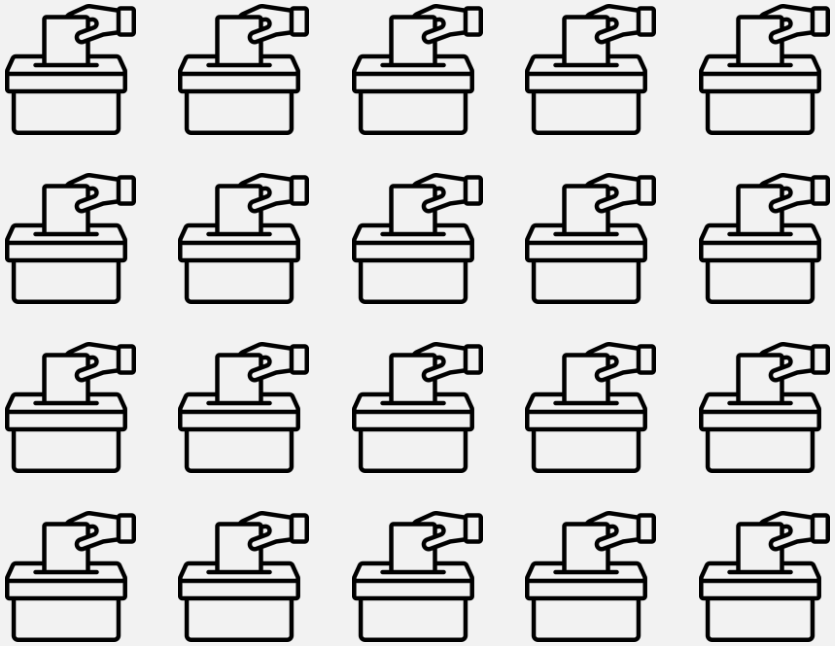


Candidates

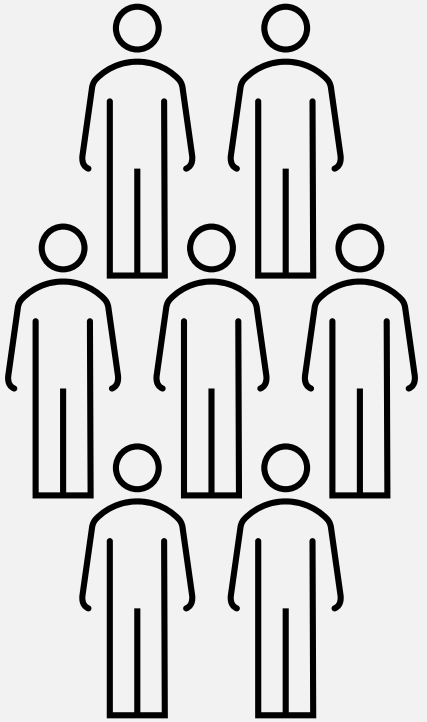


Voting systems

Voters give their preferences over candidates



Candidates



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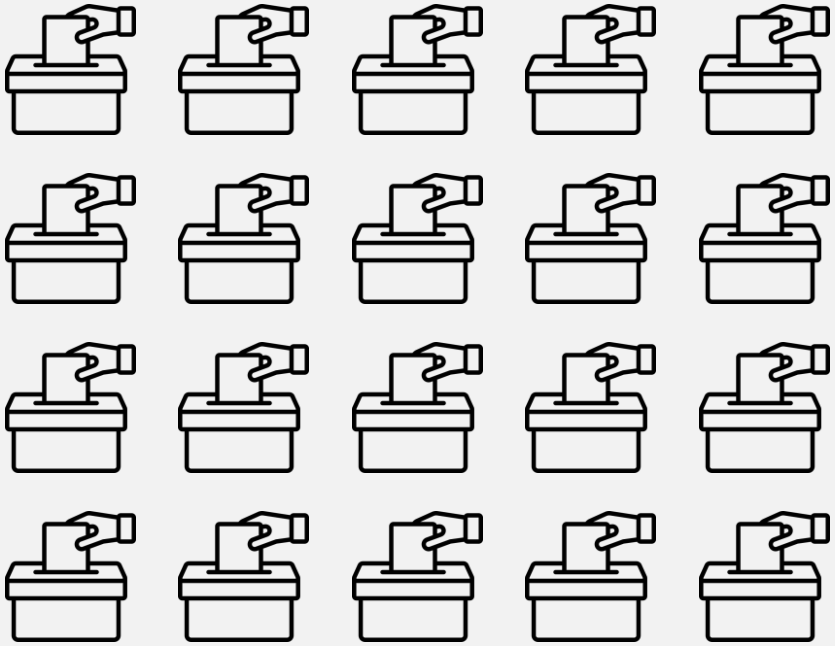
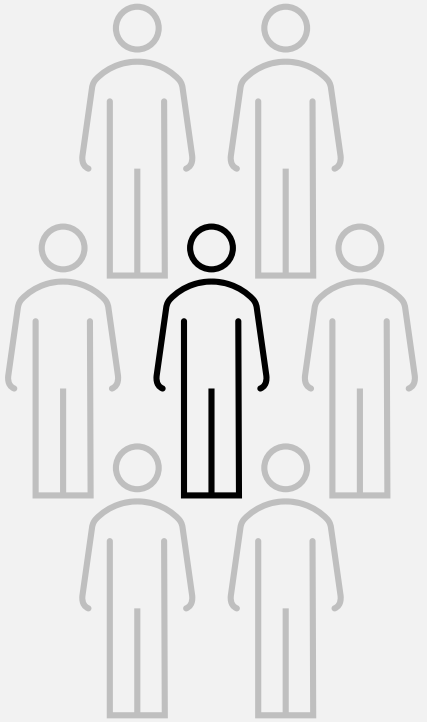
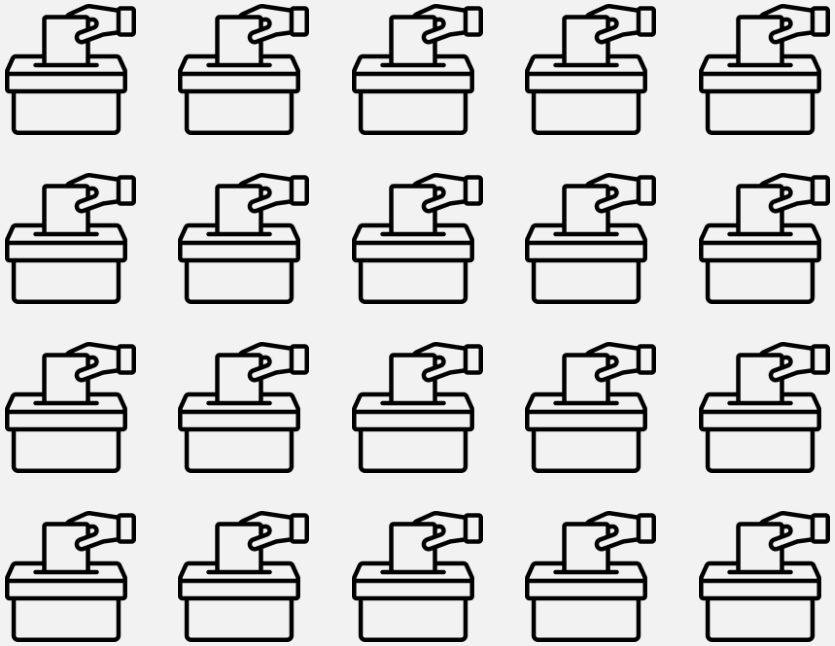


A **winner** is selected

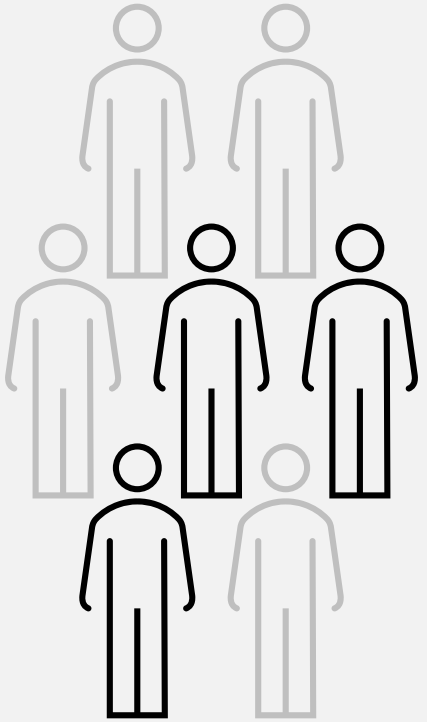


Voting systems

Voters give their preferences over candidates

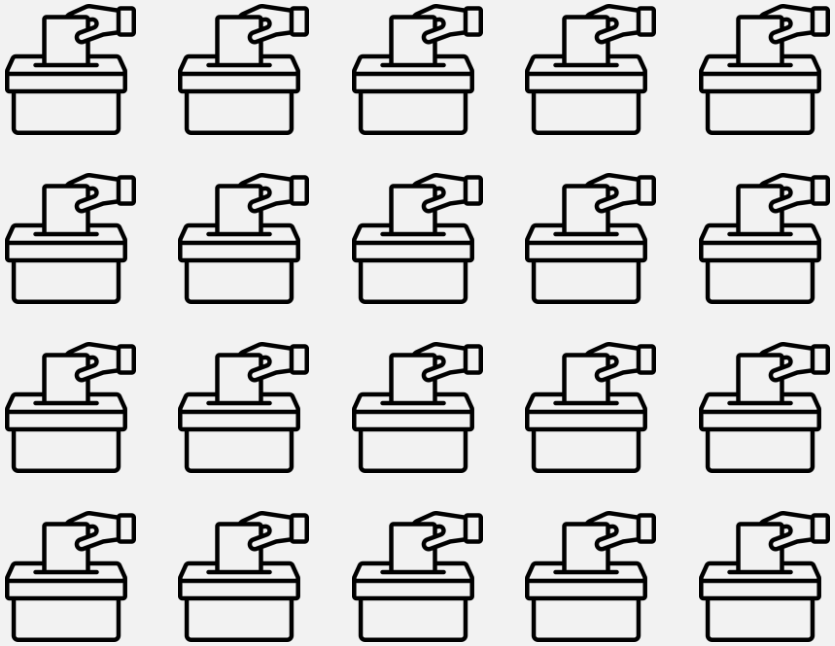


A **committee** is selected



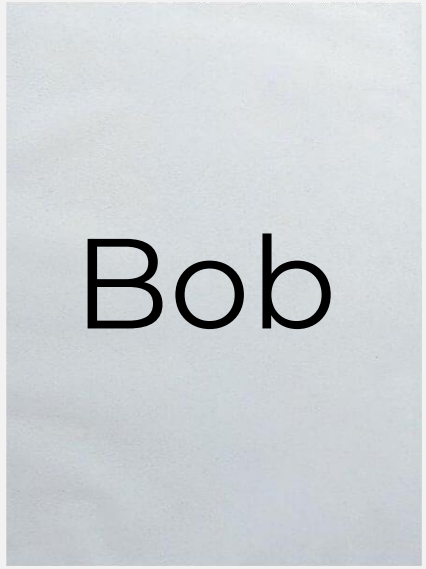
Voting systems

Voters give their preferences over candidates



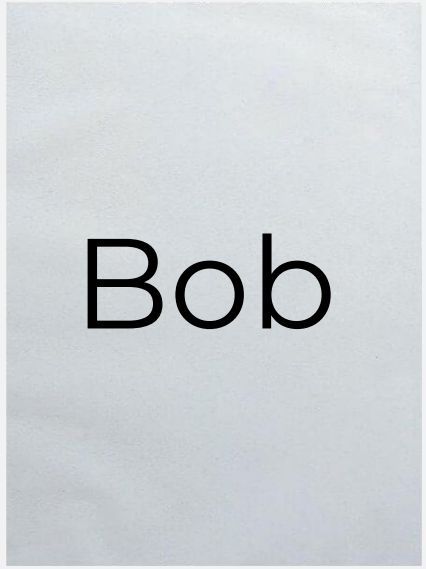
A **parliament** is elected

Voting ballots

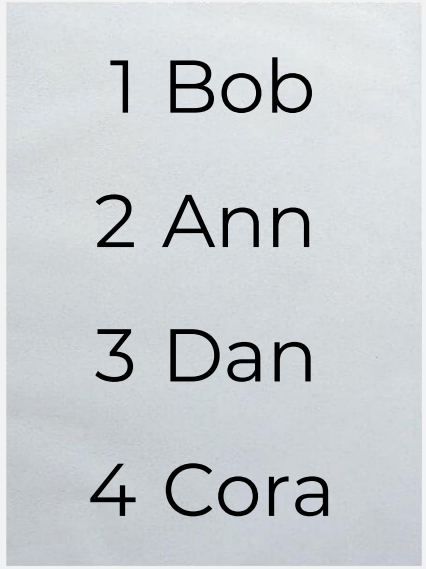


Uninominal Ballots

Voting ballots

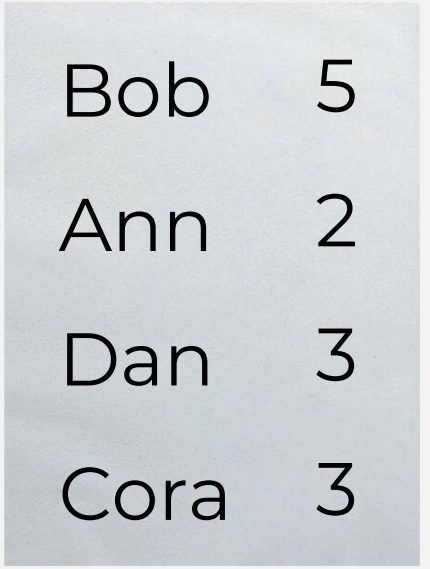


Uninominal Ballots



Rankings

Approval Ballots



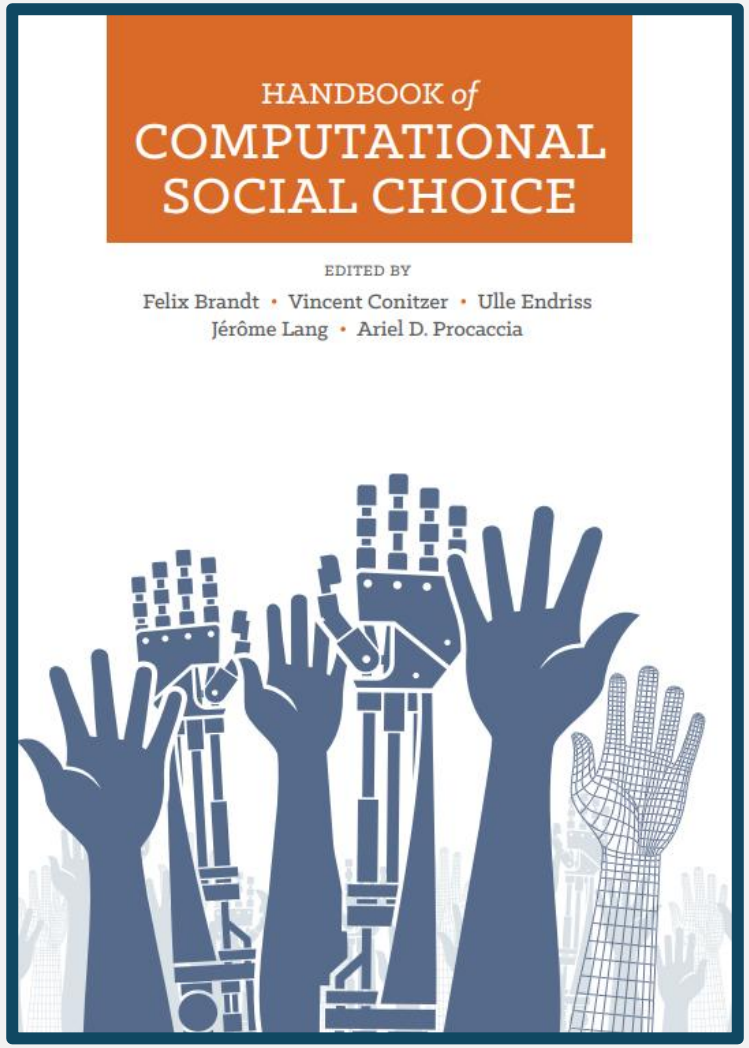
Scores

Our main question

Question: How can we reduce flaws of current voting systems by using more expressive voting ballots?

Computational Social Choice

- From the field of **Computational Social Choice**
- Studying social choice problems (e.g. voting) **with tools and methods from TCS and AI**
- Strongly connected to **Multi-agent systems**



Outline of the thesis

I. Expressive Ballots for Voting Systems

1. Approval with Runoff
2. Instant Runoff Voting with indifferences
3. Rankings in proportional election with thresholds

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5. Identify conflicting pairs of candidates via rankings

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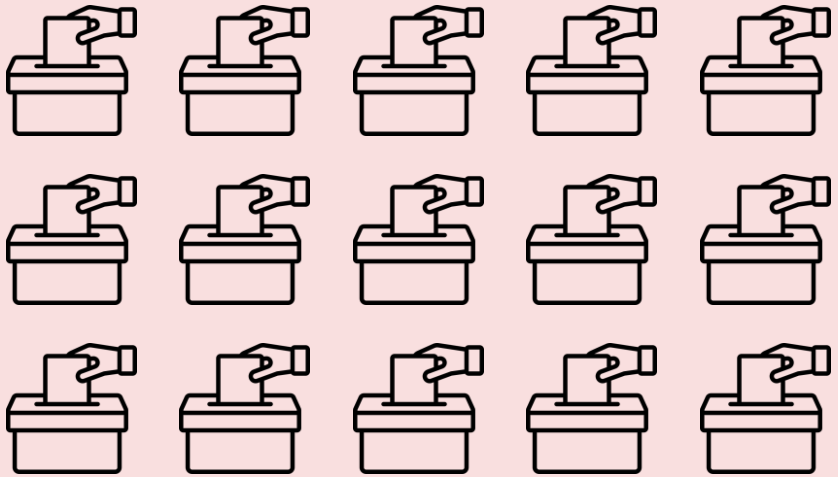
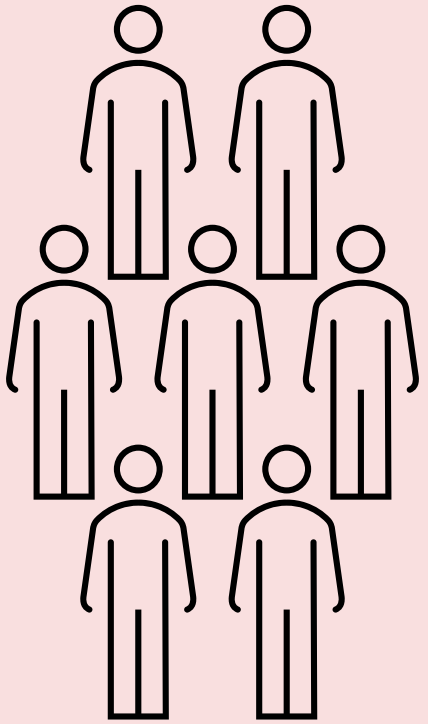
- 1 -
Approval With Runoff

Approval with Runoff
Théo Delemazure, Jérôme Lang, Jean-François Laslier, Remzi Sanver
IJCAI-2022

The Plurality with Runoff voting system

1st Round: Voters can vote for one candidate among the possible ones.

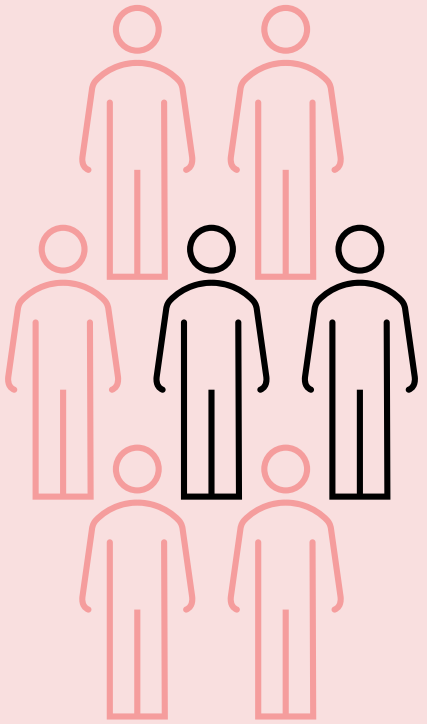




The Plurality with Runoff voting system

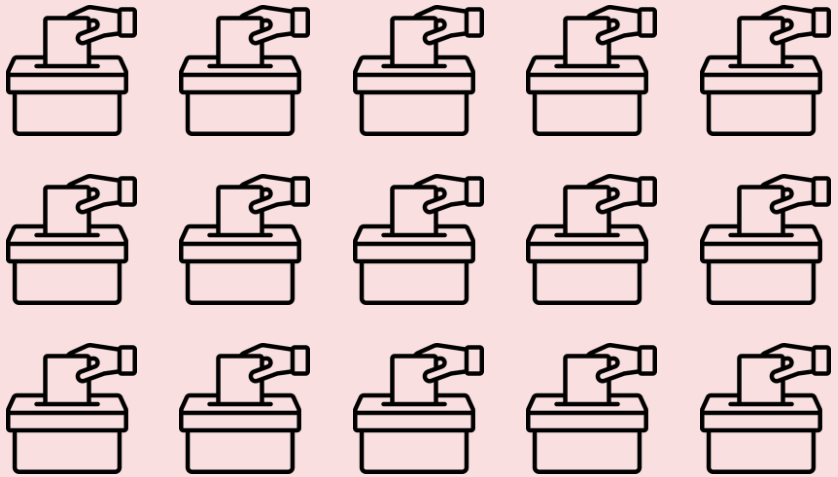
1st Round: Voters can vote for one candidate among the possible ones.

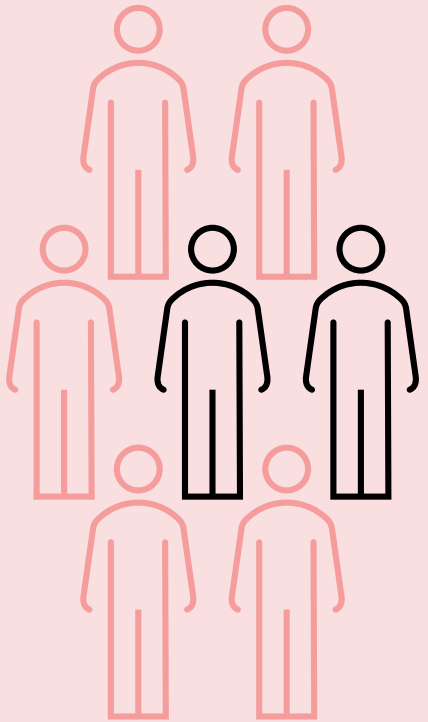
The **two candidates** with the most votes are selected.



The Plurality with Runoff voting system

2nd Round: Voters can now only vote for one of the finalists.

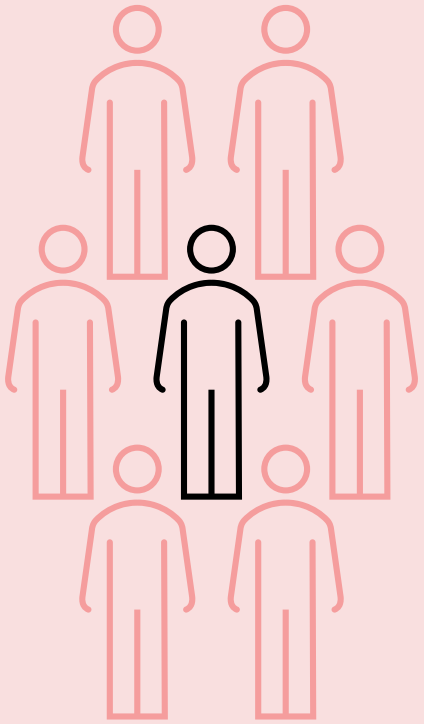




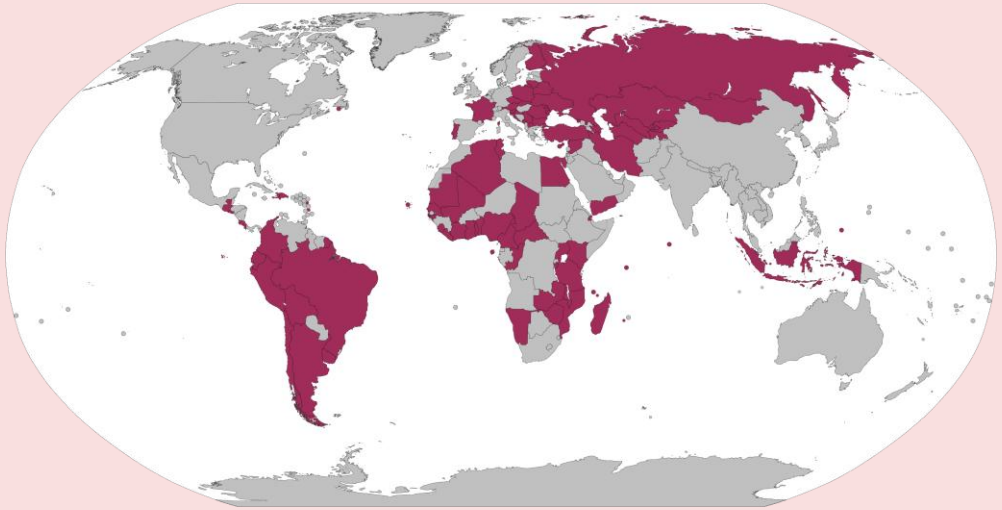
The Plurality with Runoff voting system

2nd Round: Voters can now only vote for one of the finalists.

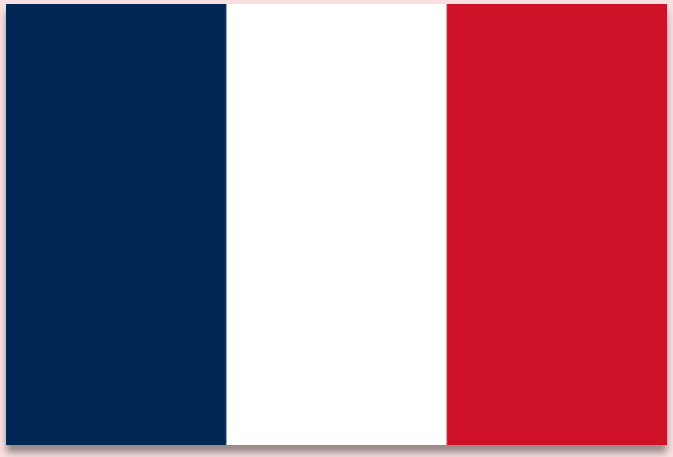
The candidate with the most votes is the **winner**.



Plurality with Runoff in practice



84 countries use it to elect their head of state



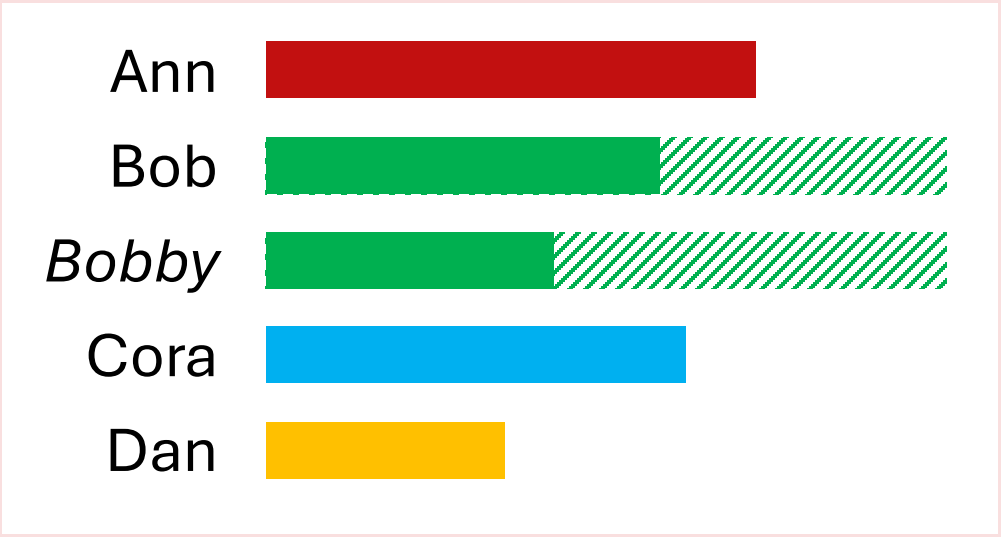
In France, it is used in almost every high-stakes election

The flaws of Plurality with Runoff

Axiom: Independence of clones *(Tideman, 1987)*

Adding a *"clone"* of a candidate should not change significantly the result of the election.

✗ **Plurality with Runoff**



2nd Round: Ann & Bob

2nd Round: Ann & Cora

The flaws of Plurality with Runoff

Axiom: Independence of clones *(Tideman, 1987)*

Adding a *"clone"* of a candidate should not change significantly the result of the election.

✗ Plurality with Runoff

Axiom: Monotonicity *(Fishburn, 1982)*

If some candidate is the winner, and we increase their support, this candidate should still win.

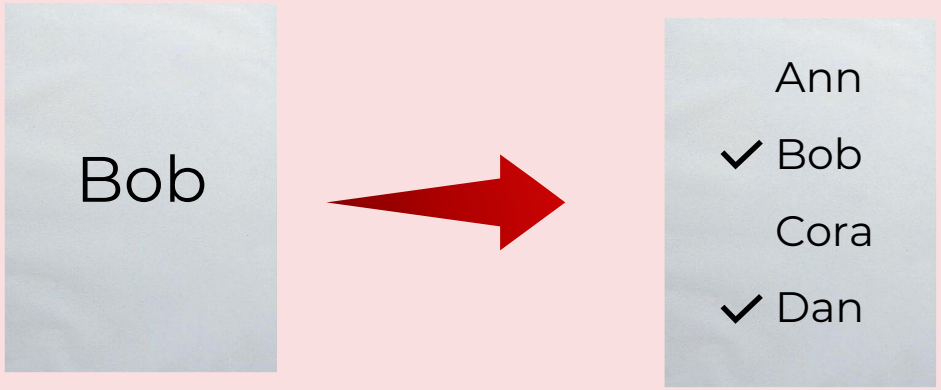
✗ Plurality with Runoff

Our proposal: *Approval* with Runoff

1st round: Voters can vote for **several candidates** among the possible ones.

Two finalists are selected based on these votes.

2nd round: Same as before.



Question: Which candidates should go to the 2nd round?

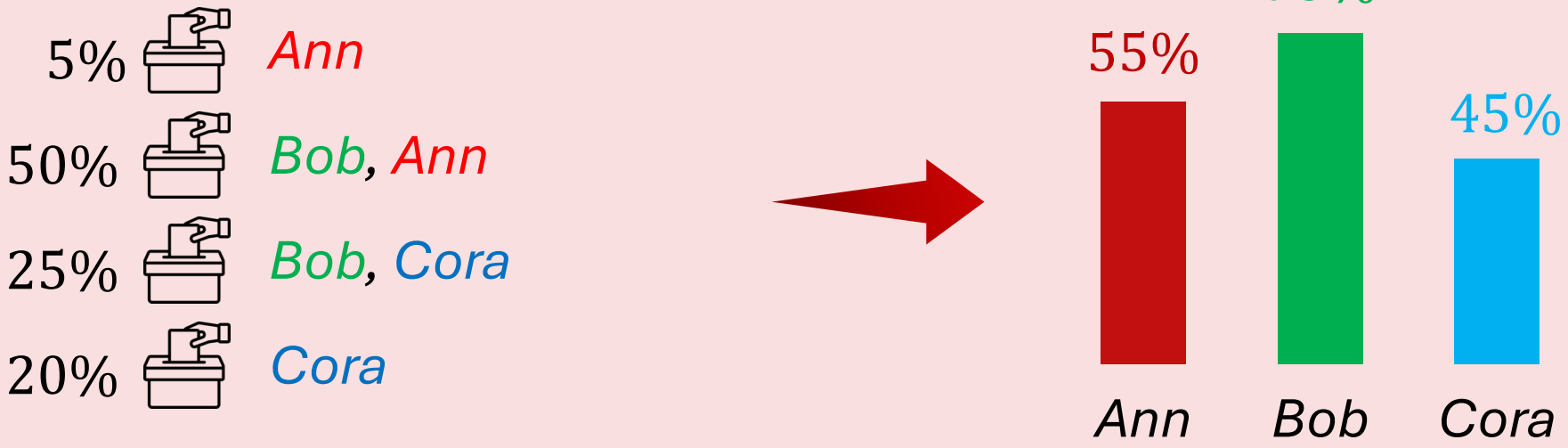
The approval voting rule

Rule: Approval Voting (AV)

The two finalists are the candidates with the most votes in the first round.

✓ **Monotonicity**

✗ **Ind. of clones**



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The two finalists are the candidates with the most votes in the first round.

✓ **Monotonicity**

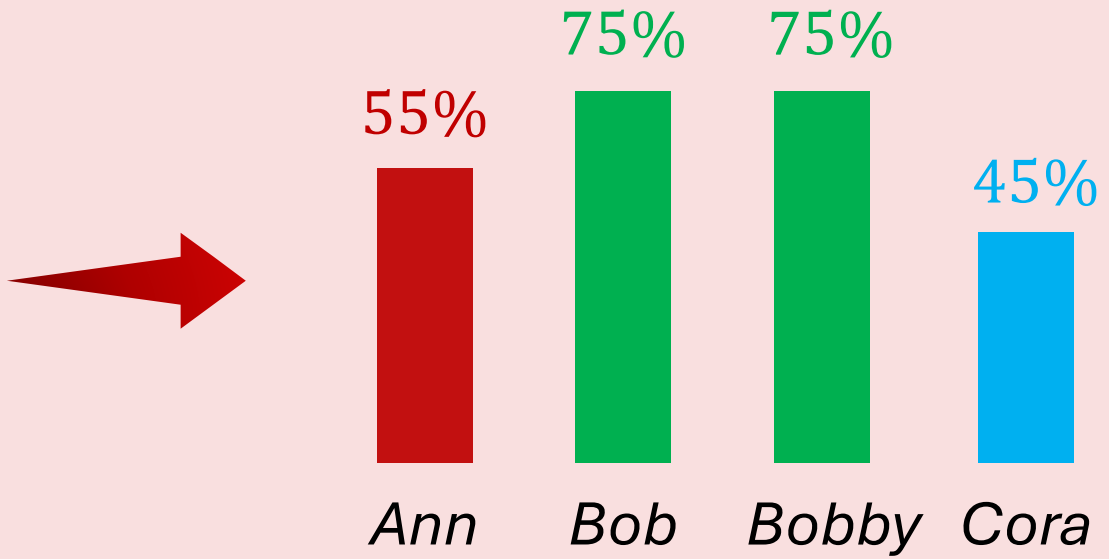
✗ **Ind. of clones**

5% *Ann*

50% *Bob*, *Bobby*, *Ann*

25% *Bob*, *Bobby*, *Cora*

20% *Cora*



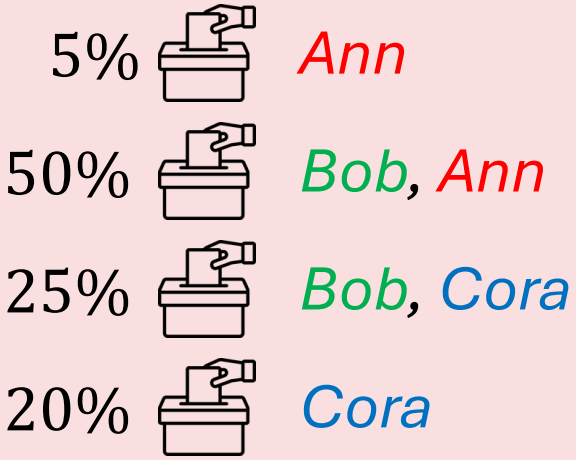
The Chamberlin-Courant rule

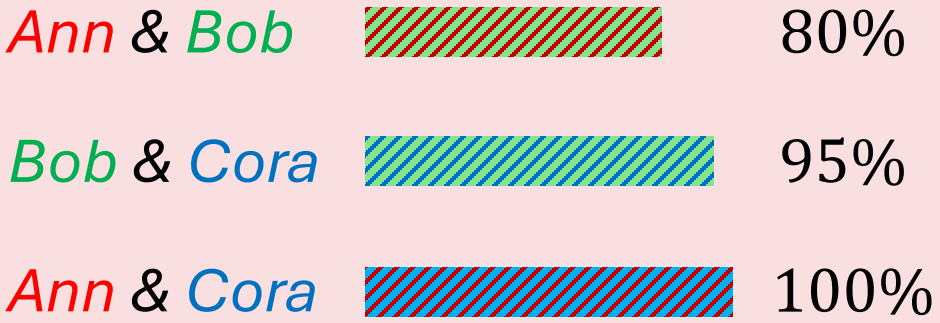
Rule: Chamberlin-Courant Approval Voting (CCAV)

The two finalists are the candidates maximizing the number of voters approving at least one of them.

✗ **Monotonicity**

✓ **Ind. of clones**





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✗ **Monotonicity**

✓ **Ind. of clones**

Impossibility Theorem

No neutral approval with runoff rule satisfies both monotonicity and independence of clones.

The favorite consistency issue

Axiom: Favorite consistency

The candidate that received the most votes in the first round should be in the second round.

✓ AV

✗ CCAV

5%	*Ann*
50%	*Bob*, *Ann*
25%	*Bob*, *Cora*
20%	*Cora*

CCAV selects *Ann* and *Cora* but *Bob* receives the most votes.

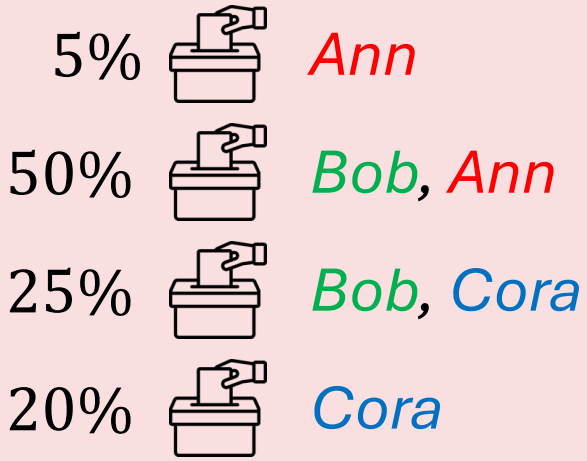
Sequential Chamberlin-Courant

Rule: Sequential CCAV

The **first** finalist is the candidate that received the most votes.
The **second** finalist is the candidate that received the most votes *among voters who did not vote for the first finalist*.

✓ Fav.-consistency

✓ Ind. of clones

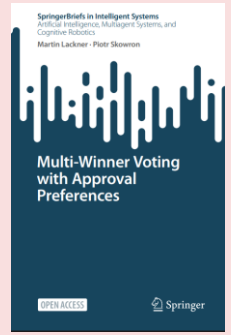


Summary of the axiomatic analysis

	AV	PAV	CCAV	S-PAV	S-CCAV	EnePhr	S-Phr	SAV	TRIV
Pareto-efficiency	✓	✓	✗	✓	✗	✗	✓	✓	✗
strategy-proofness	✓	✗	✓	✗	✗	✗	✗	✗	✓
strong strategy-proofness	✗	✗	✗	✗	✗	✗	✗	✗	✓
monotonicity	✓	✗	✗	✗	✗	✗	✗	✗	✓
weak ind. of clones	✗	✗	✓	✗	✓	✗	✗	✗	✗
approval-efficiency	✓	✗	✗	✓	✓	✓	✓	✗	✗

Fig. Axioms satisfied by the different rules.

[Lackner & Skowron, 2023] Multi-Winner Voting with Approval Preferences.



Experimental analysis: *Voter Autrement*

- Statistical analysis in a 1D Euclidean space
- 38 datasets of approval preferences
- 18 of which are political datasets of the ***Voter Autrement*** collection

French presidential elections (since 2002)

Outside polling station, or online

Testing alternative voting methods

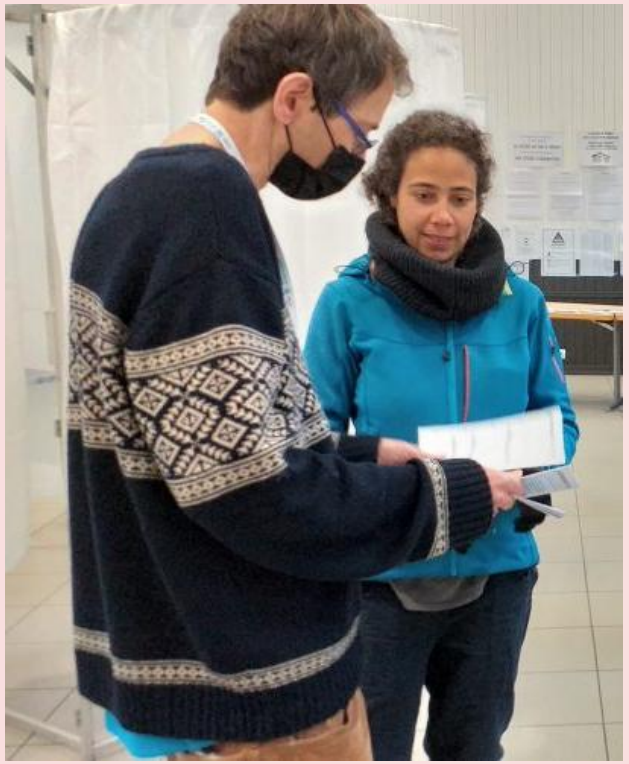


Fig. A researcher explaining how to vote with approval voting to a participant of the experiment.

Experimental analysis: *Voter Autrement*

1. Approval with runoff rules generally return different finalists than plurality with runoff.
2. AV generally selects similar candidates while CCAV selects ideologically distant ones.

Voter Autrement – *Grenoble (2017)*

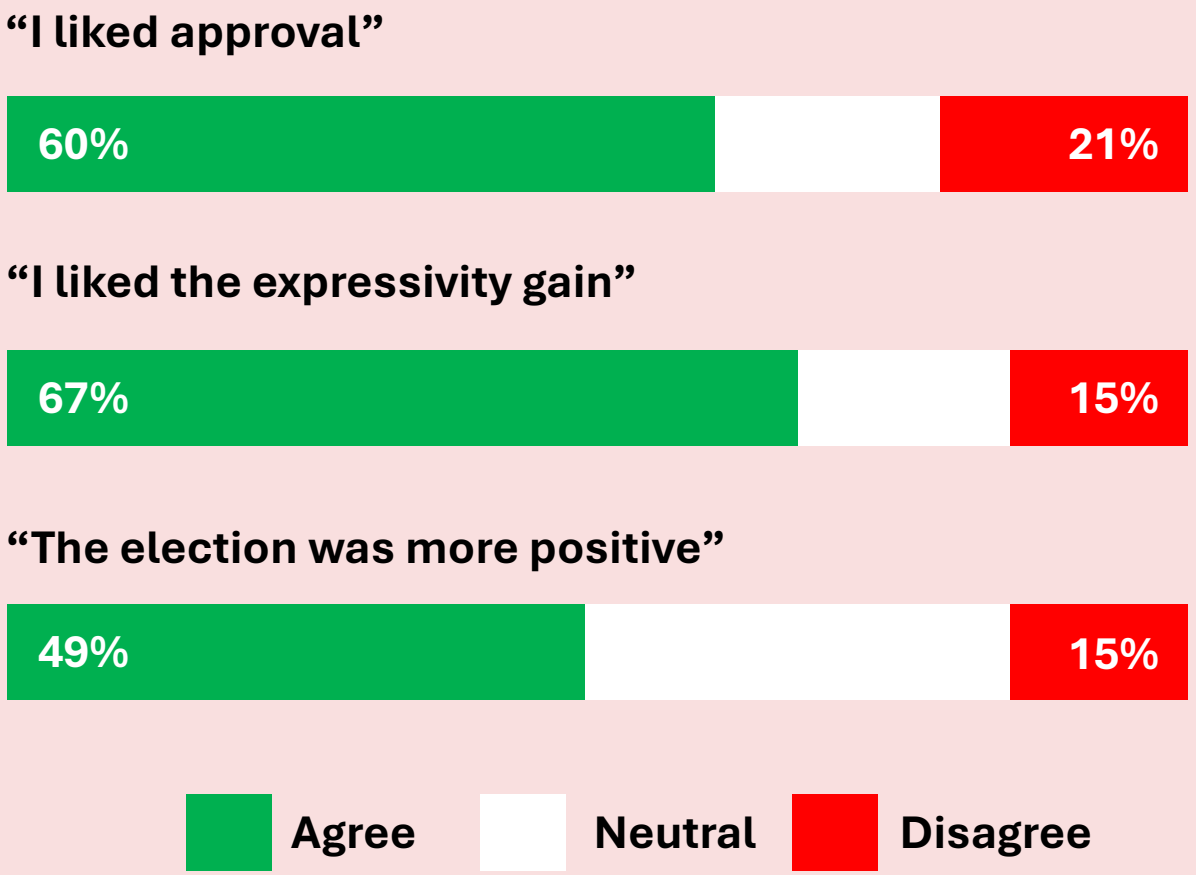
1 069 11

Plurality	LO	NPA	LFI	SOC	EM	LR	DLF	FN
AV	LO	NPA	LFI	SOC	EM	LR	DLF	FN
(Seq-)CCAV	LO	NPA	LFI	SOC	EM	LR	DLF	FN

Can this be implemented?

Approval with runoff has been used in *St. Louis, Missouri* since 2021 for city elections.

Fig. St. Louis, Missouri



- We can enhance **plurality with runoff** using approval ballots, but there is not a single way to do it.

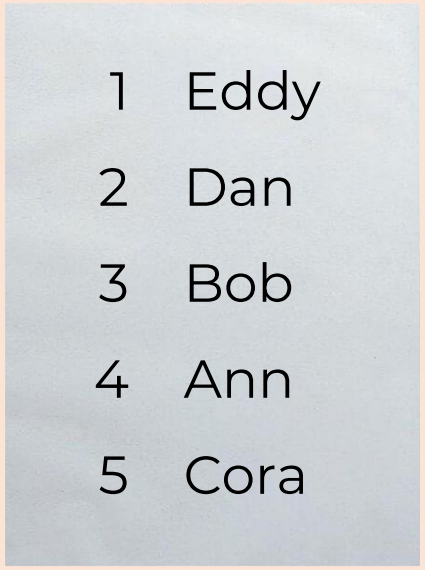
- Let us now see if we can improve another single-winner voting system: **Instant Runoff Voting (IRV).**

- 2 -
Instant Runoff Voting with Indifferences

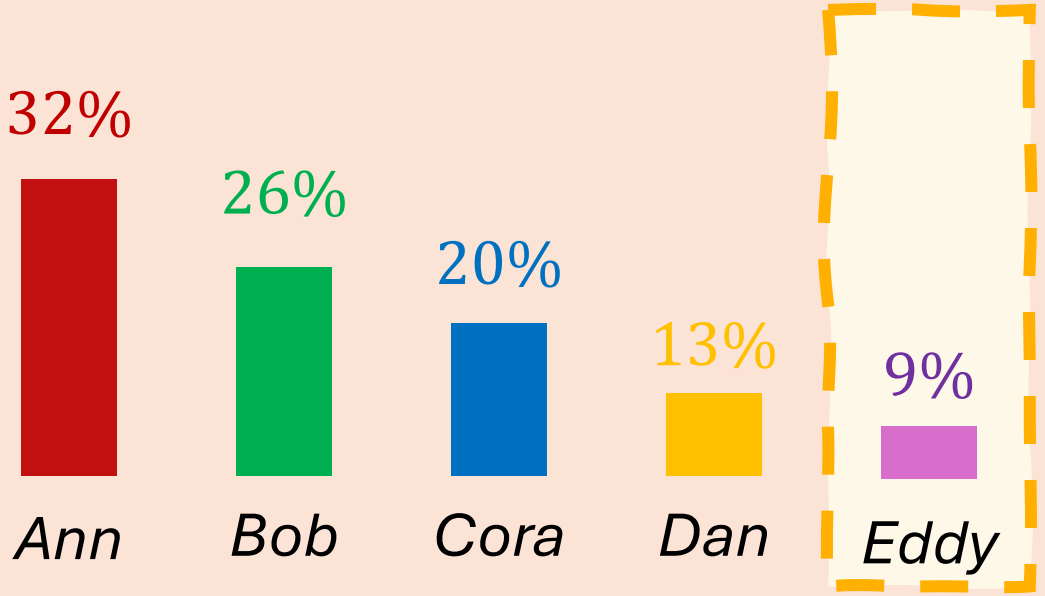
Generalizing Instant Runoff Voting to Allow Indifferences
Théo Delemazure, Dominik Peters
EC-2024

The Instant Runoff Voting system (IRV)

Voters provide
a **ranking** of the candidates



We count the number of **first-place votes** each candidate receives



The candidate with the lowest score is **eliminated**, and their votes are transferred

The Instant Runoff Voting system (IRV)

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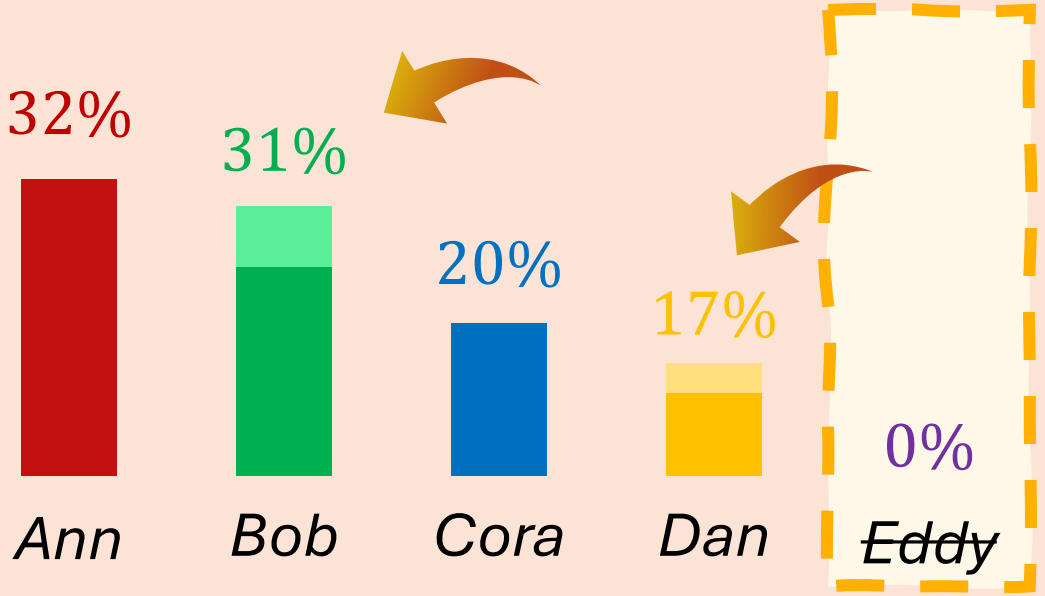


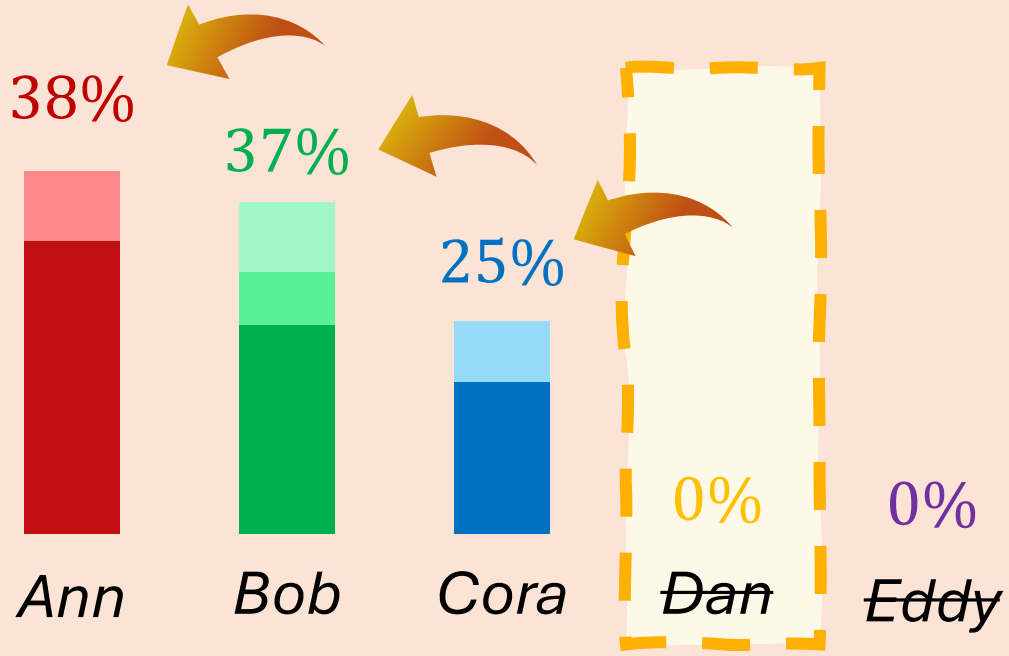
The candidate with the lowest score is **eliminated**, and their votes are transferred

The Instant Runoff Voting system (IRV)

Voters provide
a **ranking** of the candidates

~~Eddy~~
~~Dan~~
1 Bob
2 Ann
3 Cora

We count the number of **first-place votes** each candidate receives



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The Instant Runoff Voting system (IRV)

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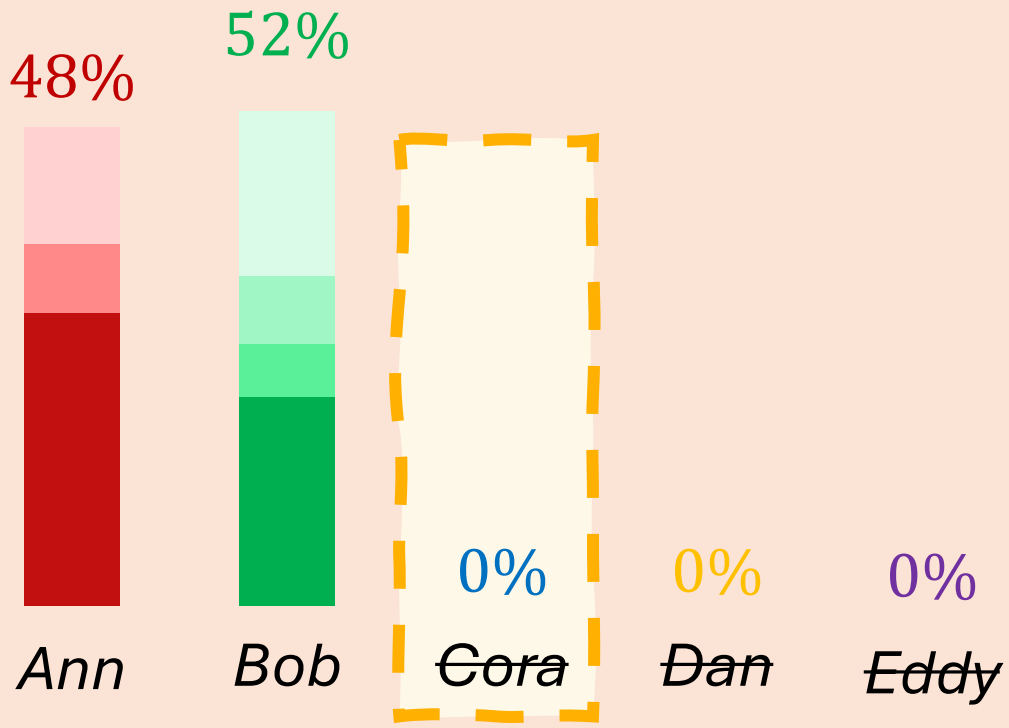
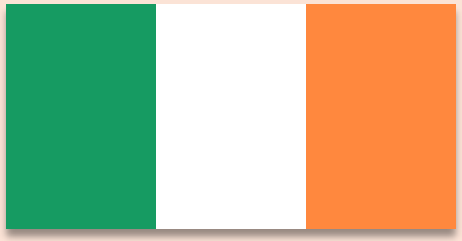


We stop when one **candidate get the majority** of the votes.

The Instant Runoff Voting system (IRV)

Ireland (since 1937)



Australia (since 1918)

Maine, USA (since 2018)

And others…

Properties of IRV

✓ Indep. of clones

✓ Majority criterion

✗ Monotonicity

Axiom: Majority criterion *(Lepelley, 1992)*

If a majority of voters rank one candidate first, this candidate should be the winner.

1	**Ann**	1	**Ann**	1	Bob
2	Bob	2	Dan	2	Cora
3	Cora	3	Cora	3	Ann
4	Dan	4	Bob	4	Dan
31%		20%		49%	

What if we are indifferent?

What if a voter is **indifferent** between several candidates?

1 Eddy
| Dan
2 Bob
| Ann
3 Cora

Voters can cast **weak orders**

Question: How to generalize IRV to weak orders?

Two generalizations of IRV to weak orders

Rule: Split-IRV *(Meek and Hill, 1994)*

Each voter gives $1/k$ point to the k candidates that are tied as first among the remaining candidates in their ranking.

1 Eddy
| Dan
2 Bob
| Ann
3 Cora

1/2 **Eddy**

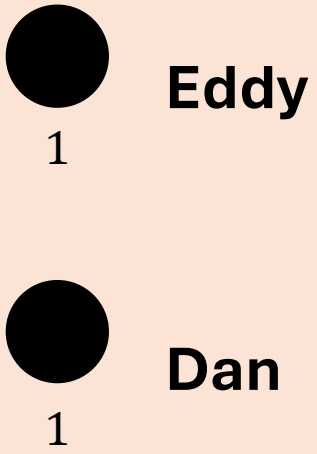
1/2 **Dan**

Two generalizations of IRV to weak orders

Rule: Approval-IRV

Each voter gives 1 point to the k candidates that are tied as first among the remaining candidates in their ranking.

1	Eddy
|	Dan
2	Bob
|	Ann
3	Cora



Generalizing IRV characteristic axioms

Instant Runoff Voting satisfies the following two axioms:

Axiom: Independence of clones *(Tideman, 1987)*

Axiom: Majority Criterion *(Lepelley, 1992)*

If a majority of voters rank one candidate first, this candidate should be the winner.

1	**Ann**	1	**Ann**	1	Bob
2	Bob	2	Dan	2	Cora
3	Cora	3	Cora	3	Ann
4	Dan	4	Bob	4	Dan
31%		20%		49%	

Generalizing IRV characteristic axioms

We can generalize these axioms to weak orders:

Axiom: Independence of clones *(Tideman, 1987)*

Axiom: Respect for Cohesive Majorities

If a majority of voters rank one candidate first, **the winner should also be ranked first by one of these voters.**

1	**Ann**		1	**Ann**		1	Bob	
|	**Bob**		2	Dan		|	Cora	
2	Cora		|	Cora		2	Ann	
3	Dan		3	Bob		|	Dan	
31%			20%			49%		

Generalizing IRV characteristic axioms

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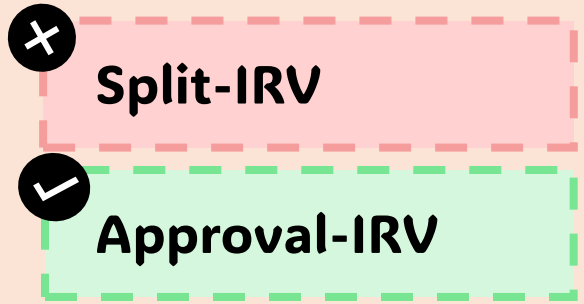

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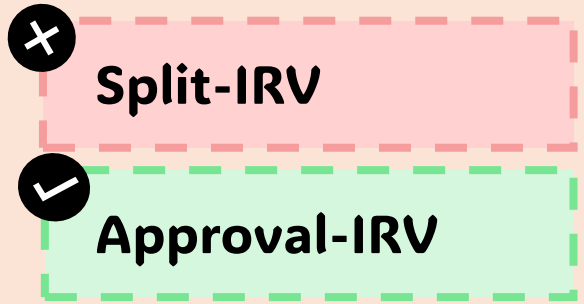


Generalizing IRV characteristic axioms

First characterization of Approval-IRV

Approval-IRV is the **only** runoff scoring rule for weak orders that satisfies **both** independence of clones and respect for cohesive majorities.

The weak monotonicity axiom

Axiom: Monotonicity *(Fishburn, 1982)*

Axiom: Weak monotonicity

✗ IRV

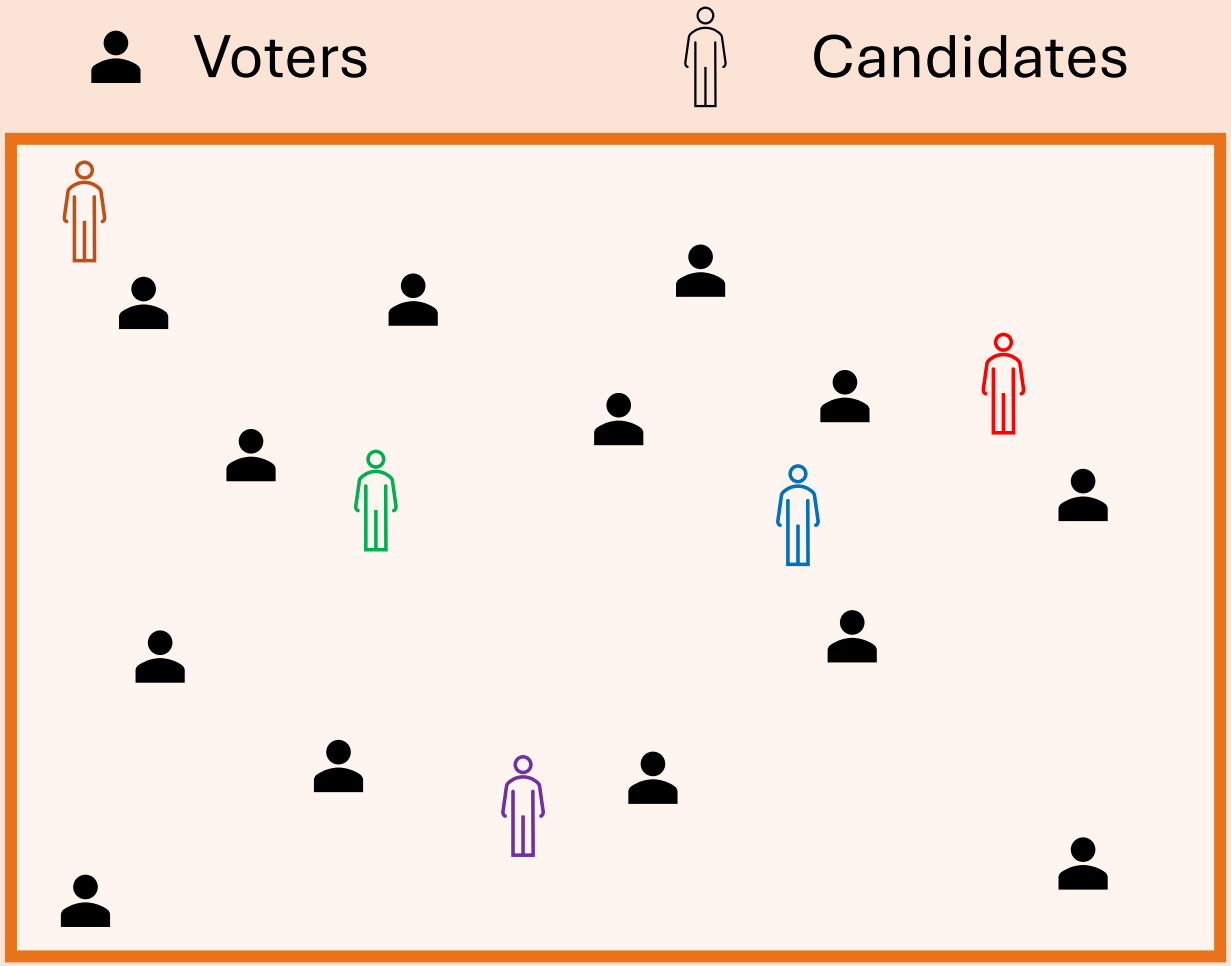
✗ Split-IRV

✓ Approval-IRV

Second characterization of Approval-IRV

Approval-IRV is the **only** runoff scoring rule for weak orders that generalizes IRV and satisfies weak monotonicity.

Experiments: Euclidean preferences

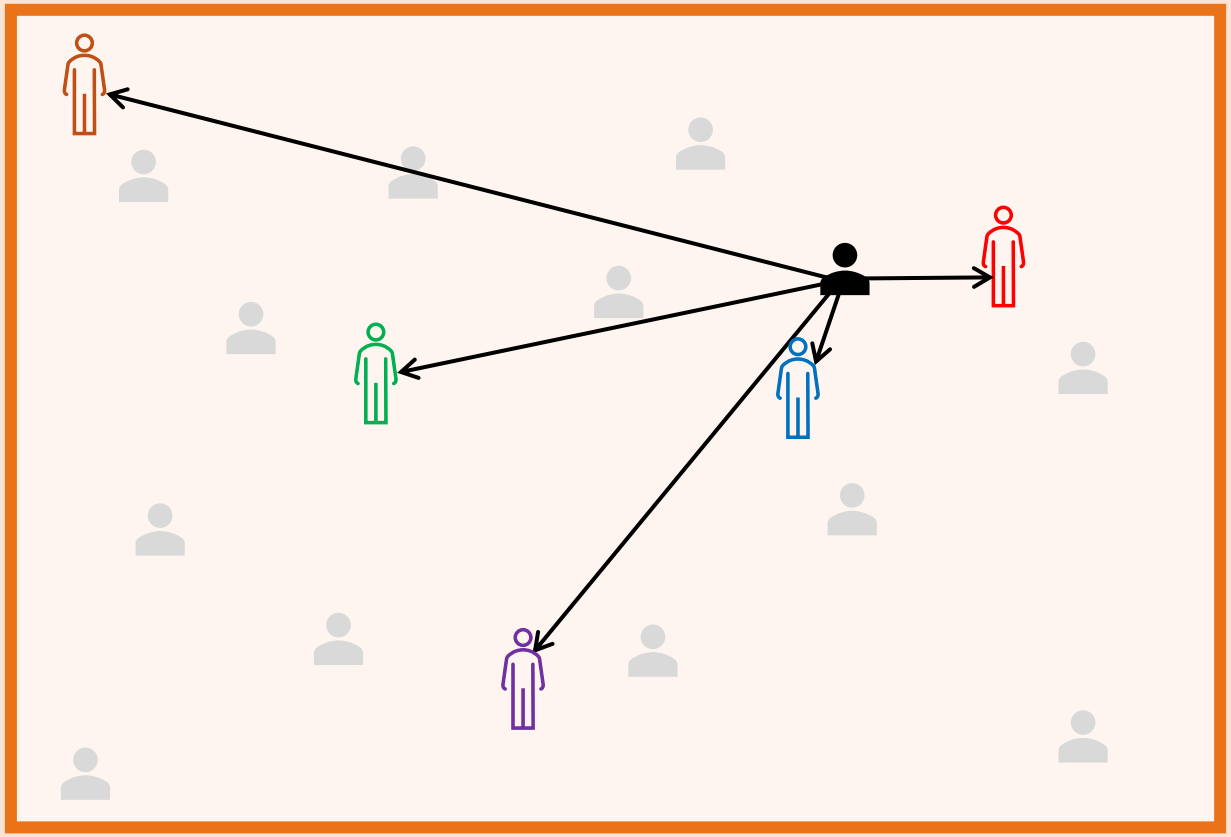


Euclidean Preferences
(Enelow and Hinich, 1984)

» Positions of voters and candidates are sampled randomly in the space.

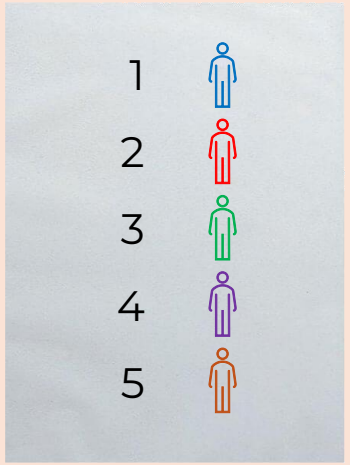
Experiments: Euclidean preferences

Voters Candidates

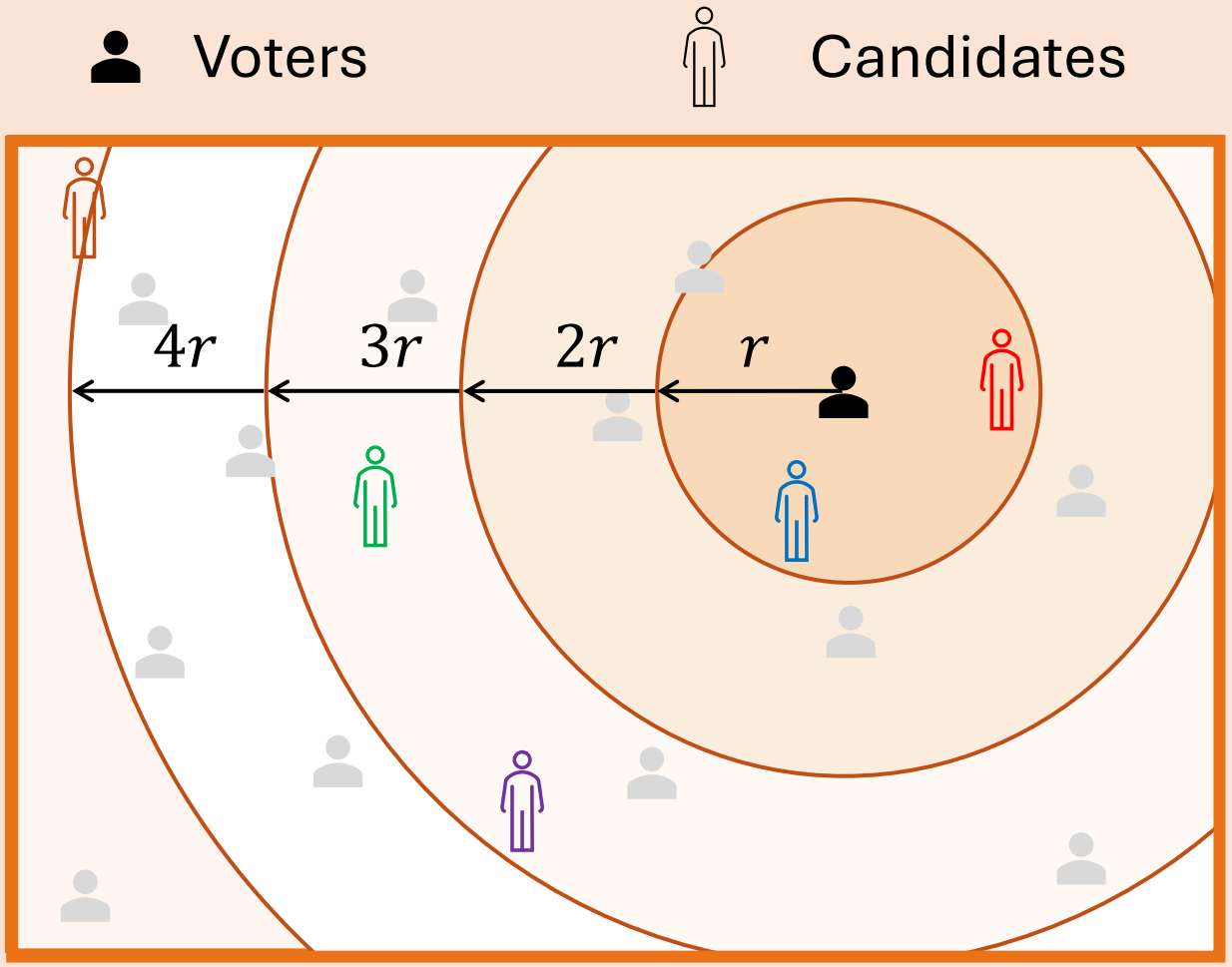


Euclidean Preferences
(Enelow and Hinich, 1984)

» Voters prefer candidates that are closer to them:

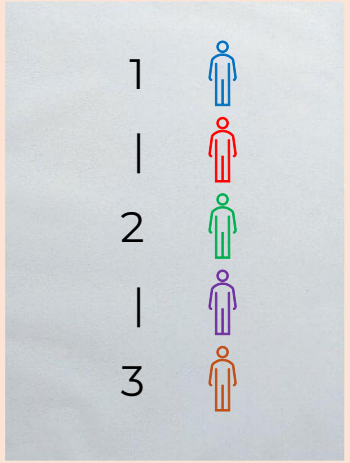


Experiments: Euclidean preferences

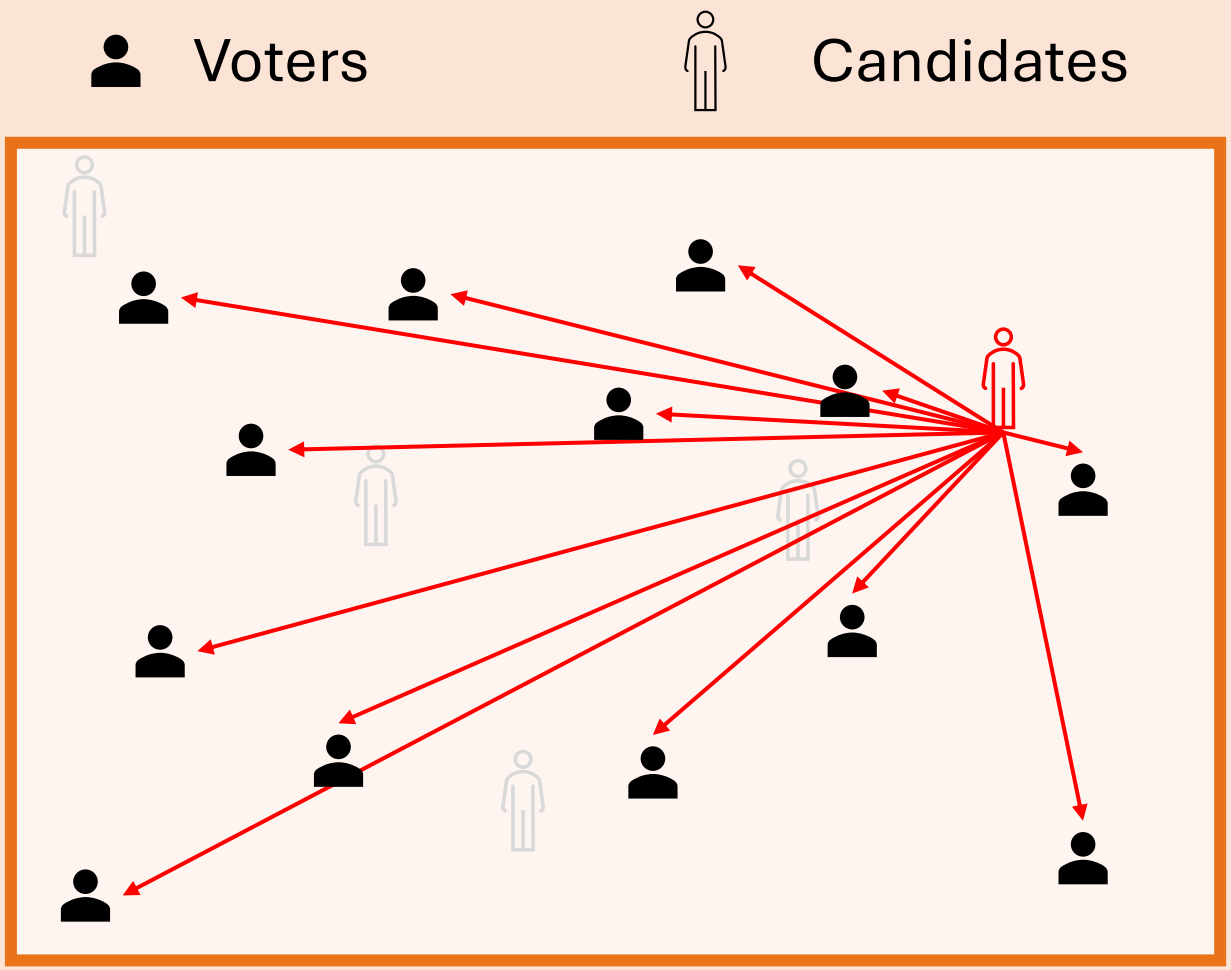


Euclidean Preferences
(Enelow and Hinich, 1984)

» We can also obtain weak orders:



Experiments: Euclidean preferences

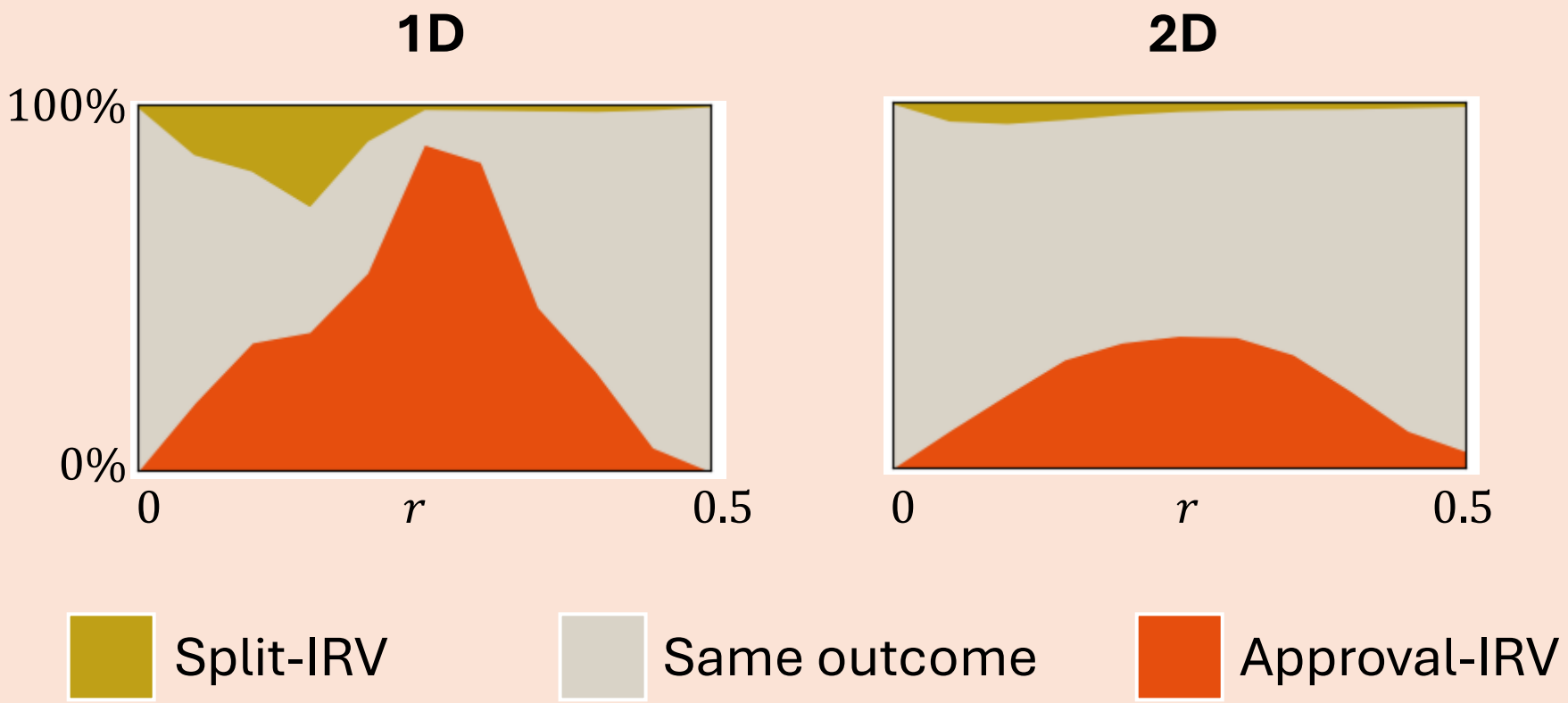


Euclidean Preferences
(Enelow and Hinich, 1984)

- The **cost** of candidates are their average distance to the voters.
- **The lower the cost, the better.**

Experiments: Euclidean preferences

Which rule returns the candidate **with lowest cost**?
(proportion over 10 000 instances)



- Approval-IRV appears to be a better generalization of IRV to weak orders than Split-IRV.

- Can expressive ballots also improve the way we vote in **parliamentary elections**?

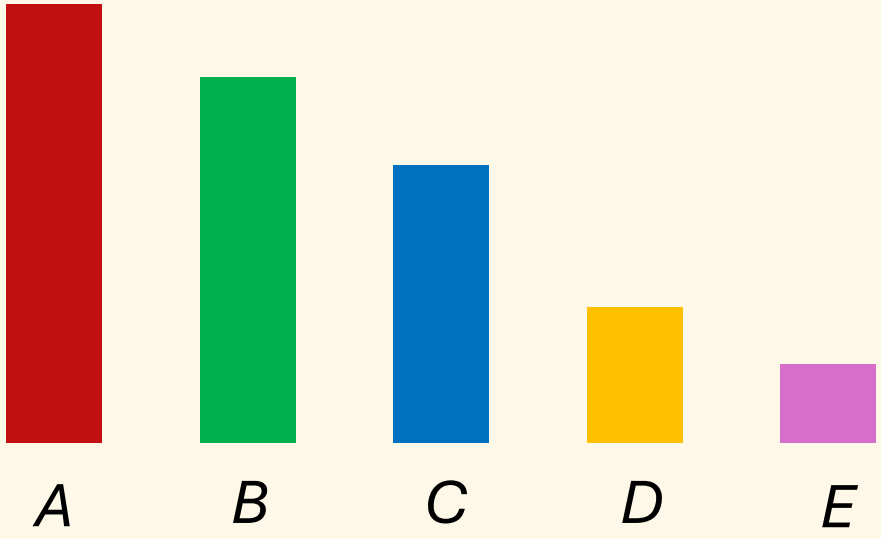
- 3 -
Rankings in Parliamentary Elections with Threshold

Reallocating Wasted Votes in Proportional Parliamentary Elections with Thresholds
Théo Delemazure, Rupert Freeman, Jérôme Lang, Jean-François Laslier, Dominik Peters
EC-2025

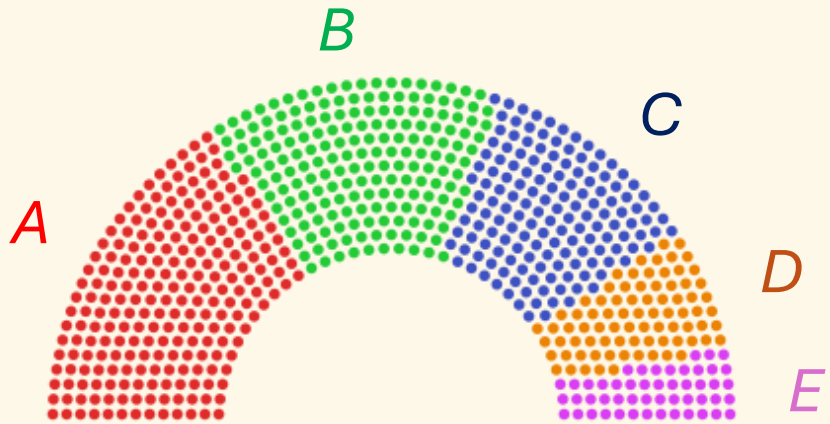
Voting systems for parliamentary elections

Proportional representation

Voters vote for one of the parties.

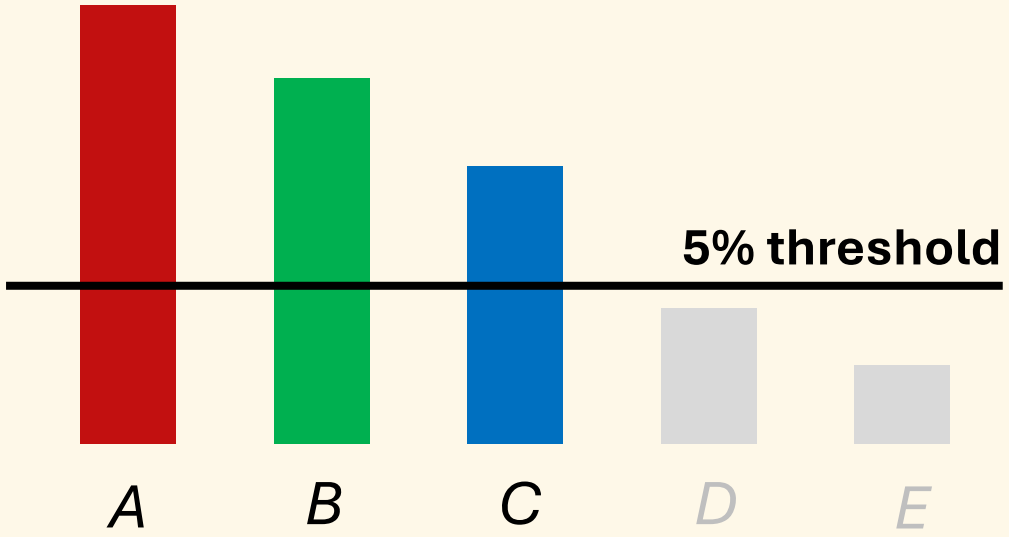


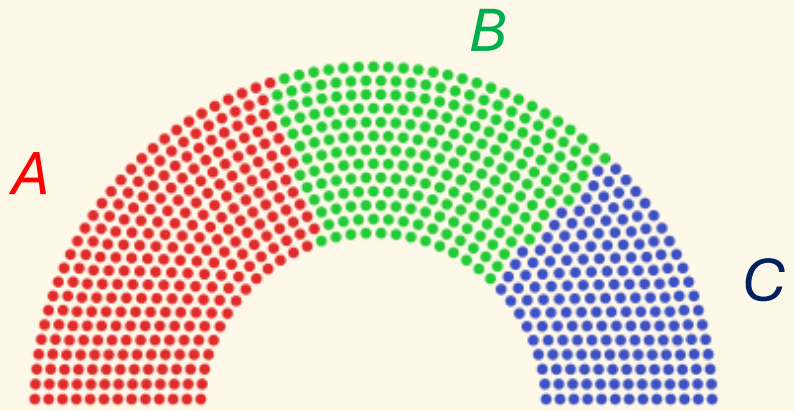
Seats are allocated to the parties **proportionally to their scores**.



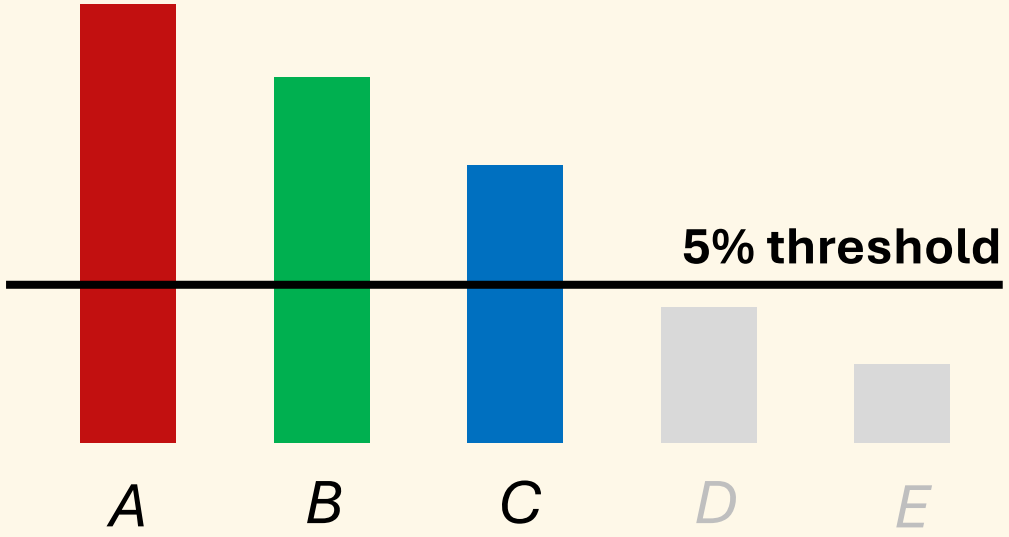
Proportional representation *with Threshold*

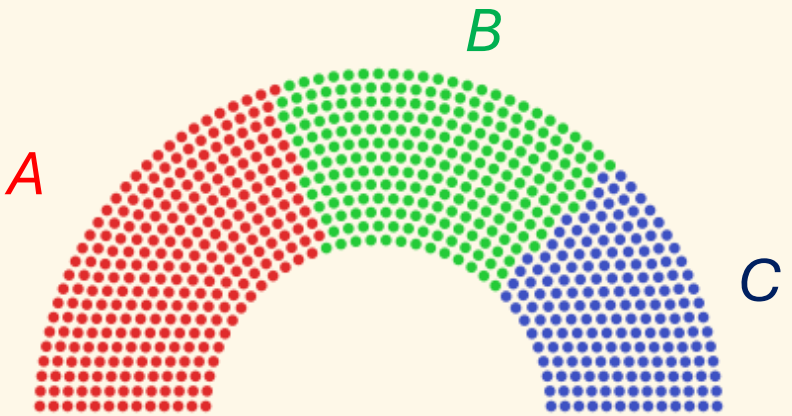
Many countries impose and **electoral threshold** to reduce political fragmentation.





Proportional representation *with Threshold*





- **Some votes are "lost":** *D* and *E* supporters have no influence on the seat distribution.
- This incentivizes forms of **tactical voting.**

Proportional representation *with Threshold*

2019 election of the French *representative to the EU Parliament*.

2025 election of the *Bundestag* members.

2002 election of the *Turkish Parliament* members.

Threshold	"Lost" votes
5%	20%
5%	14%
	increasing in recent decades
10%	46%

Second-chance voting

We could ask voters to rank **two parties**

1 Party B

2 Party D

We could even ask for a **truncated ranking**

1 Party B

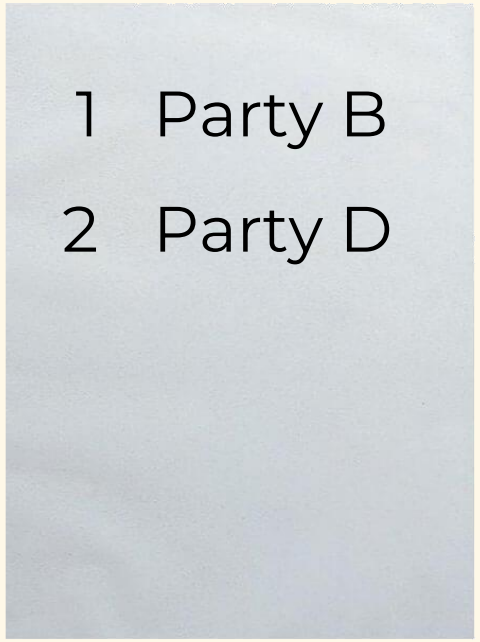
2 Party D

3 Party A

4 Party C

Second-chance voting

We could ask voters to rank **two parties**



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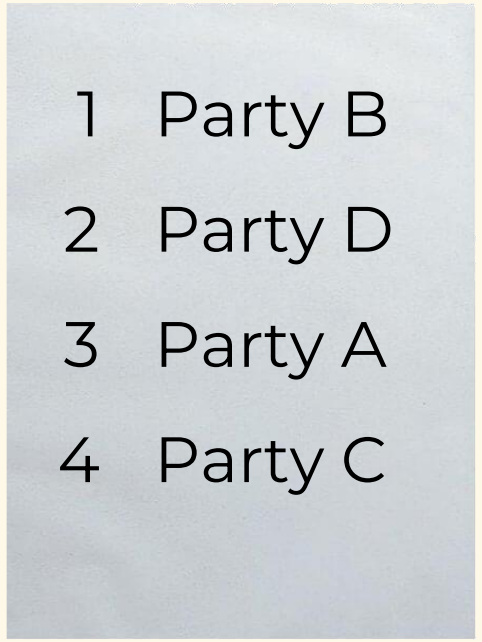


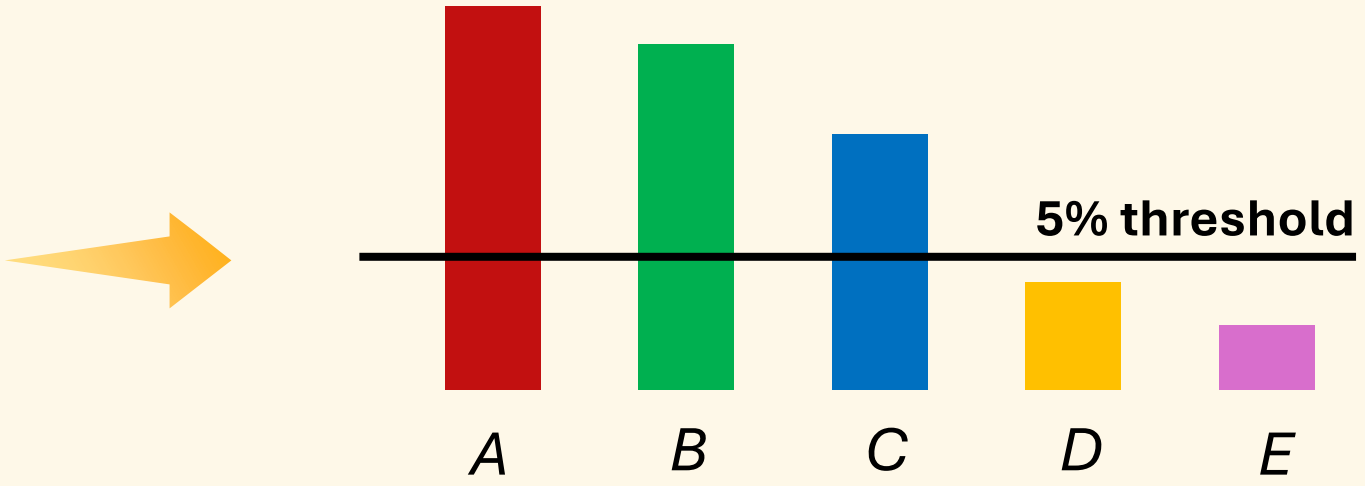
Question: How to select the parties that are above the threshold?

1st possibility: Direct Winners Only (DO)

Rule: Direct winners Only (DO)

The selected parties are all those which receive more first-place votes than required by the threshold.

1. A 2. B	1. B	1. C 2. D 3. A
34%	32%	15%
1. C 2. B	1. D 2. C	1. E 2. D 3. B
12%	4%	3%



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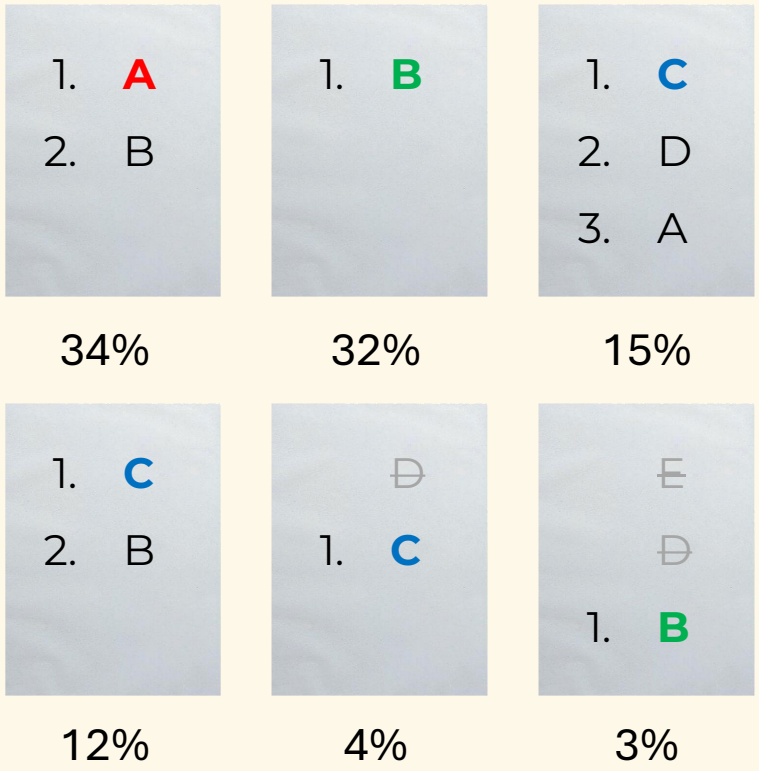

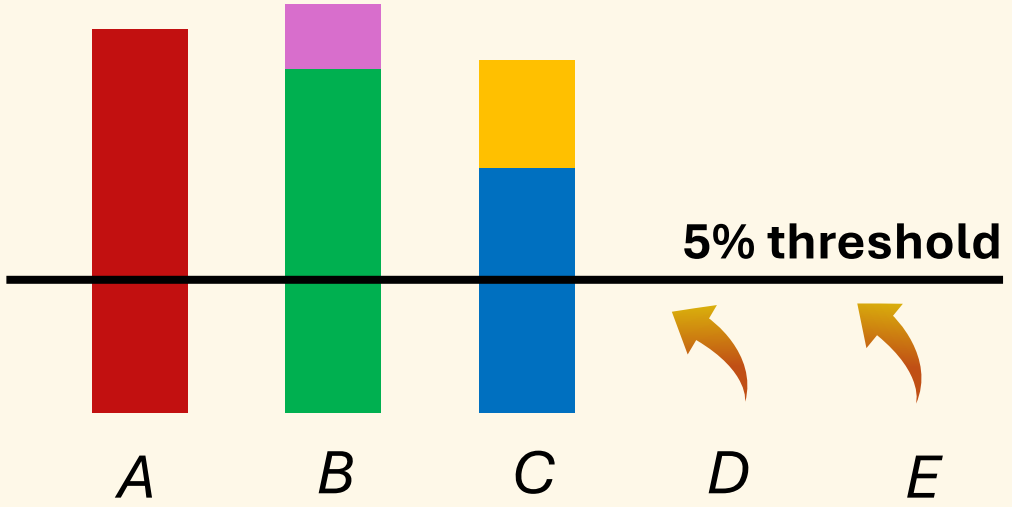

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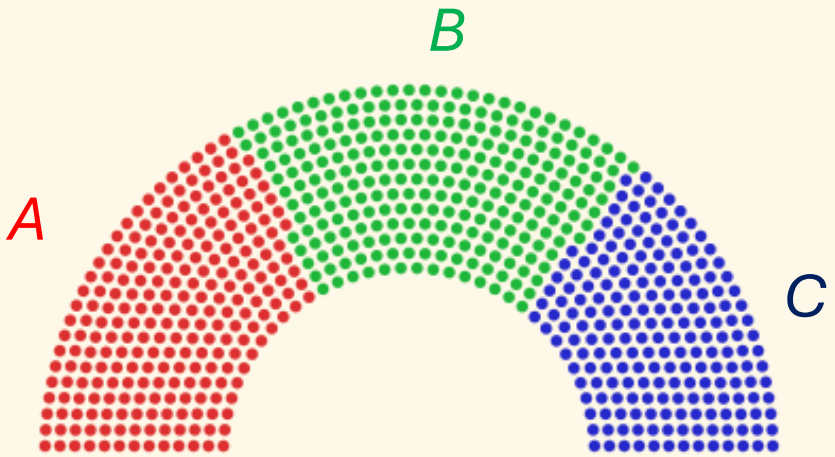

2nd possibility: Single Transferable Vote (STV)

Rule: Single Transferable Vote (STV)

Parties that receive the fewest votes are successively eliminated until all parties receive more votes than required by the threshold.

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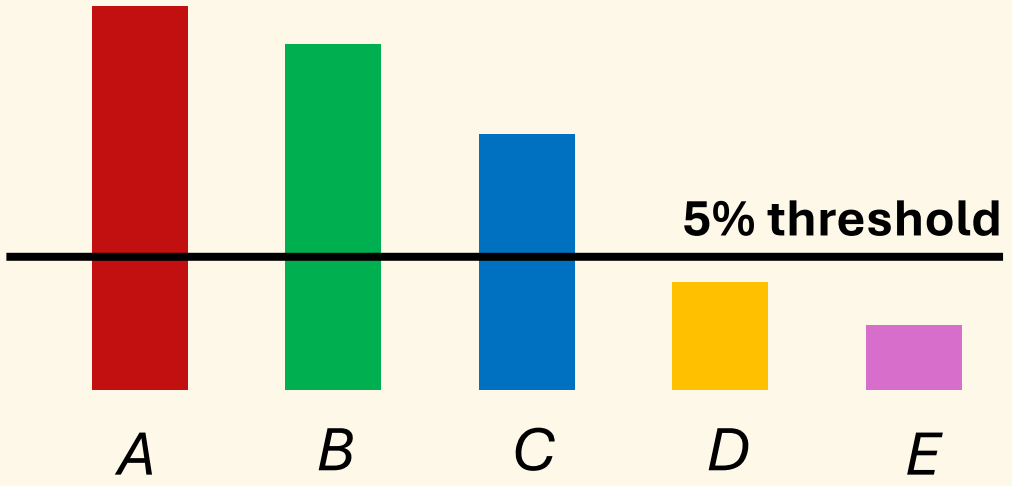

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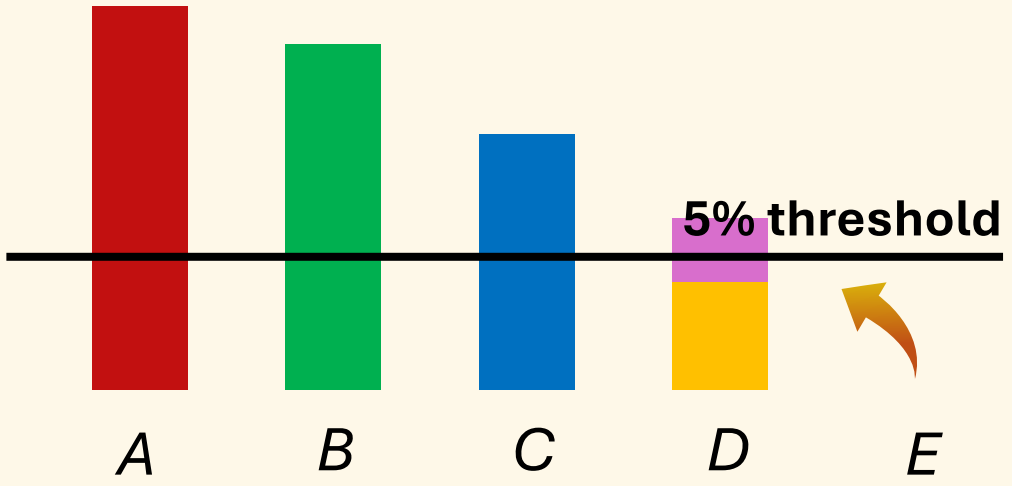

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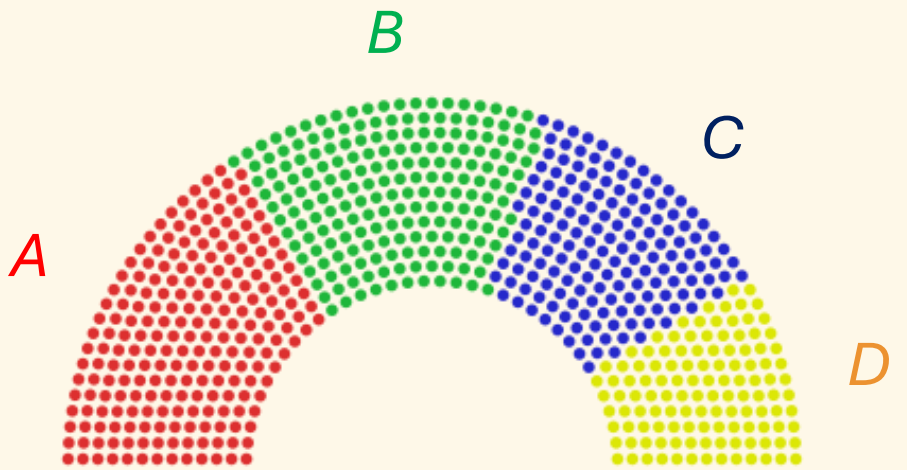


Five rules

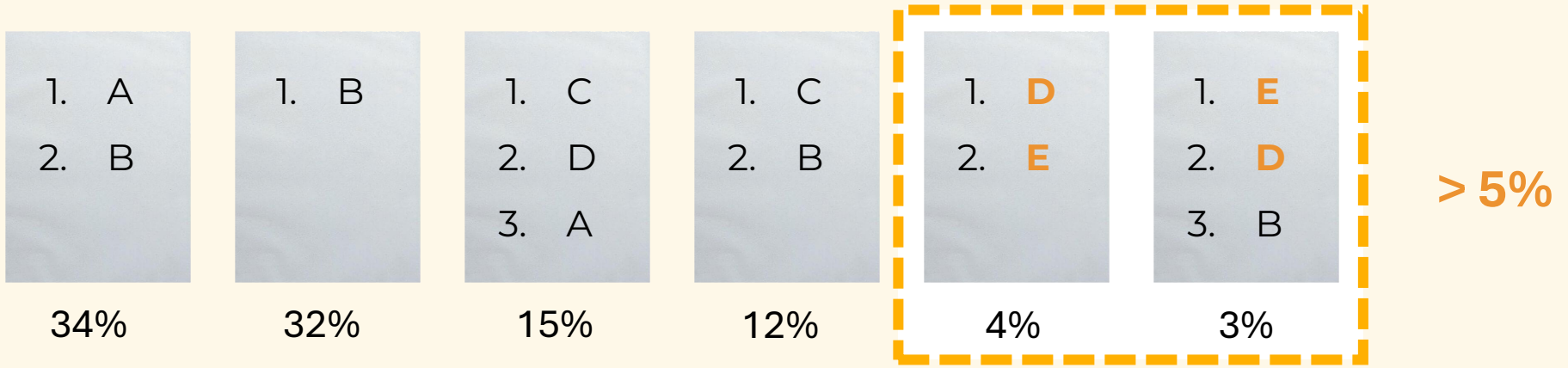
- **Rule:** Direct winners Only (DO)
- **Rule:** Single Transferable Vote (STV)
- **Rule:** Greedy Plurality (GP)
- **Rule:** MaxPlurality (MaxP)
- **Rule:** MaxRepresentation (MaxR)

Axiomatic analysis

Axiom: Representation of Solid Coalitions

If more voters than required by the threshold all rank a set of parties *S* **on top of their rankings**, at least one of these parties should be selected.

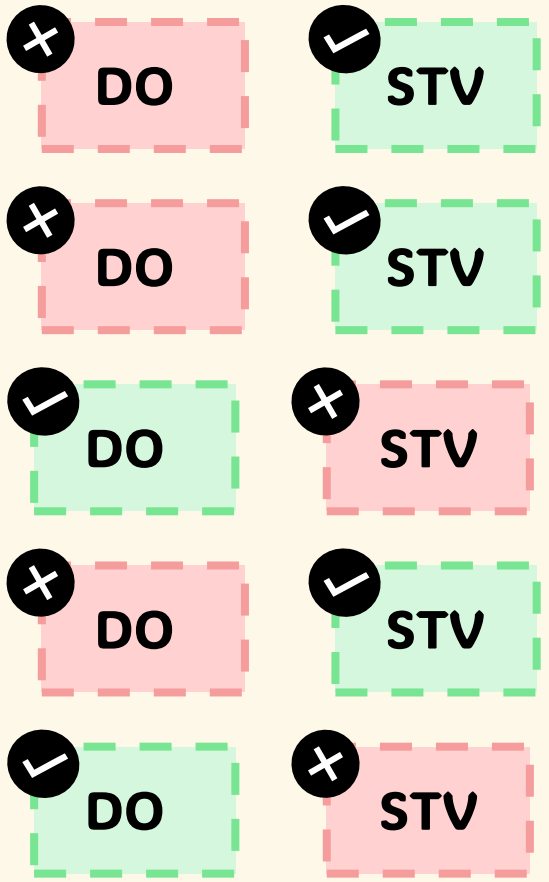
✗ DO ✓ STV



Axiomatic analysis

The rules satisfy different sets of axioms

- **Axiom:** Representation of Solid Coalitions
- **Axiom:** Independence of Clones
- **Axiom:** Monotonicity
- **Axiom:** Independence of definitely losing parties
- **Axiom:** Reinforcement for winning parties



Axiomatic analysis

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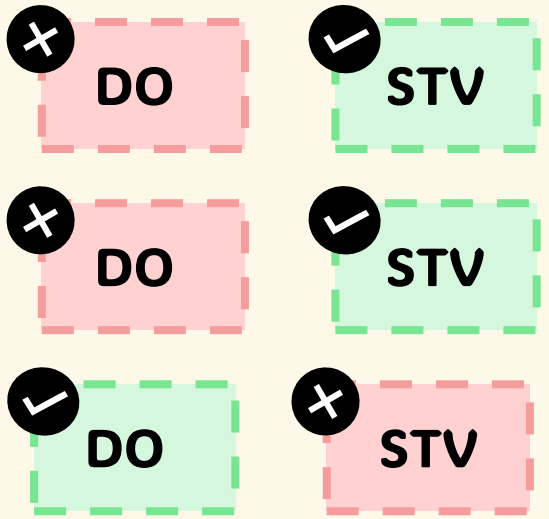


Characterization of STV with independence of definitely losing parties

Characterization of DO with reinforcement for winning parties

Axiomatic analysis

	DO	STV	GP	MaxP	MaxR
Set-maximal	✗	✗	✓	✓	✓
Inclusion of direct winners	✓	✓	✓	✗	✗
Representation of solid coalitions	✗	✓	✗	✗	✗
Representation of unrepresented voters	✗	✗	✗	✗	✗
Threshold monotonicity	✓	✓	✗	✗	✗
Independence of definitely losing parties	✗	✓	✗	✗	✗
Independence of clones	✗	✓	✗	✗	✓
Reinforcement for winning parties	✓	✗	✗	✗	✗
Monotonicity	✓	✗	✗	✗	✗
Representative-strategyproof (one risky party)	✗	✗	✓	✓	✓
Share-strategyproof (safe first or second)	✓	✗	✗	✗	✗
Share-strategyproof (representative ranked first)	✓	✗	✓	✗	✗

Fig. Axioms satisfied by the different rules.

The experimental setup

1. Explanation of the issues caused by the threshold.
2. Presentation of the candidate lists.
3. **Vote with alternative voting methods.**
4. Questionnaire.

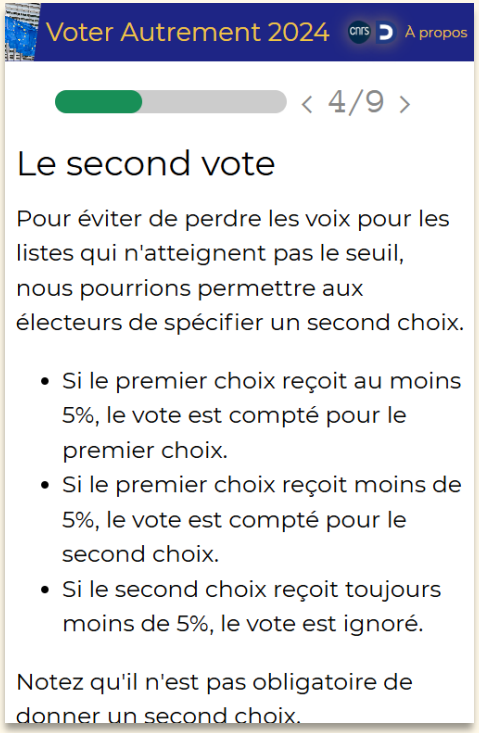


Fig. Screenshot of the website of the experiment conducted during the 2024 election of the French representative to the EU Parliament.

Two samples of participants

1 Self-selected sample

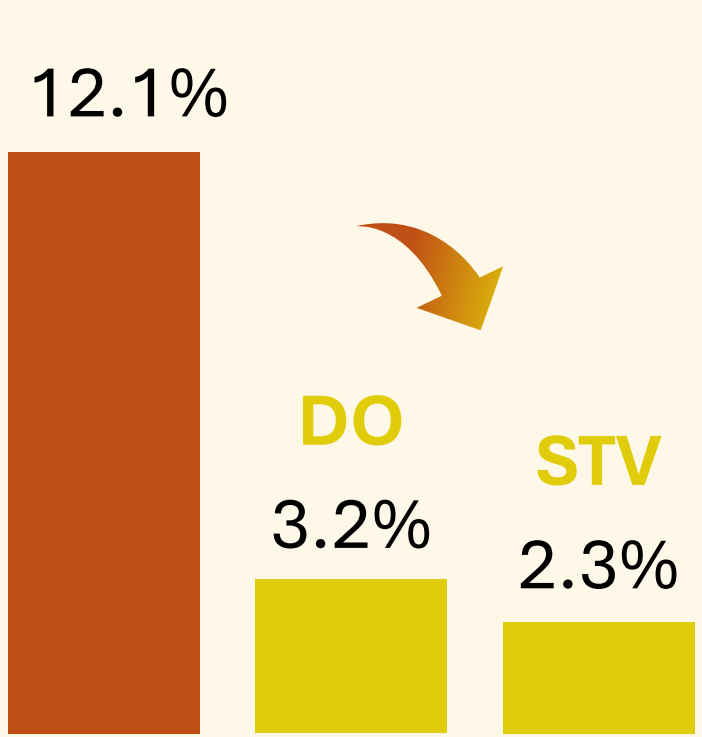
- 3046 participants in a week.
- Recruited through social media.
- Overrepresentation of left-wing, young and educated people.

2 Representative sample

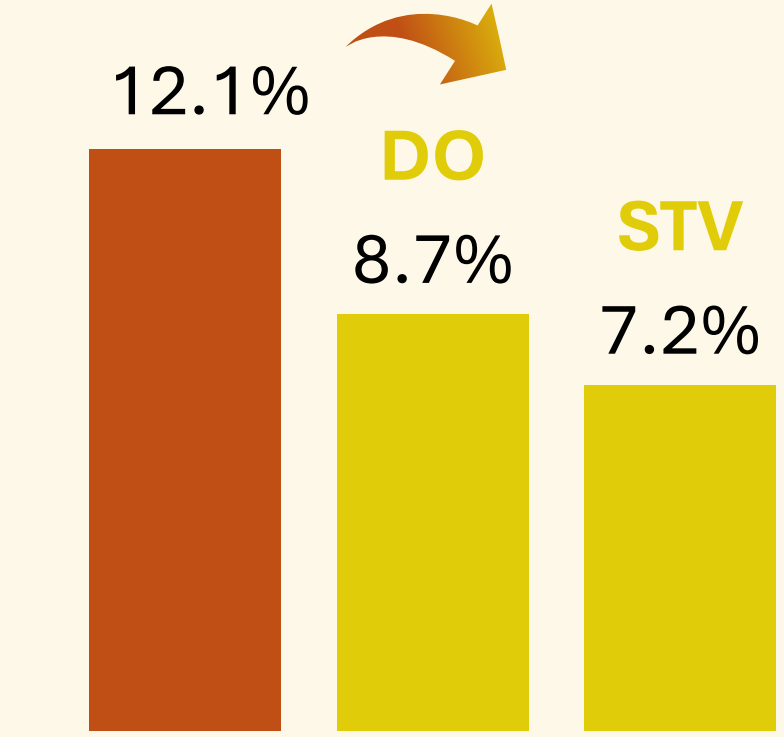
- 1000 participants.
- Recruited via a polling institute and paid a fixed amount to participate.
- Representative of the French population.

Observation 1: fewer votes are “lost”

» Fewer votes are “lost” and not taken into account



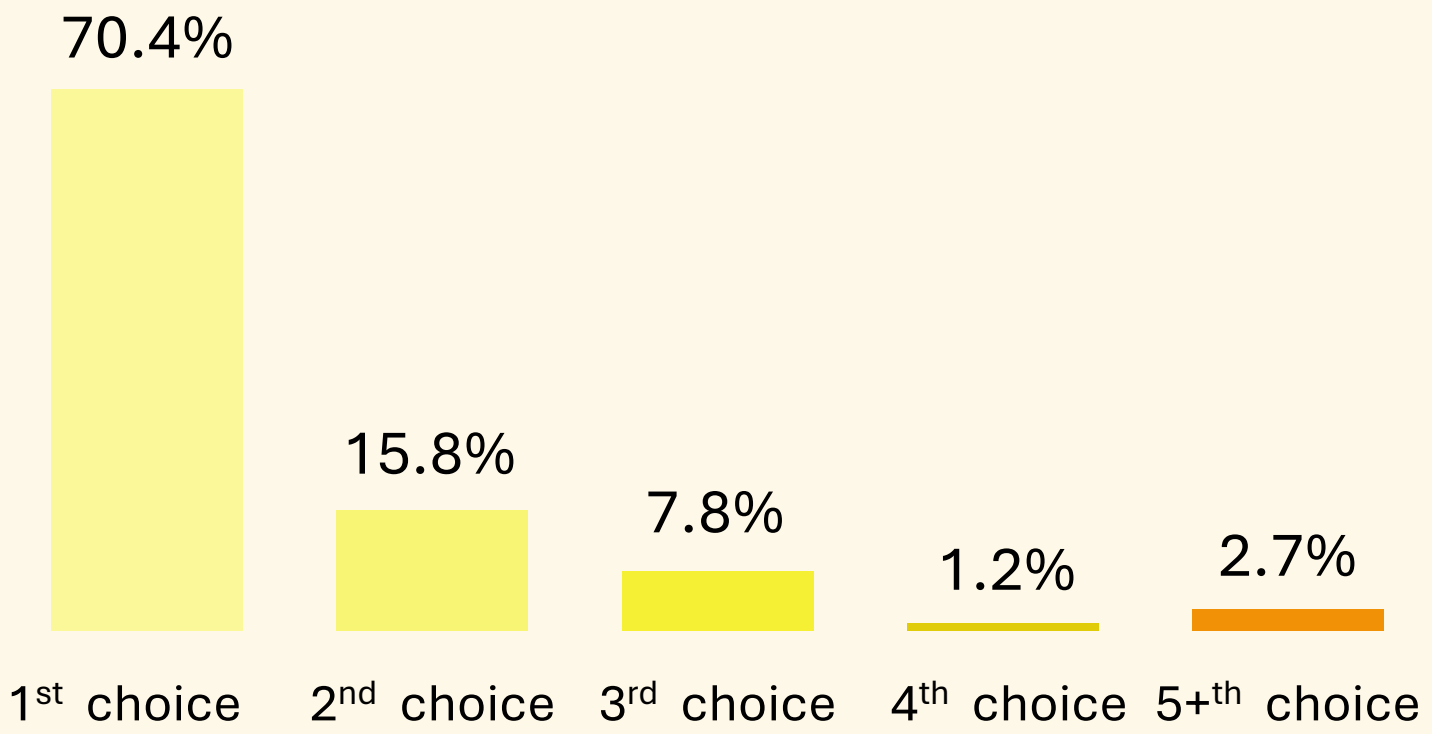
1 *Self-selected* sample



2 *Representative* sample

Observation 2: We can ask for short rankings

» We still reduce the number of lost votes if we impose short rankings

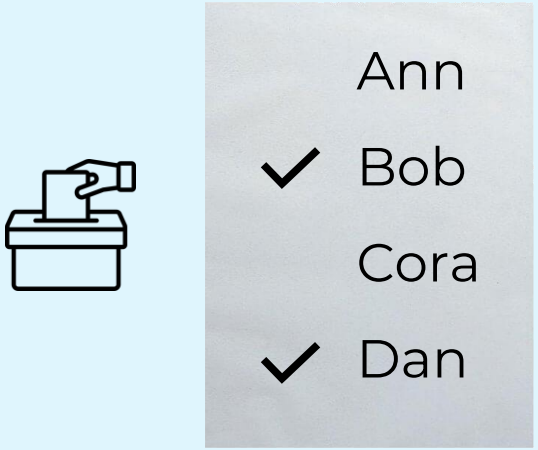


● *Self-selected* sample with the ***STV*** rule

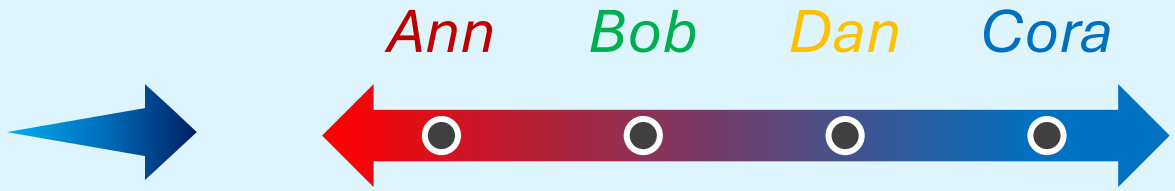
- 4 -
Learning Structures from Preferences

Learning candidate axes from approvals

We know voters' **approval ballots**



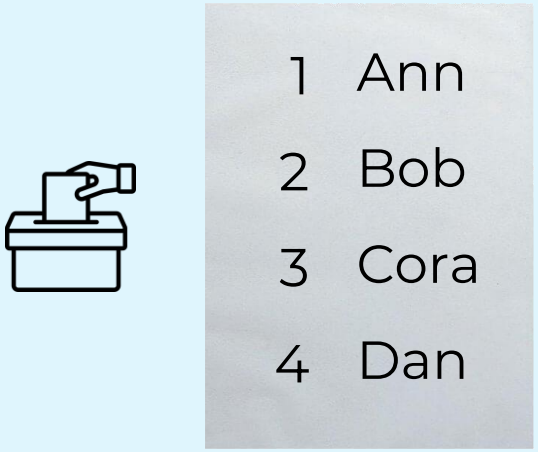
We want to find an **ordering** of the candidates



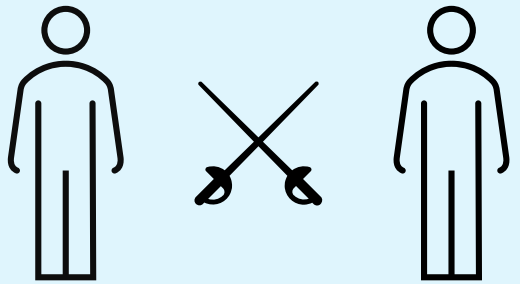
Comparing Ways of Obtaining Candidate Orderings from Approval Ballots
Théo Delemazure, Chris Dong, Dominik Peters, Magdalena Tydrichova
IJCAI-2024

Measuring conflict and polarization

We know voters' **rankings**



We want to measure the **conflict/polarization induced** by pairs of candidates



Selecting the Most Conflicting Pair of Candidates
Théo Delemazure, Łukasz Janeczko, Andrzej Kaczmarczyk and Stanisław Szufa
IJCAI-2024

- 5 -
Concluding remarks

Dissemination & further work

- Inform **the policy makers** on alternative voting methods (via think tanks or directly talking to them).
- Make the **non-academic public** aware of our works and convince them of the value of our methods (events, blog posts, social media).
- Use the axiomatic and experimental approaches to analyze **other forms** of collective decision processes, like *citizens' assemblies*?

Special thanks

Thanks a lot to my **supervisors**...

Jérôme Lang

Dominik Peters

...and to my **co-authors** !

Antoinette Baujard ● Sylvain Bouveret ● Markus Brill ● Roberto Brunetti
Rachael Colley ● Tom Demeulemeester ● Chris Dong ● François Durand
Manuel Eberl ● Rupert Freeman ● Anne-Marie George ● Hugo Gilbert ● Jonas Israel
Łukasz Janeczko ● Andrzej Kaczmarczyk ● Martin Lackner ● Jean-François Laslier
Patrick Lederer ● Simone Marsilio ● Fabien Mathieu ● Grzegorz Pierczyński
Remzi Sanver ● Ulrike Schmidt-Kraepelin ● Stanisław Szufa ● Magdalena Tydrichova