Approval with Runoff

Théo Delemazure [1] Jérôme Lang [1] Jean-François Laslier [2] Remzi M. Sanver [1]

[1]CNRS, LAMSADE, Université Paris Dauphine–PSL [2]CNRS, Paris School of Economics–PSL

Plurality with Runoff

First round: Voters vote for their favorite candidate. The two candidates with the highest scores advance to the second round

Second round: Majority vote

Used in more than 80 countries, but fails most good theoretical properties because of the use of plurality

Can we keep **the benefits of the two-round protocol** without having to bear all the **drawbacks of plurality** in the first round?

	Ann	Bob	Carol	Dan
scores	30%	27%	26%	17%

⇓

	Ann	Bob
scores	54%	46%

⇓

Ann

Monotonicity ⇒ Failed

If $a \in \mathcal{C}$ is the winner of an election, and one voter changes his vote in favor of a then a should remain the winner.

Resistance to cloning ⇒ Failed

Introducing a clone of an existing candidate should not change significantly the result of the election.

Condorcet-loser criterion ⇒ Satisfied

A candidate who can be defeated in a head-to-head competition against each other candidate should not win.

Approval with Runoff (AVR)

First round: Voters approve as many candidates as they like. We use an approval-based committee rule to select the two finalists

Approval ballot
$10\times$ Bob,
$20\times$ Ann, Bob, Carol
$30\times$ Ann, Bob
$20\times$ Carol, Dan
$5\times$ Dan

score	MAV ($\alpha = 0$)	PAV ($\alpha = \frac{1}{2}$)	CCAV ($\alpha = 1$)
Bob, Ann	110	85	60
Bob, Carol	100	90	80
Bob, Dan	85	85	85
...	...	...	...

Second round: Majority vote

αAV-rules

A family of rules that select pairs of candidates maximizing:

$$\alpha\mathrm{AV}(V) = \mathrm{argmax}_{x,y\in\mathcal{C}} \left(S_V(x) + S_V(y) - \alpha S_V(xy)\right)$$

$S_V(x)$ = number of voters who approve x

$S_V(xy)$ = number of voters who approve both x and y

$\alpha = 0$ **Multi-winner Approval Voting (MAV)**
Select the two candidates with highest number of approvals.

$\alpha = \frac{1}{2}$ **Proportional Approval Voting (PAV)**

$\alpha = 1$ **Chamberlin-Courant Approval Voting (CCAV)**
Select the pair of candidates that maximizes the number of voters approving at least one of them.

Favorite-consistency

Approval ballot
$10\times$ Bob,
$40\times$ Ann, Bob
$40\times$ Ann, Carol
$10\times$ Carol

	CCAV score
Bob, Carol	100
Ann, Bob	90
Ann, Carol	90

With CCAV, Bob and Carol are finalists, but Ann is the approval winner with 80% approvals

⇒ We might want the approval winner to be among the finalists

Favorite-consistency: At least one finalist is an approval winner

⇒ MAV satisfies it, but not CCAV and PAV

We define sequential αAV-rules as a family of rules such that:

1. The first finalist x maximizes the approval score $S_V(x)$
2. The second finalist y maximizes $S_V(x) + S_V(y) - \alpha S_V(xy)$

Note: MAV and sequential MAV are equivalent

Experimental results on real data

We used datasets of approval ballots from various sources:

- Datasets collected during the **2017 French presidential election** in several cities, each dataset with ~ 1000 voters and 11 candidates
- **Poster competition votes**, collected at a Summer School. ~ 60 voters per dataset and 17 candidates.

	MAV	PAV	S-PAV	CCAV	S-CCAV
2017-Strasbourg	Lib/ Left	Lib/ Left	Lib/ Left	Lib/ Left	Lib/ Left
2017-Grenoble	Soc/ Lib	Lib/ Left	Lib/Soc	Soc/ Cons	Soc/ Cons
2017-Crolles	Lib/ Left	Lib/ Left	Lib/ Left	Lib/ Nat	Lib/ Nat
Best-Poster-A	P. 1/P. 2	P. 1/P. 4	P. 1/P. 4	P. 1/P. 6	P. 1/P. 6
Best-Poster-B	P. 1/P. 2	P. 1/P. 2	P. 1/P. 2	P. 1/P. 2	P. 1/P. 2

If we use the following political scale, the ideological distance between the two finalists **increases** when we go from MAV to CCAV:

Left Soc Lib Cons Nat

Summary of axiomatic results

	MAV^R	$\mathrm{S\text{-}PAV}^R$	$\mathrm{S\text{-}CCAV}^R$	PAV^R	CCAV^R
Pareto-efficiency	✓	✓	✓*	✓	✓*
monotonic	✓				
resistant to cloning			✓		✓
favorite-consistency	✓	✓	✓		

* Depends on the tie-breaking used

Impossibilities:

1. No AVR rule is monotonic, weakly clone-proof and neutral
2. No AVR rule is clone-proof and Pareto-efficient
3. No AVR rule is weakly strategy-proof and Pareto-efficient

Conclusion

- Approval with runoff is not one rule but a **family of rules**, parameterized by the ABC rule chosen for determining the finalists
- **Axiomatic** and **experimental** results show that this choice actually makes a big difference

31st International Joint Conference on Artificial Intelligence — July, 2022 theo.delemazure@dauphine.eu