Measuring a Priori Voting Power

Taking Delegations Seriously

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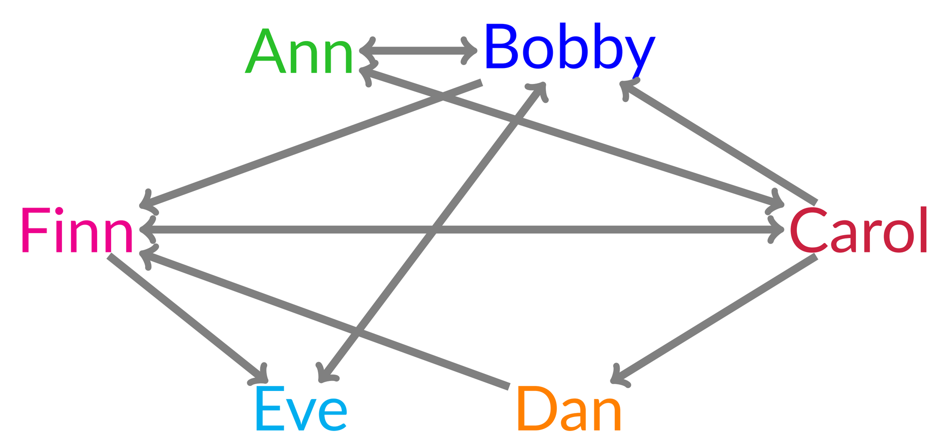
The Setting

We have a board of members, with:

1. An underlying **network** of trust:

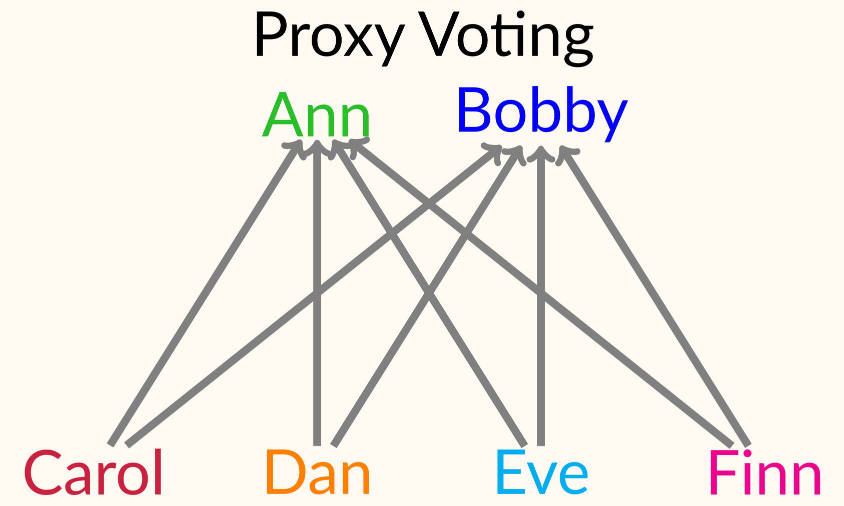


2. **A voting game:** The vote is a success iff...

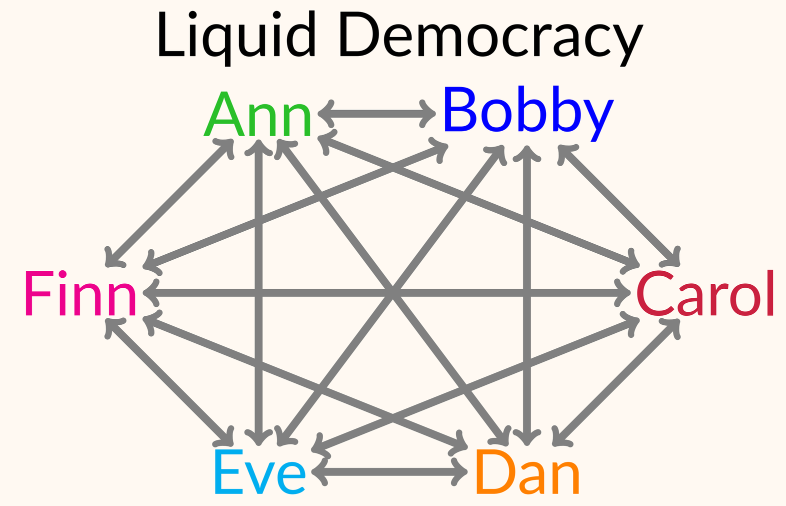
- ...there are more votes in favor than against.
- ...there are at least 4 voters in favor.
- ...Dan and Bobby both vote in favor.
- ...the total weight of voters in favor is greater than a quota q times the total voting weight.
  $\Rightarrow$ Weighted Voting Game (WVG).

Special Cases

Proxy Voting

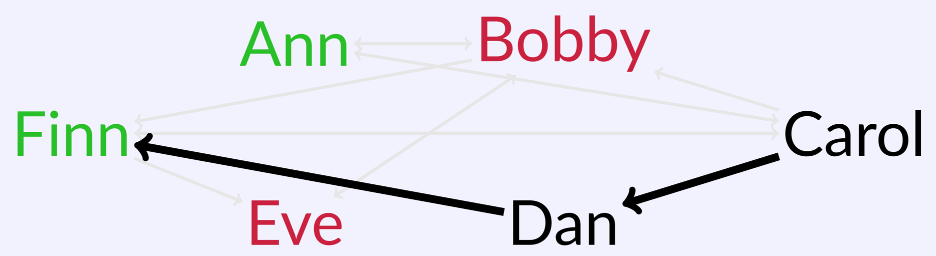


Liquid Democracy

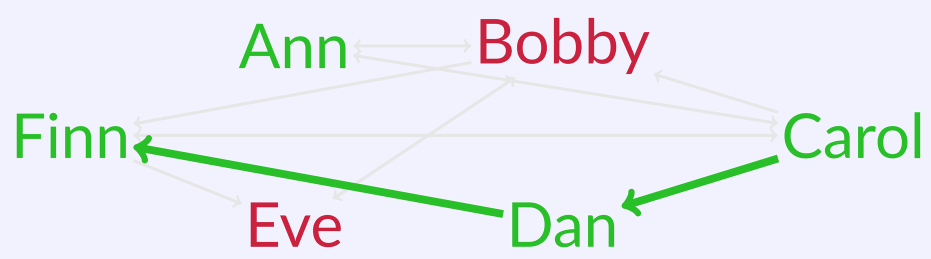


Example

1. Everyone vote or delegate their vote to a neighbour.



2. We resolve delegations and apply the voting rule.



Question: Given a voting game and a social network, and *without assuming anything* about the bill and the voters, what is the *a priori voting power* of each voter in the network?

The *LD* Penrose-Banzhaf Measure of Voting Power

- **Network structure** $G = (V, E)$.
- G**-delegation partition** D: map voters to votes (in favor, against, **delegate** to a neighbour).
- **Direct vote partition** T_D: map voters to direct votes (by resolving delegations in D).
- **Voting game** W: map *direct vote partitions* T_D to outcome (accept/reject).

LD Penrose-Banzhaf measure of voter i

$$\mathcal{M}_i^{ld}(W) = \sum_{D \in \mathcal{D}} \mathbb{P}(D) \frac{W(T_{D_i^+}) - W(T_{D_i^-})}{2}$$

The *LD* Penrose-Banzhaf measure is defined as the probability that the voter is **critical**, i.e., that they can *affect the outcome of the vote by changing their vote.*

Our probability model

Similarly to the intuitions behind the classical Penrose-Banzhaf measure, we invoke **the principle of insufficient reason**: We ignore everything of voters' opinions or dependencies.

- Each voter *delegates* with probability p_d^i and *votes* with probability p_v^i (in our experiments, voters share the same probability to delegate: $p_d^i = p_d$)
- If *vote*: The probabilities to vote in favor and against are the same $p_y = p_n = 1/2$
- If *delegate*: The probability to delegate to some neighbour $j \in \texttt{NB}_{out}(i)$ is the same for all neighbours: $1/|\texttt{NB}_{out}(i)|$.

If $p_d^i = 0$ for all voters, this is a classical voting game.

Complexity Results

General case

Computing the *LD* Penrose-Banzhaf measure is #P-Hard in general, even for weighted voting games.

Proxy Voting

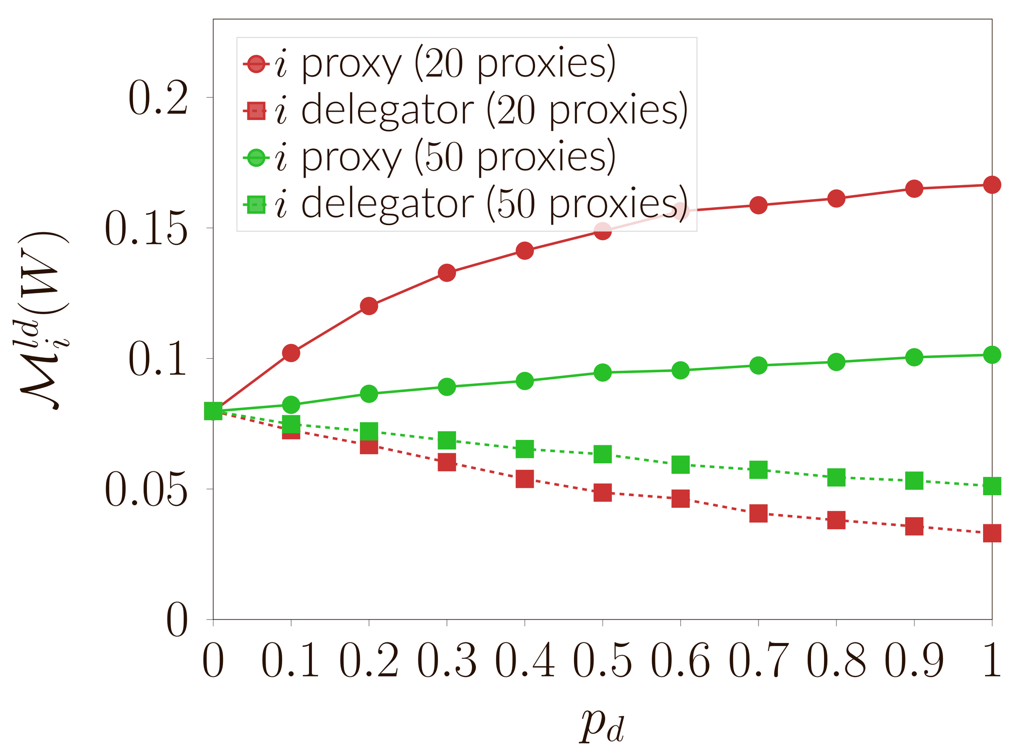
For weighted voting games, if the underlying graph is **bipartite**, it can be computed by a pseudo-polynomial algorithm that runs in polynomial time w.r.t $|V|$ and $\max_{i \in V} w(i)$.

Liquid Democracy

For weighted voting games, if the underlying graph is **complete**, it can be computed by a pseudo-polynomial algorithm that runs in polynomial time w.r.t $|V|$ and $\max_{i \in V} w(i)$.

Experimental Results

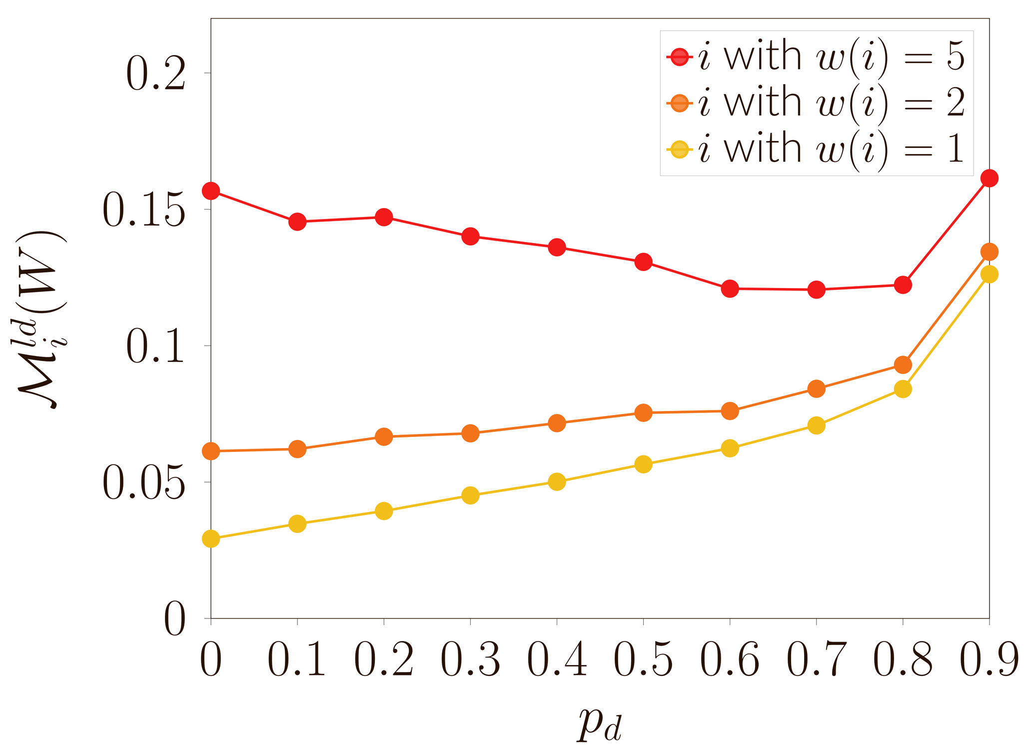
Proxy Voting



WVG with $|V| = 100$ voters of weight $w(i) = 1$, and a quota $q = 0.5$, with 20 proxies and 50 proxies.

$\Rightarrow$ The lower the number of proxies, the **more unequal the voting power of the voters.**

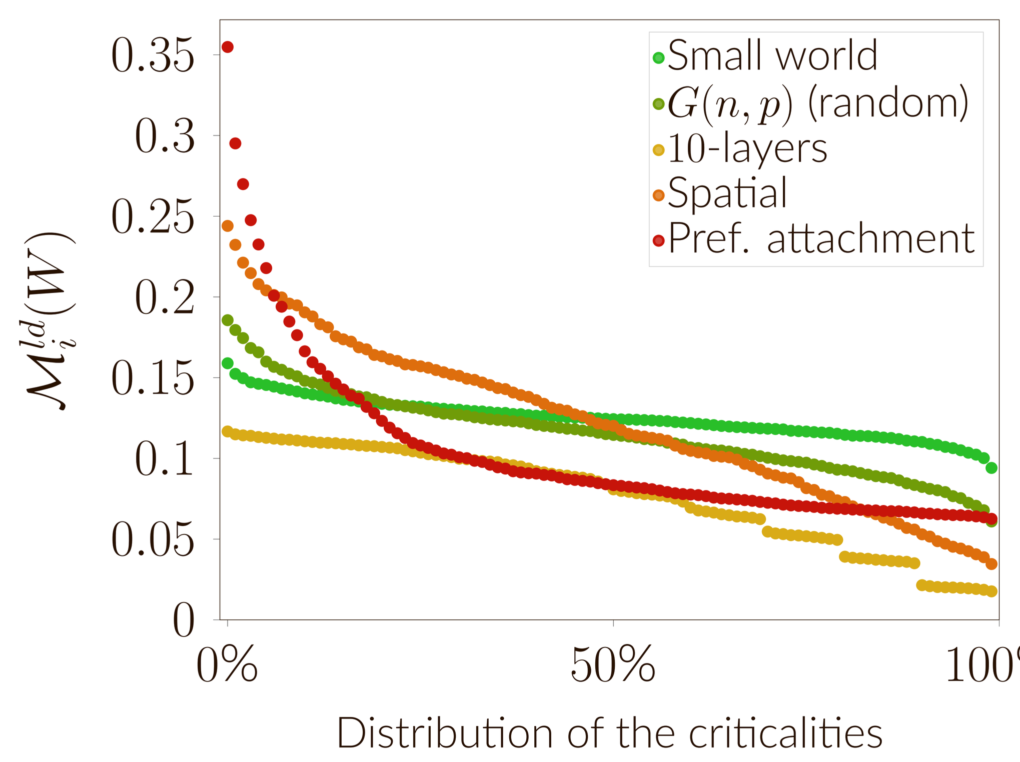
Liquid Democracy



WVG with $|V| = 100$ voters of weight $w(i) = 1$ (50%), $w(i) = 2$ (30%) or $w(i) = 5$ (20%), and a quota $q = 0.5$.

$\Rightarrow$ When the probability to delegate p_d gets higher, the **voting weight** has less influence on the voting power.

Network Structure



WVG with $|V| = 100$ voters of weight $w(i) = 1$ and a quota $q = 0.5$, with various network structures.

$\Rightarrow$ Different structures give different inequalities, **and the criticality of the voter is correlated to its in-degree.**